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Efficacy of teaching mathematics with method of didactical games in a–didactic situation

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Abstract

This article is describing continuance and results of research on efficacy of mathematics' teaching by the method of didactical games in a-didactic situation. Usage of didactical games is part of many modern theories of education (Progressivism (J. Dewey), Waldorf Education (R. Steiner), GFEN (Groupe Francais d'éducation nouvelle), etc.). Also many psychologists stated importance of games for children's development (Groos, Volpicelli, Piaget, Bärbel, etc.). So the question about efficacy of teaching with usage of didactical games is actual and essential.

Author has been trying to check potential contributions of didactical games for the teaching of mathematics. The research was realized by the experimental approach. Efficacy of teaching mathematics with method of didactical games was inquired in real teaching conditions of common school classes. Results of experiment were evaluated by statistical analyse, in the concrete by parametrical t-test of two sets of data. By this statistical evaluation we tried to verify hypothesis of experiment. This hypothesis assumed that teaching of mathematics with using of didactical games will be more effective as that without them, because active, playful and motivational elements included in games will develop abilities and motivation of pupils.

In generally we can say that the hypothesis appeared to be true. In this article the reader will find more about progress of research and about influences of games on teaching process.

Abstrakt

Tento článok opisuje priebeh a výsledky výskumu efektívnosti vyucovania matematiky metódou didaktických hier v rámci a-didaktickej situácie. Používanie didaktických hier je súcastou viacerých moderných teórií edukácie (Progresivizmu (J. Dewey), Walfdorskej školy (R. Steiner), GFEN (Groupe Francais d'éducation nouvelle), atd.) Tiež viacerí psychológovia konštatovali dôležitost hier pre rozvoj dietata (Groos, Volpicelli, Piaget, Bärbel, atd.). Preto otázka efektívnosti vyucovania s metódou didaktických hier je aktuálna a dôležitá.

Autor sa experimentálnym postupom snažil overit potenciálne prínosy didaktických hier pre vyucovanie matematiky. Efektívnost vyucovania matematiky metódou didaktických hier bola skúmaná v reálnych ucebných podmienkach bežných školských tried. Výsledky výskumu boli štatisticky vyhodnocované, konkrétne parametrickým t-testom pre dve skupiny dát. Pomocou tohto štatistického vyhodnocovania sme sa snažili verifikovat hypotézu výskumu. Táto hypotéza predpokladala, že vyucovanie, ktorého súcastou sú didaktické hry je efektívnejšie ako vyucovanie bez používania týchto hier, pretože samostatná, aktívna, hravá cinnost rozvíja rozumové schopnosti žiakov a zvyšuje ich motiváciu.

Celkovo môžeme povedat, že hypotéza výskumu sa javí pravdivou. V tomto clánku nájde citatel popis priebehu výskumu ako aj postrehy o vplyve didaktických hier na vyucovací proces.

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1. Introduction

In studies which analysed the efficacy of teaching was emphasised the need to use teaching methods which will lead students to an active work during lessons. These methods should motivate students and develop their social abilities and personal qualities. Didactical game is one of possible methods suitable to fulfil these needs.

Task in our experiment was to consider whether using of didactical games[2] in teaching process had improved efficacy of teaching mathematics. Answer to this question is important for more often use of didactical games during mathematical lessons.

The need of this experiment was given by requirements of teaching praxis. Many psychologists and pedagogues observed positive influences of games. If didactical games show ability to effectively realise educational goals, possibilities of theirs often usage during the lessons will be given.

Our personal experience with usage of didactical games (**Vankúš, 2002)** and experience of people that have used didactical games during mathematics' lessons resulted in thus hypothesis of our experiment:

> H: The teaching of mathematics with using of didactical games will be more effective as that without them, because active, playful and motivational elements included in games will develop abilities and motivation of pupils.

Results of experiment that tried to verify this hypothesis are included in our article.

2. Theoretical issues of experiment

Now we will clarify to the reader our meaning of notions *didactical game* and *efficacy of teaching*. This chapter also deals with our conception of measuring efficacy of teaching methods.

The notion *didactical game* denotes in pedagogical literature pupils' activity, that brings fun and pleasure for pupils and also realise stated educational goals. The main differences between normal meaning of word *game* (**F. Spagnolo, J. Cižmár, 2003, p. 58–59**) and between *didactical games* are:

- Normal game is totally free, in didactical game all pupils have to participate.
- Didactical game is used to realise educational goals, the main aim of normal game is just fun and pleasure.
- Didactical game has its external management (teacher).

