WHAT STUDENTS WANT: AN ENVIRONMENT WHERE LEARNING TO BE IN MATHEMATICS

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Abstract: In this paper we analyse an ongoing teaching experiment with students of secondary school, working within the new technological environment provided by the TI-Navigator. Our purpose is to investigate students’ processes from a semiotic point of view, particularly referred to signs and meanings supported by this environment. Here we present examples of some activities with excerpts of video-taped discussions. Our results suggest that this new technology offers to didactical practice something more than an usual environment.

INTRODUCTION AND THEORETICAL FRAME

Our research project is framed on a semiotic-cultural approach, referring to the theory of objectification (Radford, 2006). This theory gives rise to an anthropological conception of thinking on the one hand, and a socio-cultural conception of learning on the other. According to Radford’s perspective, thinking is characterized by way of its existence as a reflexive praxis (praxis cogitans), which is a practical reflection on the external world, practical because it takes the forms and the modes of the activities of the individuals. This concept of reflection differs both from empiricism and rationalism. Thinking as re-flection is a dialectical process between a historically and culturally constituted reality and an individual who receives it, as well as revises it, according to his/her own subjective feelings and interpretations. Thinking is not only a reflexive practice, but also a mediated one, if we refer to the role played by artefacts and signs used in order to carry the cognitive praxis itself. Artefacts and signs are not simple aids or catalysts to achieve knowledge, but rather constitutive and consubstantial parts of thinking. The individual thinks with and through semiotic means, which are bearers of an “embodied intelligence” and hide cultural-historical experiences in themselves, conceptions and meanings of the past generations’ cognitive activity. The importance of the role and the mediation of the artefacts is an important theme of many recent researches, following a common direction framed in a post-Vygotskijan perspective.

According to this theoretical perspective, learning is a matter of endowing the mathematical objects that the student finds in his/her culture with meaning, it’s a matter of making visible something that was invisible. Then, learning mathematics is not so much a matter of learning to do mathematics; rather it is a matter of learning to be in mathematics. This re-conceptualization of learning implies a re-organization of the class itself and of the roles that students and teachers play within it. What is important is to learn how to live in the classroom as a community, to interact with others, namely: to be-with-others. Learning is considered as a social praxis, based on a common and active reflection about the environment students are living in. What we study as researchers is when and how the students are making visible something that was not before, and in doing this we observe and analyse not only speech but also gestures and whatever sign is introduced by the students in their activity: symbols, drawings, graphics, actions, bodily movements and glances. Recent research (Arzarello, 2006; Robutti, 2006) has stressed the
prominent role of bodily and artefact-mediated actions, linguistic and symbolic activity in the process of generalization and meaning production.

Another theoretical element of our frame is taken by a recent study (Borba & Villarreal, 2006), regarding the role played by technology in mathematics education. Within the mathematics education community, there is a common awareness about the fact that computers and calculators by themselves cannot bring any improvement of the quality of learning: an intense pedagogical discussion is necessary to understand how to use technological tools in the classroom. Borba’s book is an attempt to give a valid answer to why technology should be used in education. This answer is the same that allows us to overcome the deep-rooted dichotomy between humans and technology: humans are users of technology, they build or develop technology, but humans and technology are always seen as disjoint sets. This recent perspective suggests that knowledge is always collectively produced by humans-with-media, and mathematics is part of this process of course. Learning is a social undertaking, a process of interaction among humans, including tools. Media, such as computers and their evolving interfaces, reorganize mathematical thinking; insofar they are not simply substitutes for humans or supplements to them, rather they are ‘actors’ in a collective thinking. Media interact with humans, in the double sense that, as technologies transform and modify humans’ reasoning, so humans are continuously transforming technologies.

This perspective does not collide with the previous of “being with others”; in the community-class ‘others’ are not only the other students and the teacher, but also the media, so that the collective composed of humans and tools becomes the basic cognitive unit.

TEACHING EXPERIMENT, TECHNOLOGY AND METHODOLOGY

The data presented in this paper comes from an ongoing teaching experiment involving two classes of the second year of secondary school (10th degree, 15-16 years old). The activities have been carried out with TI-Navigator, which provides wireless communication between students’ graphic calculators and the teacher’s computer. In TI-Navigator, the public display is the most innovative part of the connectivity system: it consists in a common Cartesian plane (called Activity Center), to which each student and also the teacher can bring their personal contribution. Another environment offered by the system is the Capture Screen, which let be possible to capture simultaneously the screens of the students’ calculators. Both the environments can be projected on a big screen, if the computer where they appear is connected to a video-projector.

Students work together in small groups (two or three members); each group uses a graphic calculator (TI-84) connected to a network hub; hubs communicate with the access point connected to the teacher’s computer. The activities are followed by collective discussions conducted by the teacher (or by the researcher). In addition to the written materials (worksheets) and data from the software, we also have the videos of the activities. Video-recorded material and transcriptions give us the data to be analysed in terms of the student’s use of semiotic resources (language, signs, gestures, actions on artefacts).

