

Anthropo-didactic study of arithmetic teaching the among pupils of 9-10 years

Case of teacher-pupil interactions

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Abstract

Which didactical functions are assumed by interactions between the teacher and pupils in mathematical teaching ? These interactions are in the present case approached as adaptations of modes of the teachers' action to two types of subservience : the first case, defined within anthropological limits, allows identification of a certain number of non-didactical conditions (the teachers' pedagogical conceptions, for instance) ; the second case, strictly didactical, permits the identification of the objective conditions of teaching. This two-way approach (anthropo-didactical) shows that the different forms of interaction can be explained as a result of the adequacy (or inadequacy) of these two types of subserviencies ; therefore, a unique interactive form gives evidence of very different didactical intentions and generates varying cognitive effects according to the different didactical contexts.

Résumé

Quelles fonctions didactiques assurent les interactions maître-élèves dans l'enseignement des mathématiques ? Les interactions seront ici appréhendées comme des modes d'adaptation de l'action des professeurs à deux types d'assujettissements : le premier, défini dans un cadre anthropologique, permet de repérer un certain nombre de conditions non-didactiques (les conceptions pédagogiques des professeurs, par exemple); le second, strictement didactique, permet d'identifier les conditions objectives de l'enseignement. Ce double cadrage (anthropo-didactique) permet de montrer que les diverses formes d'interaction s'expliquent comme une sorte de résultante de l'adéquation (ou non) de ces deux types d'assujettissements ; ainsi, une même forme interactive témoigne d'intentions didactiques fort différentes et engendre des effets cognitifs variables selon les différents contextes didactiques.

This article discusses teacher-pupil interactions as a component of the didactic action of the teacher, and therefore as an object of study of the conditions in which knowledge is diffused in school institutions.

1 | PROBLEM

- 1) In classical teaching, the organisation of the milieu is really a-didactic (in the sense of G. Brousseau, 1997). Here it is very rare that retroactions are sent back to the pupils regarding decisions that they take with respect to a problem already given to them. Yet it is known that these retroactions are necessary for pupils to be able to transform the implicit model of the situation that they have.
- 2) The teacher is therefore subjected to the contractual obligation to teach (in a classical sense of the term), i.e. to set up didactic strategies which are not necessarily intentional or conscious, and which are compatible both with the non a-didactic nature of the milieu and a set of background factors ; for example, his implicit epistemology, his pedagogic convictions, and whether they are associated with a pedagogic ideology or institutional standpoint, etc.

It is therefore important to study the interactive verbal forms which occur in these teaching contexts. They are taken here as being forms of adapting to these two types of subjection (anthropologic and didactic).

The aim of the study is to characterise these forms in three strongly contrasted didactic settings : “devolving”, “intermediary”, and “institutionalising”, to demonstrate the functions that they have in the didactic system and to study their cognitive effects.

- a) The “devolving” context : in this setting teachers regularly practise group work, but do not necessarily always group their pupils together. Their classroom is a highly interactive place and these teachers maintain a sense of uncertainty, since the institutionalisation of a model of resolution is normally put off to a later date in this form of teaching.
- b) The “institutionalising” context is characterised by a limited sense of openness and little variety of situations. These teachers very quickly institutionalise a procedure or a model of resolution that they require the pupils to utilise in various problems. Briefly, they try to control as much as possible all the parameters of the learning situation.
- c) The “intermediary” context is quite close to the institutionalising style, particularly with weak pupils, even if these teachers do sometimes open up the situations. In any case, these pupils have a greater probability than those working with institutionalising teachers of being confronted with research situations, but which are always non a-didactic.

2 | OBSERVATIONAL CONDITIONS

The research concerned 7 classes in elementary school (CM1) N=142 pupils. Each teacher did two lessons (L1 and L2) on the resolution of TTT problems (Vergnaud, 1983). There was an interval of 10 days between the lessons, and they were preceded

and followed by a pre-test and a post-test comprising 22 problems involving only two numbers (lower than 10).

OBSERVATION GRID FOR INTERACTIONS

We selected only interactions with a didactic objective, i.e. those for which :

- 1) it was clearly possible to identify a link with the object of the lesson and
- 2) those for which the interactors were clearly identifiable.