Very comprehensive description of didactical game gives this quotation (**Prucha, Walterová, Mareš, 1998, p. 48**)**:**

[2] Didactical game can establish situation, when pupils draw knowledge from their own experience or practise using mathematical knowledge from their own interest outside any teaching context and in the absence of any intentional direction. Such situation is called *a-didactic situation* (Brousseau, 1997, p.30). (Mentioned experience and interest come from interaction with milieu of didactical game.)

Didactical game: Analogy of spontaneous children's activity, which realise (for children not ever evidently) educational goals. Can take place in classroom, sport-hall, playground, or in nature. Has its rules, needs continuous management and final evaluation. Is suitable for single child either for group of children. Teacher has various roles: from main organiser to an onlooker. Its advantage is simulative charge: it raises interest, makes higher children's involvement in teaching activities, encourages children's creativity, spontaneity, co-operation and also competitiveness. Children can use their knowledge, abilities and experience. Some didactical games approach to model situations from real life.

This definition of didactical games describes their structure. Each didactical game has:

? Milieu of the game
? Goals of game
? Activities of teacher and pupils, which are determine with rules of game
? Final evaluation

More about structure of arbitrary didactical situation see in **Trencanský, 2001**. Didactical game can be considered as special kind of *a–didactic* situation.

Interactions between pupils and **milieu** of didactical game should motivate pupils to the work. This work leads to the realisation of the **goals of game**. Goals of game are dedicated to educational goals, which have to be realised by the game. The goals of game determine form of game. Usage of didactical games has value only if it enables to reach educational goals.

Activities of teacher and pupils, which are determined with rules of game, should be for pupils attractive and motivational. These activities have to be suitable for age of pupils and their abilities. **Rules of game** determine the form and organisation of pupils' work. These rules include gamesome elements (e.g. competitiveness between teams of pupils).

Final evaluation verifies realisation of game's goals and have to reward pupils and motivate them for the next activities.

For illustration we will now describe concrete example of didactical game:

We are between children in the fifth year of primary school. Pupils are 11-12 years old. The theme is *Area of plane object (Square and rectangle).* The lesson is about finding area of plane objects in square grid (e.g. fig. 1).

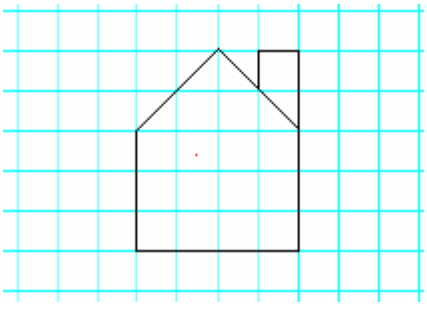

fig. 1 problem is to find area of object
(one square of grid has 1 cm^2)

In order to practise taught skill we will use didactical game. The goals of game will be besides the practise also feedback from pupils how they manage lesson's activities. We will choose suitable didactical game on the basis of the game's goals, age and abilities of pupils.

Name of game: Mathematical fishing (**Foltinová and Novotná, 1997**)

Number of players: Whole class divided to the teams of 5 children.

Instruments: For each team set of cards with problems for pupils. Problems are about finding area of plane objects in square grid. These problems have various difficulties. Cards have shape like fishes (see fig. 2). Cards have also various colours, depending from difficulty of problem on concrete card (e.g. a green card – the least difficult problem, a blue card – mild difficulty, a red card – the most difficult problem)

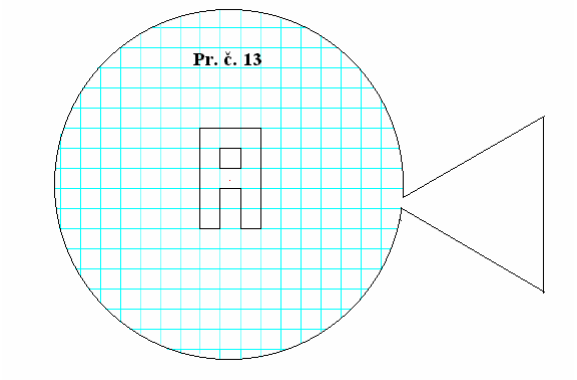

fig. 2 *playing card from the game Mathematical fishing*

Rules: Each team of pupils obtains set of cards with problems. Pupils are solving problems. Pupil can choose difficulty of problem – it depends on the colour of chosen card. Team obtain points for every good solved problem. The number of points depends also on difficulty of problem (for the most difficult problems the biggest number of points). At the end teacher says which solutions are correct and adds up points for each team. In order to make evaluation faster each problem has its number (is displayed on card). Teacher has correct results for each problem's number so can easily check correct answers. Aim of teams is to gather the biggest possible number of points.

