The present study reports a part of a teaching experiment that took place in two secondary school classes. The basic aim was to raise the students’ consciousness on the role that descriptions of geometrical objects play while one moves from a visual to a verbal representation and vice-versa. The analysis of the transcripts was based on social semiotics and our findings suggest that students experience great difficulties in moving between different semiotic representations.

Geometry has some features that differentiate it from the other mathematical themes taught in secondary schools. The most important is the role of the visual representations involved. Although Algebra and Statistics use some visual representations as well, in Geometry these representations are sometimes themselves the object of teaching. This fact has made Geometry one of the difficult subjects for students, since it is related to some cognitively complex processes for them (Duval, 2006, p. 108). However, concerning school Geometry in Greece, it is interesting to observe that it becomes “independent” as a subject in the 1st grade of “Unified Lyceum” as it is called in Greece (14-15 year-old students). In the previous grades’ mathematics textbooks, we find only few scattered chapters related to Geometry. Thus, in this grade students are essentially faced for the first time in their school lives with Euclidean Geometry as a strict axiomatic system containing definitions of concepts, axioms that define the relations between these concepts and theorems.

The present study was inspired by a teacher’s initiative to implement a student-centred strategy for mathematics teaching. Based on the principles of situated learning (Lave & Wenger, 1991), Maria had her students involved in various activities, including team work and whole classroom discussions on mathematical topics. The activity described in the present paper was inspired by a game called “Chinese whispers” in English or “σπασμένο τηλέφωνο” (which means broken phone) in Greek. According to this game, a person communicates a message to his/her fellow person, without anybody else knowing the message content. Then, the second person has to communicate the message to a third person and so on. Based on this, we created an activity in which the students could actually participate by constructing their own definitions of some geometrical objects and their relations. The purpose of this activity was to improve the students’ consciousness on the use of mathematical language and on its communicative function. Students sometimes treat (mathematical) language as the means to express some ideas without considering the fact that the messages described are intended to be read and interpreted by someone else. The teacher’s attempt to raise her students’ consciousness on that fact was expressed in most lessons, when students were asked to read aloud and discuss their classmates’ work.

THEORETICAL FRAMEWORK

Teaching and learning in any classroom is made possible through interactions between the teacher and the students or the students themselves. Mathematics in particular can be communicated in various ways; Pirie (1998) has identified the following means (or semiotic systems) of mathematical communication: ordinary language, mathematical verbal language, symbolic language, visual representations, unspoken but shared assumptions and quasi-
mathematical language. Whether we accept or not the dichotomy between “ordinary” and “mathematical” language implied, we cannot ignore that these two are interrelated; in fact, everyday language is the means through which mathematical meanings can be expressed, interpreted, developed and modified. Whether this means distorts mathematical meanings (Kanes, 1998) or becomes a part of them (Moschkovich, 2003; Pirie, 1998) remains an open issue in the continuous discussion between researchers. However, the most important challenge remains the clarification of the procedures that are related to the establishment of mathematical knowledge. The so-called semiotic perspective attempts to face this challenge. According to this perspective, the learning of mathematics, i.e. the development of mathematical meanings, is embedded in the communication that takes place between the teacher and the students, or between the students. Sáenz-Ludlow (2006) describes the abilities related to mathematical knowledge:

a) represent in order to communicate;
b) deal simultaneously with several semiotic systems;
c) recognize a mathematical object embodied in different representations without conflating the object with any of its representations;
d) transform representations of mathematical objects within and between representational systems;
e) construct and interpret meanings mediated by signs.

The above features are inherent in most Geometry lessons and that is the reason why this research was conducted in Geometry. In the next section we describe the procedure followed and the basic elements of our analytical procedure.

METHODOLOGY – SAMPLE ANALYSIS

The teaching experiments took place in two different classes during two consecutive days. Class 1 consisted of ten female and fourteen male students and Class 2 consisted of fourteen female and ten male students. All students had the same knowledge of Greek language, although two of them were not born in Greece. The initial plan comprised of two steps: firstly engage all students in the “broken phone” game and then, after collecting the anonymous working sheets, discuss the results in whole classroom discussions. In the present paper only a part of the first step is presented together with the analysis of students’ work. The students were asked to describe three figures (one at a time) to their fellow-students and based on these definitions the corresponding figures were made. This process was repeated with the new figures. The diagram that follows describes this process. F stands for figure, S stands for student and D stands for description. For example, Figure 1 is given to Student 1 who produces Description 1. Description 1 is given to Student 2 who produces Figure 1’, and so on. Maria did her best to ensure that Students 1 and 2 for example, were not sitting in neighbouring desks, so Student 2 could not see Figure 1 at the moment it was given to Student 1.