Five modes of interaction were defined :

- **Spontaneous interventions**, coded SI (2 modes) : the pupil intervenes without asking to be able to speak and without the teacher requesting him to speak ; such interventions could be followed (SI+) or not (SI-) by the teacher ;
- **Requests to participate**, coded R (2 modes) : the pupil requests to speak ; these requests may be satisfied or not (R+ and R-) ;
- **Orders to participate**, coded X (1 mode) : the teacher orders the pupil to intervene without the latter requesting to speak.

3 | RESULTS

3.1 DIDACTIC FUNCTION OF VARIOUS FORMS OF INTERACTION

Result 1a : The volume of didactic interaction and the rate of non-participation of pupils across all modes was very variable from one class to another, and from one style to another.

Result 1b : The second lesson (L2) was significantly less interactive than (L1) for all classes and styles. The volume of pupil-initiated interaction (SI and R) significantly diminished from L1 to L2: (Wilcoxon : z_{is} , s.; $p. < .01$ – z_D , s. ; $p. < .001$).

This may be explained on one hand by the increased volume of speaking time of the teacher at the end of the lesson for reasons of institutionalisation, and on the other, by the progressive reduction of uncertainty for the pupils regarding the object taught (because knowledge had increased), leading to a significant drop in their interventions.

Result 1c : The volume of interaction across all modes was not stable according to the degree to which the teaching process had advanced. There was no correlation between the two lessons ($\rho = .33$; n.s. ; $p. = .34$ for any of the 7 classes). This was true for all three styles. This phenomenon may be explained by the change in behaviour of the teachers in relation to the questioning process (Overall : $\rho = .39$; n.s. ; $p. = .38$).

The teachers tended to question the good pupils more at L1 than at L2. On the other hand, weak pupils were questioned more at L2 than at L1.

There was a strong correlation between L1 and L2 regarding the frequencies of SI and R — $\rho_{SI} = .96$; s. ; $p. < .001$ et $\rho_D = .89$; s. ; $p. < .006$).

Interactive forms initiated by the pupil, and over which the teacher had no control, remained stable between L1 and L2. However, those initiated by the teacher were not stable, and the two distributions (X_{L1} and X_{L2}) were not correlated.

Comment : it is therefore possible to distinguish two types of interactive modality :

1. The first composed of interactive forms initiated by the teacher and over which he exercises control ;
2. The second corresponding to interactive forms that the teacher allows and even encourages without being able to control the pupils to initiate them. These interactive forms are didactically functional for certain teachers. Thanks to them, the lesson may advance :
3. Either by a functional reduction of the gap between two types of subjection to their pedagogic convictions, on the one hand, (i.e. the pupils must to be active, autonomous), and on the other, to a non a-didactic organisation of the milieu, which does not allow these ideals to be attained *de facto*. In other words, these milieux do not allow validation by retroaction, but only allow evaluation by the teacher. This phenomenon is very clear in devolving teachers, and is more discreet in the intermediary type.
4. Or by limitation and the reduction of these types of interactions, as with the institutionalising teachers, who experience them as if they were a sort of didactic noise which is to be limited as much as possible, whereas such interventions are didactically vital for the devolving teachers.

Conclusion : An interactive form therefore does not in itself have a didactic meaning and function which are inherit to it. An examination of the various registers of intentionality in teachers gives insight into their mode of didactic action as being the product of an adaptation to many subjections or to an “extended milieu”.

3. 2. INTERACTIVE STRATEGIES : THE PARTNERS OF INTERACTION

How may the reasons for the teachers’ “decisions” relative to the interactors be explained ?

Result 2a – There was a strong variability between the styles and the different levels of performance of the pupils regarding mean scores of requests for them to take the initiative. For example, good pupils in devolving classes requested to speak 6.35 times (on average) overall in both lessons, compared to only 3.63 times for good pupils in institutionalising classes.

Whatever their level of school performance, the pupils in institutionalising classes participated significantly more than they requested to (Wilcoxon : “Good” $p. < .14$; “Moy” $p. < .008$ et “Weak” $p. < .001$), contrary to what was observed in the devolving or intermediary classes.