Evaluation: After the end of game teacher says final score of each team. Pupils are rewarded. (The best team obtains the best reward, but also the last team should be rewarded.)

Advantages of this game:

– differential difficulties of problems, pupils can choose problems that are for them suitable
– pupils' motivation with competitiveness between teams
– thanks to various problems' difficulties also not so gifted pupils how possibility to contribute to the success of their team.

Didactical games can have form of competition of teams or single person; can have form of interesting strategic game **(Vankúš, 2002**). In some didactical games pupils work with puzzles (e.g. Tangram – see in **Uhercíková, 2002**). There are didactical games that imitate situation of real life (e.g. *Play to be businessmen* see in **Korenová, 1997**)

The second key word of our article is *efficacy*. For us is important to know how is defined efficacy of teaching method. Meaning of this word in didactic and pedagogic is taken from economic. There is efficacy of some work defined as effect from used resources. This effect is measured in terms of work's results. So work is more effective when the value of used resources is the lower and the value of work's results is the higher.

In teaching is result of work a fulfilment of educational goals. So efficacy of teaching method is given by the extant of this fulfilment. Teaching method will be more effective when the value of used resources is the lower and the extant of educational goals' fulfilment is the higher. There are many resources and conditions in teaching process (Resources: teachers, textbooks, school buildings, teaching aids and tools... Conditions: number of pupils in class, duration of teaching process, number of pupils abilities and skills which are being developed, influences on children's feelings and attitudes...). Not every of these resources and conditions can be easily measured or observed. For instance is very difficult to say how big is value of some educational goal. Therefor efficacy is in didactic and pedagogic seldom measured in terms of money.

The efficacy of teaching method is usually stated just as the extant of educational goals' fulfilment. From two teaching methods is more effective that one, which reaches educational goals more proper and has better attributes in some criteria. After the study of another experiments, which measured efficacy of teaching methods, we have chosen some criteria. If one of two teaching methods is better in these criteria we will take it as more effective. So our definition of efficacy of teaching method is:

Efficacy of teaching method is rate of:
– how properly are educational goals realised
– how good are influences on student's feelings and attitudes
– how many people can be taught, what type and level of people is method suitable for
– how long is duration of teaching process needed to realise educational goals.

Now we can answer question how to find out efficacy of teaching method. When we consider our definition we have to measure forementioned criteria. First one can be measured by didactical tests (they can measure knowledge, skills, psychological and psychical abilities). The second criterion is possible inquire by interviews or questionnaires. The third and the fourth criterion can be observed directly.

3 Methodology and organisation of experiment

In our experiment we tried to inquire efficacy of mathematics' teaching with method of didactical games. So our variable was usage of didactical games. In one class we used didactical games, in one didn't. Then we were able to compare teaching process in these classes and state efficacy of teaching with and without didactical games. Both classes had to be the same age of children, and equivalent number of pupils. The same teacher and theme have been taught in both classes during experiment. Only significant difference between these classes was our variable (didactical games yes or not).

Experiment was held in two classes of fifth year of primary school (Together 51 pupils). Pupils were 11-12 years old. The theme was *Area of plane object (Square and rectangle).* Whole experiment took 17 lessons in each class (one lesson = 45 minutes). During this time we used four didactical games: *Circles, Dominoes, Decryption* and *Lottery* (see **Vankúš, 2004**)

The main hypothesis of our experiment is very generally formulated. In order to verify it was needed to divide it into partial working hypotheses. They are given by our main hypothesis and our conception of efficacy of teaching method.

H1A: Pupils that will be taught with usage of didactical games will obtain better knowledge as pupils taught without didactical games.

H1B: The usage of didactical games will result in better pupils' abilities of solving mathematical problems.