Our study is focused on the construction of the meaning of function, starting from different representations: graphical, algebraic and numeric. The activities are centred on families of functions, principally linear, quadratic, exponential and start from modelling problems. As cognitive roots for the description of a function we choose the qualitative concept of invariance and the quantitative concept of slope and its variation, as a ratio of increments. Related to these roots, we also use other concepts, as: domain, sign, intersection, zero, parallelism, and so on.
According to us, this setting, shared by all the community-class and composed by the overlapping of each local setting, could effectively be considered in the framework of humans-with-media previously described. We worked with three people in the classroom: the researcher as participant observer (Ornella Robutti), the mathematics teacher (Silvia Ghirardi in one school and Marialuisa Manassero in another) and a master degree student as participant observer and manager of the technological equipment (Maria Teresa Ravera). Collected data have been transcribed by the students and analysed by the research group.

Our research questions are really numerous, because this new equipment is substantially different from the usual, made of computers or calculators used by singles or groups. What changes is the dynamic of the class activity. In a usual laboratory setting, in fact, each group of students follows the screen of the calculator and does not have information of what is happening in the other groups. The teacher herself, if wants to have information of the processing made by the students on their screen, has to pass from one group to the other and look, supervise, discuss. But the rest of the class looses this discussion. In our setting with TI-Navigator things are happening differently from both the points of view: of the students and of the teacher. In fact, each group may follow his job but simultaneously also other groups’ job, looking at the big screen where all the jobs are projected. And the teacher herself may remain in a central position, following every job on the big screen, discussing with a single group or guiding a class discussion where everyone can take part because information is shared, both through Activity Center and Capture Screen. It is both a matter of roles in the classroom and sharing of common information, and of time and dynamics. For example, feedback does not come only from the teacher, but also from the environment. So feedback acts in a more rapid time than without a shared environment, and determines a rapid change in the activity, or correction of mistakes, or comparison of more solutions.

Questions and issues of our work are partially common to the analysis made by Trouche, who is experimenting the same equipment during the last year. The most important questions of our research are the following: What are the potentialities and the constraints of a new environment aimed at facilitating connectivity among students? In particular, which educational profit can be offered to students by a shared display? What are the effects on the learning process? What is the kind of mediation of this equipment in the learning processes? How is teacher’s role in the classroom with this equipment? What are the new difficulties and opportunities for the teacher in designing and managing mathematical activities? How do the rhythms change with respect to a more traditional activity (with or without technology)?

Our purpose is to show some activities of the teaching experiment, in order to discuss about the previous research points. Our research hypothesis is that the possibility to share a common mathematical domain (given by the two environments Activity Center and Screen Capture) at different, but complementary levels, offers something of significantly new with respect to other technology, when used by a group, and not shared with other groups. And these new features should be investigated in the frameworks of learning to be in mathematics and humans with media. These features should offer to students more opportunities of creating and managing signs than a more traditional technology, where there is no possibility of sharing the mathematical domain of the activities.
EXEMPLES OF ACTIVITIES

In Figure 1 an example of Activity Center (almost at the beginning of the teaching experiment), with the task to trace a straight line parallel to a given line of equation $y=2x-1$. The group of students who send the equation of the purple line (of equation $y=1$) to the Activity Center, looking at the big screen, immediately said (T means the teacher):

$L.$: a line parallel to the $x$ axis ... it is because we solved in a wrong way...

$T.$: And what equation did you write? The immediate reaction is to erase the previous input and to digit the new and correct equation for the line (remember that students do not know the rule of the $x$-coefficient about parallel line, and this activity is aimed at finding one rule through some examples).

In Figure 2 we see what is projected on the big screen as a result of Screen Capture during the task to write the equation of a line intersecting the origin of the Cartesian plane and passing through the second and fourth quadrant. One of the group made a mistake, sending the equation of a line through the first and third quadrant (the last screen in the second line of Figure 2) and, immediately, conscious of this while looking at the big screen, corrected and sent the right equation.

In Figure 3 we see a table with the data for the experiment of plant growth.

<table>
<thead>
<tr>
<th>giorno</th>
<th>altezza della pianta (mm)</th>
<th>giorno</th>
<th>altezza della pianta (mm)</th>
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</thead>
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<tr>
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<td>7</td>
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<td>12</td>
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</table>
In Figure 3 the Cartesian plane from the Activity Center, as a result of the following task:

In a wonderful spring day the Prince of MatMat planet decides to plant a little seed. The following day he sees a 2 mm high plant sprouting. Then the prince observes the plant growing each day by 2 mm. Model this situation sending some points of coordinates (day, height) to Activity Center.