Discussion : this result underlines the value of an anthropo-didactic framework for understanding this dual inter-pupil and inter-style variability. Indeed, although the didactic position of the good pupil (or the weak pupil) may be defined as being that to which is associated the strongest or the weakest probability of replying to the expectations of the teacher, such participation is always dependent on the objective conditions in which it may occur. In other words, the terms “good pupil” or “weak pupil” are to be considered as forms of differentiation which are necessary to the functioning of any didactic system. They correspond to didactic roles attributed to the individuals in order for the didactic situation to function ; their meaning is to be found in the functions that these roles play in the setting and not in the personal characteristics

(social, psychological or other) of those to whom they are attributed (Cf. For this aspect, see the work of the educational anthropologists Mc Dermott, 1977 ; Gumperz, 1981 ; Gearing, 1971).

Finally the inter-style variability that we observed demonstrates that the scripts associated with the same role may greatly vary from one context to another ; “good pupils” take on responsibilities (obligations, ways of being...) which are attached to these roles which are not equivalent from the point of view of their didactic function.

Result 2b – Requests for the pupil to take the initiative (*SS*) and the questions from the teacher (*X*) were not significantly linked ($\rho = .17$; n.s.) for any of the two lessons or for any sub-group. This result was stable for all three styles (“Devolving” : $\rho = .23$; n.s. ; “Intermediary” : $\rho = .28$; n.s. ; “Institutionalising” : $\rho = .15$; n.s.)

Therefore it is not necessarily because the pupils do not ask to speak that they are called upon by the teacher. For all 3 styles, it is not the weak pupils who are the most called upon by the teachers, although they are the ones who ask to speak the least.

Discussion

The teachers’ requests probably testify to a didactic strategy which is very probably unconscious and which aims to make the lesson progress, and not as one might believe to give a sort of didactic compassion or a form of compensation demonstrated by asking those who do not wish to be called upon. Such a scenario is very well described by Mc Dermott (1976), when he described the case of Rosa.

Rosa is a little Mexican girl in first grade. Despite her low reading level, she has no didactic interaction with her teacher. Yet Rosa seems to request her “turn to read” by regularly raising her finger. On the other hand, her teacher *wants* to teach her how to read. Mc Dermott shows very well how a reciprocal adjustment is constructed by very fine non-verbal interactions. This leads to a form of understanding that I would call didactic, between, on one hand, the teacher who does not really want to call upon the girl for very deep didactic (vs pedagogic) reasons, but does not want to create the conditions of this very upsetting pedagogic confession ; and on the other, Rosa who gives her teacher the chance to call upon her by raising her finger, whereas her teacher has just called upon another pupil.

It is not our contention that teachers are insensitive to the ethical dimension of their action. We simply wish to demonstrate how the way in which they consider their didactic mission (trying to impart as much knowledge as possible to the greatest number of pupils in a limited time) leads to a different way of formulating the ethical issue. To ignore the didactic dimension and the pressure of the institutional contract which obliges the teachers to teach and to move on in the program, would seem to suggest that the question of equity is nothing more than a question of “good will” in the Kantian sense of the term. Our discussions with the teachers showed that the ethical question is sometimes a painful matter for them, because it involves a contradictory issue : imparting new knowledge without abandoning certain pupils en route.

3.3 COGNITIVE EFFECTS OF TEACHER-PUPIL INTERACTIONS

It is now widely accepted that interactions with the teacher allow pupils to progress in their school performance. However, the present results do not confirm this :

Result 3a – No correlation was found between the volume of interactions (SI+, R and X) and the progress accomplished after the test ($\rho = -.04$; n.s. ; $p = .64$) throughout the sample. This was true for each level and for all three styles.

Result 3b – Interactions initiated by the teacher had a cognitive effect on the pupils. These effects were apparent in the good pupils of the “institutionalising” and “devolving” classes :