H2A: The teaching of mathematics with using of didactical games will improve pupils' attitude towards mathematics.

H2B: Lessons with using of didactical games will improve pupils' attitude towards process of mathematics' teaching.

H3: Due to the fact that didactical games are teaching method fancied by children and suitable for using during lessons there will be no problem to use them in classes of normal number and level of pupils. Realisation of educational goals will take the same time or less as in classes where games are not used.

The first A and the first B hypotheses come out from fact that if pupils are motivated they would work more and thus would obtain more knowledge and would better develop their skills and abilities. The first hypotheses were verified by didactical test (see app. 1).

The second A and the second B working hypotheses are linked with expectations that didactical games are for pupils an attractive way of teaching. Positive experience of this method could improve general pupils' attitude towards mathematics. These second hypotheses were tested by questionnaire (see app. 2).

The third hypothesis regards two last criteria of our definition of efficacy of teaching method. Because experiment was held in two classes of the same number and comparable level of pupils we inquired only time needed to realise educational goals. It was tested by observation.

In statistical verification of our hypotheses we tried to evaluate validity of these inequalities:

H1: $T_E > T_C$, where T_E is the average value of test's score in an experimental class (where are didactical games used) and T_C is the average value of score from the same test in a control class (teaching without didactical games).

H2: $Q_{E1} > Q_{E0}$, where Q_{E1} is the average value of questionnaire's score at the end of experiment in the class where didactical games were used. Q_{E0} is the average score's value of the same questionnaire in the same class but at the beginning of experiment.

H3: $D_C = D_E$, where D_C is time needed for realisation of an educational goal in control class and D_E is duration of this realisation in experimental class.

Against our hypothesis there were so named zero–hypotheses (see **Kerlinger, 1972, p. 178; Spagnolo, Cižmár, 2003, p. 149**). Zero–hypothesis is a statement that there are no expected influences of experiment's variables. So there will be no measurable difference between experimental and control group. In our experiment zero–hypotheses were expectations that usage of didactical games during the lesson would have no influences on efficacy of mathematics' teaching. There were:

$H1_0$: $T_E = T_C$

$H2_0$: $Q_{E1} = Q_{E0}$

$H3_0$: $D_C < D_E$

The aim of statistical verification is to prove that experimental results show statistically important divergence from zero–hypotheses towards normal hypotheses of experiment. In order to prove it we have used **t – test.** It is a parametrical statistical test. Input parameters are average values and statistical deviance of the set of data. These data we obtained in our experiment. T – test is suitable for two sets of data. (It is experimental and control class in our case.) The results of the t – test are the same as ones of Fisher's analysis of variance if we had two groups of data.

4 Results of experiment

In this chapter we will summarise results of experiment. We will answer question if our hypotheses are verified. We will discuss realisation of experiment and state steps needed for its improvement.

Following numbers describes results linked with each hypothesis.

H1:

Average value of test's score in experimental class	$T_E = 52.69\%$
Statistical deviance	$S_E = 22.18\%$
Number of pupils	$N_E = 23$
Average value of test's score in control class	$T_C = 56.36\%$
Statistical deviance	$S_C = 23.16\%$
Number of pupils	$N_C = 25$

H2:

Average value of questionnaire's score at the beginning of experiment in experimental class	$Q_{E0} = 1$
Statistical deviance	$S_{E0} = 4.34$
Number of pupils	$N_{E0} = 24$
Average value of questionnaire's score at the end of experiment in experimental class	$Q_{E1} = 3$
Statistical deviance	$S_{E1} = 4.98$
Number of pupils	$N_{E1} = 21$

H3:

Duration of experiment was the same in both classes: $D_C = D_E$

Results linked with H1 showed that H1 is not true. There were no significant differences in tests' scores in experimental and control class. The value of t-test was t = 0,56, it showed that differences were made by accident. These results could be explained by the fact that at the beginning of experiment experimental class was better in mathematics as control class (considered marks of pupils). If classes had been the same the results in experimental class would have been better. So at the moment we could say that pupils taught with didactical games had equal results at tests as pupils taught without games.

Results for verification of hypothesis H2 confirmed that $Q_{E1} > Q_{E0}$. So there were improvement of student's feelings and attitudes towards mathematics and towards its teaching. Statistical verification by the t-test gave value t = 1.43. With probability 85% it means that hypothesis H2 is correct.