The students send those points (x,y), where x is the day and y is the corresponding height of the plant, in mm.

Looking at the Cartesian plane in Figure 3, we immediately see that some students were wrong to interpret the situation, because they sent points with exchanged coordinates (x instead of y). Then, if we examine the right model (the points on the green line, of equation $y=2x$), we may find out on Activity Center that not all the students sent the point (0,0). In the following an excerpt of the discussion about this fact (T means the teacher):

- T.: How did you start to put numbers in the table?
  - A.: The zero day, zero
  - A.: The first day...
  - T.: What did you put?
  - A.: Two zeroes...
  - T.: How many of you sent (0, 0)? ...Three groups. Fine. And the others? Which is their first point?
  - A.: (1, 2).
  - T.: [...] Why did some of you send point (0, 0) and some other did not? What is the reason to send (0, 0)?
  - A.: As soon as he plants the plant... It's zero because it does not grow... It's underground.
  - A.: ... As soon as he plants it, he saw that his plant was zero.

The situation presents an immediate feedback of the screen for those students who made a mistake in the coordinates of the growing plant, and also to whom who did not send as first point (0,0). The teacher here has the purpose to make a reflection on the correct model, that begins with the point (0,0), considering the first instant as time=0.
In Figure 5 one of the last activities (taken from Hershkowitz & Kieran, 2001):

*You have three families of rectangles (A, B and C) whose sides are growing up according to different rules, as depicted in figure (Figure 5). Represent the areas of the rectangles of the three families as functions. Which family has an area greater than the others and when?*

Students here have to discuss in pairs in order to find the right model for each function, write it in a symbolic way in the function environment of the calculators and represent them graphically on the same Cartesian graph. Then they have to compare the different kinds of growing of the functions. Observing Figure 5, we can notice that in Family A the height does not change, while the basis increases of 1 at each step. In Family B we have squares with the side that increases of 1 at each step. And in Family C the height does not change, while the basis doubles at each step. The three models corresponding to the three Families are respectively linear, quadratic and exponential. While the first two are relatively simple to be found, the third presents some difficulties, because the students are not usual with the exponential function.

In the following we may observe the discussion in a group, referring to Family C (O means the observer):

O: What can you multiply by two?
A: The area.
O: What area?
A: The area of the two rectangles, …is always multiplied by two.
O: In which sense? Can you explain me?
A: Because … well, one side remains constant, then the height remains constant, while the basis doubles at every step, so also the area, consequently, doubles at every step.
O: Then to find the 184th area, how do you do?
C: You have always to find the previous one … Every time.

The difficulty of finding the right model for expressing the area of the third family as a function of the step is evident in the previous protocol, where emerges the awareness of the recursive law, but not more. Later, during the class discussion in front of the projection of all the screens of the groups, that function will appear as “a power of 2”.

From a semiotic-cultural approach, this new technology offers something more than an usual one, because deeply influences the use of signs coming from the activity of the single components of the groups, but also from the sharing of the data processing, in a continuous exchange of information and feedback among humans and media.
Observing the protocol (and the video of the situation) we may analyse different signs present in the context of this discussion: gestures and words made by the students, gestures and words made by the teacher or the observer, words and symbols used by the students on the worksheet, mathematical signs on the calculators of the groups, and, last but not least, all the signs on the big screen, shared by all the people who take part in the discussion. Taken as framework the humans-with-media approach previously described, we can notice a double influence: of humans on the technology and of technology on humans. Particularly the second one seems to be important for our research, because it acts in different ways, linked together: in the use of the environment Activity Center or Screen Capture, to share symbols and signs that acts as catalyst of attention or elements of feedback, but also in modifying the role of the teacher, less involved in explaining the source of mistakes or in sharing results of the single group among the class. In this way, the teacher has more time and energy to dedicate to the students, supporting them in the construction of mathematical meanings.

Going back to our research questions, we have some evidence to demonstrate that the potentialities offered by TI-Navigator may really facilitate connectivity among students, reinforce their correct ideas and give support in correcting wrong results in their activities. In particular, such a shared display, offered by the technology, gives profit to the educational process, indirectly supporting the teacher in guiding the discussion and sharing results. The mediation given by this technology is particularly evident in the use and sharing of signs (in our activities, they are: equations, representations, coordinates, functions as models), and their meaning supported by different kinds of representations (numerical, graphical, symbolic) possible in the calculators and in the software. This mediation is also visible when the technology supports the introduction of other kind of signs (gestures, words, actions, inscriptions), as we saw in the videos.

Our workshop will be divided in three parts: in the first we introduce some theoretical elements of our research and some information about the technical equipment; in the second the participants are involved in some activities, working in small groups, and in the third a discussion about the same activities, solved by the students, is conducted by the authors.

REFERENCES


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