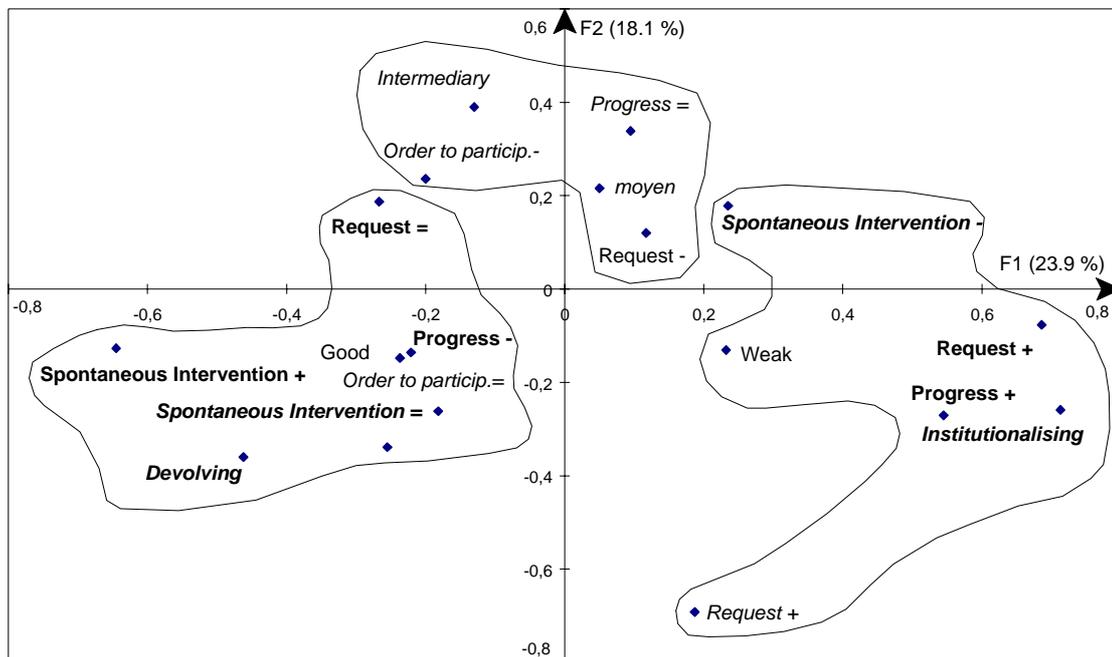
- In the “devolving” classes, the more the good pupils were called upon by the teacher, the less they progressed at the second test ($\rho = -0,54$; $p. < .06$) ;
- On the other hand, in “institutionalising” classes, the more the good pupils were called upon by the teacher, the more they progressed on the second test ($\rho = -0,55$; $p. < .08$).

Result 3c – An multifactorial analysis confirmed the differential effects of didactic styles, and the type of associations between these styles, the interactive modes, the progress accomplished on the second test, and the performance level in mathematics of the pupils.

Axis 1 demonstrates a strong opposition between the most contrasted styles, i.e. “devolving”, total contribution of 11.16%, and “institutionalising” (24.67%). This was the case for progress noted on the second test and on the preferential modes of action inherent in each teaching style : spontaneous interventions (SI) for “devolving” pupils who did not progress, and “questioning” for “institutionalising” pupils who experienced the greatest progress.

The second axis demonstrates an opposition between “intermediary” pupils (16.2%) on the positive pole and the “devolving” (8.9%) and “institutionalising” pupils (5%) on the negative pole of the axis, with respect to mode of interaction : average level pupils in the intermediary classes interacted very little and progressed moderately, while pupils in the devolving and institutionalising classes who requested to speak (R+, 20.14%) had the lowest level of progress.

Cognitive effects of the didactic interactions according to performance level of pupils and didactic style : principal findings of the multifactorial analysis



(*) Groupings of the variables were obtained by truncation of the cluster diagram with a bottom-up hierarchical classification (euclidian distance, clustering by means) on the scores of the variables of the first four axes.

(**) Variables in bold face (*italics*) are those contributing the most to explaining the inertia of axis 1 (axis 2).

Commentary

Contrary to what might have been expected regarding the cognitive effects of the verbal interactions, the bivariate and multifactorial analyses demonstrate that it is neither the pupils nor the classes which are the most interactive who have the greatest progress on the second test.

Attention must also be paid to the didactic context which has an impact on the cognitive effects of an interactive mode : e.g. the teacher’s questions (*X*) have a completely different effect on good pupils according to the didactic context, be it “devolving” or “institutionalising”.

These results would seem contradictory if the didactic function of these interactive forms were not to be taken into consideration. The task of calling upon a good pupil (according to Chevallard, 1999) does not have the same didactic intention with “devolving” pupils as it does with “institutionalising” pupils. Calling upon the best pupils is didactically functional in a “devolving” class, because it allows the teacher to pursue the lesson without demonstrating (or without admitting to do so) at the same time a deliberate desire to teach. This would be contradictory to their pedagogical will. On the other hand, in institutionalising classes, the aim of calling upon the best pupils is to intentionally modify the relationships that the pupil has with the object being taught. In other words, calling upon a good pupil in a devolving class means establishing a task centred on the teaching itself and whose function is to regulate the teaching process. In

institutionalising classes, the task is pupil-centred and its declared function (although not necessarily effective) is to regulate the learning process.