Experiment was realized in both classes coincidently and teaching of stuff took the same time. So we can say that usage of didactical games doesn't slow pace of teaching. Hypothesis H3 is true.

At all methods used in experiment seem to be proper and correct. It is necessary to make experiment on bigger number of pupils; by this way we can reach statistically more proper results.

5 Conclusion

Experiment evidently showed that usage of didactical games improves student's feelings and attitudes towards mathematics and so enhance their motivation for work during mathematics' lessons. Motivation is very important in teaching process. If students are motivated they work better and with higher interest. Good motivation is needed in order to make teaching more effective.

Didactical game is method suitable to be used during mathematics' lessons. Our results said that pupils taught with didactical games obtained the same knowledge (from statistical point of view) during the same time as pupils taught without games.

If we summarized results of our experiment then usage of didactical games during mathematics' lessons appears to be contribution for teaching. We can say that teaching with usage of didactical games is more effective as teaching without them. (We consider our

definition of efficacy.) This is challenge for mathematics' teacher to use didactical games more often. Teachers can use also games described in our work (**Vankúš, 2004**).

Generality of our results is limited by the number of pupils those took part in experiment. Problem also remains for what kind of knowledge and thinking processes are didactical games suitable method of teaching. Also important is to find some features of proper didactical game in order to make it really effective education's method. The best will be to make collection of didactical games suitable for teaching of concrete parts of mathematical thinking and knowledge. All these problems can be solved by further experimental using of didactical games in mathematics' education. It is needed to make experiments with more students and on greater number of mathematics' lessons. Such bigger research will give more proper answer to questions about efficacy of teaching mathematics with method of didactical games.

Bibliography

[1] Bereková, H., Földesiová, L., L. Hríbiková, I., Regecová, M., Trencanský, I.: Slovník teórie didaktických situácií. 1. cast. In: Zborník 4 Bratislavského seminára z teórie vyucovania matematiky. Bratislava, Univerzita Komenského 2001. p. 95–103.

[2] Bereková, H., Földesiová, L., Regecová, M., Kremžárová, L., Slávicková, M., Trencanský, I., Vankúš, P., Zámožíková, Z.: Slovník teórie didaktických situácií. 2. cast. In: Zborník 5 Bratislavského seminára z teórie vyucovania matematiky. Bratislava, Univerzita Komenského 2003. p. 113–122.

[3] Brousseau, G.: Theory of didactical situations in mathematics. Edited and translated by Balacheff, N. et al. Kluwer academic publishers 1997.

[4] Foltinová, K. – Novotná, J.: Matematické hry a souteže na druhém stupni základní školy. Praha, Pedagogické centrum 1997.

[5] Földesiová, L.: Sequence analytical and vector geometry at teaching of solid geometry at secondary school. In: Zborník 5 Bratislavského seminára z teórie vyucovania matematiky. Bratislava, Univerzita Komenského 2003. p. 27–37.

[6] Kerlinger, F. N.: Základy výzkumu chování. Praha, Academia 1972.

[7] Korenová, L.: Motivácia k uceniu a tvorivosti na hodinách matematiky. Písomná záverecná práca k 1. kvalifikacnej skúške. Bratislava, Metodické centrum 1997.

[8] Kremžárová, L.: La Géométrie comme outil de modélisation. In: Zborník 5 Bratislavského seminára z teórie vyucovania matematiky. Bratislava, Univerzita Komenského 2003. p. 49–60.

[9] Regecová, M.: Použitie vektorového poctu pri riešení planimetrických a stereometrických úloh na strednej škole. In: Zborník 5 Bratislavského seminára z teórie vyucovania matematiky. Bratislava, Univerzita Komenského 2003. p. 61–79 .

[10] Spagnolo, F. – Cižmár, J.: Komunikácia v matematike na strednej škole. Brno, Masarykova univerzita 2003.

[11] Trencanský, I.: Možností teórie didaktických situácií na zefektívnenie ucenia sa. In: Zborník 4 Bratislavského seminára z teórie vyucovania matematiky. Bratislava, Univerzita Komenského 2001. p. 81–90.

[12] Turek, I.: Zvyšovanie efektívnosti vyucovania. Bratislava, Metodické centrum v Bratislave 2002.

[13] Uhercíková, V. – Haverlík, K. I.: Tangram – poutavá hra. Praha, RAABE 2002.