In only two cases the above “chain” was actually completed, i.e. six students took part. In most cases, the process stopped at the third or the fourth student, due to practical reasons (lack of time or smaller number of students). The analysis focused on the descriptions produced; the figures were used only as an aid on our attempt to locate the elements of the descriptions that affected the drawing of the figures. Halliday’s systemic functional grammar (Halliday &
Matthiessen, 2004) was used as our main tool, as it was implemented in mathematical texts (e.g. Morgan, 2006). According to this view the basic functions of a text are:

The **ideational** function, realising the field of discourse, is represented in text by choices made within the transitivity system, that is, the types of processes, the participants in those processes and the representation of actors.

The **interpersonal** function, realising the tenor of discourse, is represented in text by the modality: the mood of verbs, the presence or absence of adjuncts and qualifiers that vary the degree of probability or the expression of attitude. It is also affected by the degree of specialism in the register.

The **textual** function, realising the mode of discourse, is represented in text by the thematic structure and the cohesive structures. (Morgan, 2006, p. 227)

Another pair of concepts we found useful were theme and rheme (Halliday & Matthiessen, 2004). Theme is what the message is concerned with; the point of departure of what the speaker is going to say. The rheme parts are the new parts and have always to be combined with the given in order to function as real communication. Our methodology will be presented in the example that follows, in which the figure given initially to the students was the following:

F1:

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 emissions.png
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A piece of paper was given to the first, the third and the fifth student by Maria, in which the following phrase was written: “You have to describe the figure to a friend by the phone. Write down what you are going to say”. The second, the fourth and the sixth student had a piece of paper with the instruction: “You have 5 minutes to draw the figure described in the message that you got”. The following represent the chain of descriptions and figures produced by six students:

D1: Make two same squares on the row, with one common side. Below the second square, with a common side make three other same squares on the row, so that each one will have a common side with the other.

F1΄: (See the appendix)

D2: Draw two parallel vertical straight lines which will be separated in them with small line segments and they will create cubes.

F1΄΄: (See the appendix)

D3: 2 vertical parallel lines which are connected to 5 horizontal parallel lines.

F1΄΄΄: (See the appendix)

By comparing the figures produced with F1, it is possible to draw some conclusions about the related descriptions, but also of the complexity of the initial figure from the students’ point of view. The fact that five equal squares are involved is considered in D1, with the use of ordinary language: “same squares”. The topological notion of neighbourhood is expressed by “common side”, while the orientation is expressed by “on the row” and “below”. It seems that the orientation expressions were responsible for figure F1΄, since five squares “on the row” were drawn, but with the same orientation. It is interesting to note that student S2 put some letters on his figure, probably influenced by the school practice.

The next description (D2) is quite interesting because it moves the focus from the squares to the process that is needed to draw them; what we see is a decomposition of figure F1΄ into “lines” and “line segments”. Using Halliday’s terminology, the theme and the rheme are quite different in D1 and D2. The outcome the process described is the creation of “cubes”, i.e. squares, a fact that is recognized by the next student, who makes figure F1΄΄. It is interesting
to note that the term “cubes” did not cause any misunderstanding on the part of student S4.
The final description (D3) has a unique feature: it does not look like a description that one
would give over the telephone. The human agent is totally missing (cf. Morgan, 2006) and the
ideational function of the text is totally affected; this may be attributed to the student’s
attitude towards mathematics as an impersonal discipline or to his inability to integrate
everyday language into the mathematical register.

By observing the transition from F1 to F1 ′′′′′, we note two main difficulties in the students’
perception of the figures; the first was related to the terminology used for the orientation of
figures and the second was the lack of precision in the description of the figures’ elements.
The first difficulty stems from the fact that orientation is not explicitly taught in Greek
Geometry classes and is taken for granted; the same is the case with topological notions like
neighbourhood, which may partially explain the transformations of F1. The second difficulty
stems from students’ unawareness of the effects of passing from verbal to visual
representations and vice-versa.

CONCLUSION

The analysis of all texts produced in this experiment has shown the need to focus on the way
different representational systems can be used to enhance students’ acquisition of Geometrical
concepts. In Duval’s (2006) words:

Changing representation register is the threshold of mathematical comprehension for learners at each
stage of the curriculum. It depends on coordination of several representation registers and it is only in
mathematics that such a register coordination is strongly needed. (p. 128)

Thus, it is only through engaging students in activities that involve more than one
representational systems that we can assist them on their Geometry learning. The activity we
have presented (followed by a whole-class discussion of the results) may serve towards the
direction of enhancing students’ awareness of the functions that language plays in
communicating mathematical concepts.

ACKNOWLEDGMENT

The author wishes to thank Dr. Maria Iatridou for performing this experiment in her classes
and for helping in all sorts of practical issues.

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APPENDIX

Figure F1’  Figure F1’’  Figure F1’’’

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