This throws light on why the cognitive effects are so different from one didactic context to another : pupils in institutionalising classes receive real feed-back with regard to implicit models of the situation (or mental representations) which may be evident in their responses, while the readjustments that devolving teachers adopted from the responses of their good pupils provide the latter with very little constructive regulations. Moreover, it may be that this quasi-absence of real feed-back could eventually give the pupils an erroneous or only partial understanding of the situation, although this is not possible to prove at present.

4. DISCUSSION

The present findings demonstrate the value of combining the two traditional research methods mentioned in the introduction, in order to better understand the phenomena associated with teaching and learning. Therefore, it seems difficult to study interactive phenomena in school situations without examining at the same time the functions that these interactive forms subsume in a school situation, and the real conditions in which these didactic and pedagogic aims are satisfied. It is not sufficient for the teacher to ask open-ended questions or to propose research situations in order to feel that his pupils are being active. Although the teacher may be active, this does not necessarily mean that his pedagogy is active. On the other hand, the didactic effects of such engineering cannot be guaranteed from an only didactic point of view. How is it possible to explain the variety of these forms and the diversity of didactic effects that they engender ? And what interpretation may be given to the fact that the teachers do not necessarily call upon the pupils who request to speak the least, while they openly say that they are attached to the principle of equal opportunities ? As mentioned above, teachers are not necessarily the absolute masters in their own classroom. However different teachers may be regarding their ideas of learning and their teaching practise, all are subjected to didactic requirements and to the institutional obligation to teach. To ignore this would inevitably lead to bringing the explanation of these teaching phenomena back to the psychological and social level (regarding the teacher or the pupil). This constitutes the already classic trap of psychologism or sociologism between which certain authors today seem to vary their approaches and explanations of these phenomena. However, this is another question.

5. BIBLIOGRAPHICAL REFERENCES

- Brousseau G. (1997). *Theory of Didactical situations in mathematics 1970-1990*, (KLUWER Academic Publishers).
- Chevallard Y. (1999). L'analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches en didactique des mathématiques*. Vol. 19/2, p. 221-266.
- Gearing FO (1971). Anthropology and education. In J.J. Honigman (ed.). *Handbook of Social and Cultural Anthropology*. Chicago : Rand Mac Nally, p. 1223-2149.
- Gumperz J. (1981). Conversational inferences and classroom learning, in J.L.Green et C. Wallat (eds), *Ethnography and language in educational settings*, New Jersey

Ablex, Norwood.

Mc Dermott R.P. (1977). Social relations as contexts for learning. *Harvard Educational Review*. 47/2, 198-213.

Vergnaud G. (1983). *L'enfant, la mathématique et la réalité*. Berne : Peter Lang, 217 p.

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Current themes of research

Mathematics teaching, didactic contract.

Most relevant publications in the field of Education :

Sarrazy, B. (1995). « Il contratto didattico », *La matematica y sua didactica*, Bologne : Pitagora editrice Bologna, 1998, n° 2, p. 132-175. (article paru dans la *Revue Française de pédagogie*, 1995, n° 112).

Sarrazy, B. (2002). Les hétérogénéités dans l'enseignement des mathématiques. *Educational Studies in Mathematics*. Kluwer Academic Publishers. (Dordrecht. Boston. London). (sous presse).

Sarrazy, B. (2002). « Effects of variability on responsiveness to the didactic contract in problem-solving among pupils of 9-10 years » *European Journal of Psychology of Education*. vol. XVII. n° 4. 321-341.

Sarrazy, B. (2002). « Contratto didattico e situazioni : analisi didattica (vs psicologica) delle risposte degli allievi nella risoluzione didattica problemi non standard ». *La matematica y sua didactica*, Bologne : Pitagora editrice Bologna, 2002, n° 3. 244-257.

Sarrazy, B. (2006). « Théorie de l'équilibration et théorie des situations : Quelques remarques sur les rapports entre constructivisme et didactique des mathématiques ». In S. Sbaragli (Prefazione B. D'Amore). *La matematica e la sua didactica*. Roma : Carocci Faber. 253-256.