[14] Vankúš, P.: Hry ako súcast vyucovania matematiky. Diplomová práca. Bratislava, UK, Fakulta matematiky, fyziky a informatiky 2002.

[15] Vankúš, P.: Možnost zefektívnenia vyucovania predmetu matematika metódou didaktických hier. Rigorózna práca. Bratislava, UK, Fakulta matematiky, fyziky a informatiky 2004.

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Appendix n. 1:

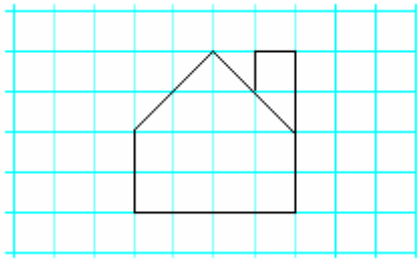
Didactical test that measures pupils' knowledge and skills from stuff that was taught during experiment.

Didactical test

1) At what activities in normal life do we measure area? Write at least one of them:

.....

2) State area of plane object in the square grid (one square of grid has area 1 cm^2).



Area of plane object is: .....

3) What units of area do you know? Write at least six different units:

.....

4) Change:

a) 720 dm^2 = ... m^2

b) 12 cm^2 = ... mm^2

c) 1,5 m^2 = ... cm^2

d) 1200 mm^2 = ... dm^2

e) 1,4 ha = ... a

f) 10 a = ... m^2

5) Write formula for area of rectangle and describe what means each letter in formula. Depict in the picture:

6) Calculate area of rectangle if its measures are 2,5 dm and 10 cm. ……

7) Area of rectangle is 32 cm^2 and length of one rectangle's side is 8 cm. State the length of adjacent side of the rectangle. ……

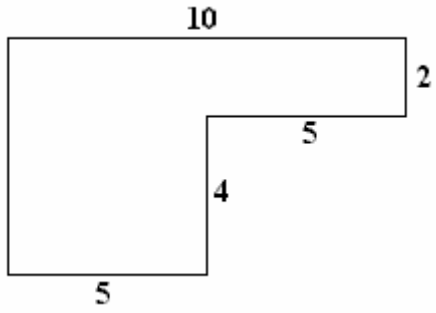
8) Write formula for area of square and describe what means each letter in formula. Depict in the picture:

9) Calculate area of square if it has one side 5 dm long. ……

10) Area of square is 36 cm^2. State the length of its side. ……

11) Area of square is 49 mm^2. State its perimeter. ……

12) We want to pave the room depicted in the picture. State what amount of floor titles in m^2 we will need. Measures of the room in the picture are in metres.



Answer: .....

13) A fence around a square playground is 40 metres long. State the area of the playground. (Write process of your solving.)

14) We want to paint with brush walls of a rectangular room with measures 4 m and 2,5 m. How many tins of painting colour do we have to buy? We know that one tin is enough for painting 5 m^2 of the wall. (Write process of your solving.)

Appendix n. 2:

Questionnaire that inquires influences on student's feelings and attitudes towards mathematics and mathematics' teaching

Questionnaire

Dear pupils, in these questionns you have possibility to express your attitudies towards mathematics and its teaching. Questionnaire is anonymous, you don't subscribe yourselves. Your answers will be used as a part of scientical research. Please, express truly and openly your opinnions.
Thank you very much for filling the questionnaire.

In following questions encircle answers that you agree with.

1) Imagine that you are teacher. Which of following subjects you would like to teach the most?

   a) Languages
   b) Geography
   c) Mathematics
   d) Physics
   e) Natural science
   f) Other (write which): ……

2) Subject mathematics is for you

   a) Very interesting
   b) Interesting
   c) Sometimes interesting, sometimes not interesting
   d) Uninteresting
   e) Very uninteresting

3) Encircle every word from following that discribes your attitudes towards mathematics

interesting	boring	worthless
monotounous	usefull	entertaining
easy	important	useless
worhfull	difficult	unimportant

4) Can you remember some activities linked with mathematics that you like?

a) Yes (Write them) ……
……
……

b) No

5) Which mark from mathematics did you have on your last school report?

a) 1 b) 2 c) 3 d) 4 e) 5

6) You look forward to having mathematics lesson:

a) always b)often c) sometimes d) rarely e) never