Polya, Mason, Brown; Conjecturing andPosing: Communities of Mathematicians

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INTRODUCTION

What does mathematical activity look like? Rather than training students at any educational level in mathematical thinking, teachers can enact a well-known maxim from Paolo Freire: “our task is not to teach students to think – they already think – but to exchange our ways of thinking with each other and look together for better ways of approaching the decodification of an object.” (Freire 1982; decodification ≡ to decode into an intelligible form). Appelbaum (1999) suggests that students bring critical thinking skills and dispositions to the classroom experience; rather than teaching students critical thinking skills, a teacher can take advantage of those skills and dispositions to enact environments in which mathematics is learned through them. Similar arguments (NCTM 2000) have been made in favor of a problem-solving context for learning mathematics, over a curriculum that explicitly teaches problem solving. Theoretical grounding for this perspective is supplied by what Moll (2000) calls “Funds of Knowledge,” the inherent cultural resources found in communities surrounding schools. Teachers of writing have long held that the way to help writers develop their craft is to have them write. At the same time, writers often study the work of other writers to help them think about ways to write. Painters often paint while also studying the work of other
artists. In such educational situations, the teachers see themselves as practitioners of their craft, whether this craft is writing, painting, or inventing mathematical ideas. For our purposes, our student are mathematicians rather than students of mathematics, and the teacher is a mathematician, rather than a teacher of mathematics. Teachers and students share ways of thinking and working as they together explore mathematical objects, using mathematics to mathematize experience broadly conceived. Both the teacher and the students make use of the Funds of Knowledge they bring to school from everyday life and past school experiences. The networking of a classroom mathematical community is critical to establishing a learning environment designed to foster the exchange of thinking strategies. The thinking strategies, more than the specific activities, define the nature of the mathematics.

The phrase “community of mathematicians” is overused in many mathematics education circles. Its meaning has become ambiguous and varied. My own incorporation of the concept emphasizes the kind of community that is developed once we focus on the mathematicians that are members of the community. My critical point is: the design of the community is more important than the planning of content in mathematics education curriculum development, because the design of the learning environment constructs the possibilities for how people work as mathematicians within the community. Whether we explicitly think this way or not, we are always fostering some ways of being a mathematician and diminishing the potential for other forms of working mathematically. In a traditional classroom, students are limited in their options for participation. I argue for including activities that intentionally center the exchange of thinking about mathematical objects; this creates increased potential for students to make use of their funds of knowledge in order to act as mathematicians. My study demonstrates the viability of this approach, and reports on the strong effects that are possible when teachers and curriculum developers learn about the funds of knowledge that their students bring with them to the mathematics classroom and enable their students to use these skills and concepts to create new mathematics.

Theoretical resources include books by Catherine Twomey Fosnot and Maarten Dolk (2001a, 2001b, 2002) and David Hawkins (1974). In the works of Fosnot and Dolk, teachers help young children to develop mathematical relationships in a workshop environment. Their work provides stories of successful mathematics education grounded in the creation of communities where mathematicians are at work. Hawkins presents the need for objects that adults and children can look at together in order for the adults and the children to develop Buberian I-Thou relationships, which he places at the center of the educational encounter; here, too, even with the establishment of a pedagogical relationship as the goal, the ways that the teachers and students are looking together at the objects of study are of paramount importance.

Further theoretical grounding for this study is supplied by Gloria Ladson-Billings’ (1997) “Six Habits of Highly Effective Teachers:”

1. Students are apprenticed rather than taught.
2. Students are enabled to be intellectual leaders in the classroom.
3. Teachers and students work within a broad conception of literacy.
4. Students’ real-life experiences are legitimized in the classroom.
5. Students and teachers together challenge the status quo.
6. Teachers conceive of themselves as political beings.

Ladson-Billings studied teachers who were identified as highly successful by students, parents, counselors, and other teachers, and observed only those teachers who appeared on all lists. These six habits were what she summarized as what the highly effective teachers have in common. I have designed a way of achieving each of these habits, and have submitted the practices to extensive repetitive tests in my own teaching and in the preparation of future teachers.
MAIN IDEA

Designed activities benefit from their placement in a community of mathematicians explicitly discussing the ways that they work mathematically. Students learn more of the prescribed content when the focus is not on the content per se, but on the ways of investigating the mathematics. It is often over-generalized that mathematical activity is essentially problem solving. However, mathematicians do things other than solve problems. For example, they pose problems, categorize problems, evaluate whether or not mathematical questions are “good” questions (i.e., worth pursuing), reflect on their own abilities as problem posers and solvers, and so on (Brown 2003). We can replace the central assumption that mathematicians solve problems with the more expansive expectation that mathematicians communicate with each other regarding mathematical questions. Students and teachers, as mathematicians working together, exchange ways of thinking mathematically, and look together for better ways of approaching the decodification of mathematical objects.

There are several ways of conversing about mathematical investigations and other mathematical work that have been highly effective in my own teaching of mathematics, and in my pre-service teachers’ work with elementary school children, including:

1. *Polya’s Problem Solving*. Students evaluate the effectiveness of Polya’s suggested phases and the questions associated with each phase for helping them move past “mathematician’s block” and making progress on a mathematical investigation.

2. *Mason’s Specializing and Generalizing*. Students approach mathematical work as scientists, collecting special cases, organizing special cases into categories and types in order to develop a conjecture, and subsequently testing their conjecture with further artfully chosen cases. This lays the groundwork for communicating a convincing argument for the strength of their conjecture to others.

3. *Brown’s Problem Posing*. Students pose their own questions through the process of attribute listing and the “what-if-not” strategy. They generate criteria for evaluating new questions in order to decide which ones to pursue.

4. *Mathematics Studio*. Students share work in progress on original, self-designed mathematical investigations for critique and advice.

5. *Taking Action*. Students identify what stands out for them in the work accomplished so far, and design an action to take that uses this prior work to make an impact on others outside of the class, or to meet with someone outside of the class who has the potential to make an impact on their own work (i.e., helps them move further in their inquiry).

METHODOLOGY

The author and six cohorts of 60-70 student teachers each led groups of students through the processes described above. The author has taught a college-level mathematics course for non-mathematics majors with these techniques through six iterations. The course meets twice weekly for 100 minutes pre meeting; classes ranged from 25-30 students, mostly first-year university students but including some students from each of the four years typical of an American University. Students enrolled in order to meet a distribution requirement for university graduation, and typically expressed no interest in mathematics at the start of the course. Most described anxiety about the course on an initial information form, and used negative metaphors to illustrate mathematics. The student-teachers, fourth-year elementary education majors, each experienced the process as a student once in a two-week orientation, and then taught groups of 8-10 elementary students using the same techniques. The elementary students ranged from grade 1-8, in grade-specific groups. Groups included students of all ability levels, special education students, and students who were considered behaviorally challenged. Student teachers were required to document the elementary children’s progress on predetermined curriculum objectives in mathematics accomplished through the open-ended inquiry format.
The work I describe led to the following results:

1. Student engagement in each of the two types of uses of the techniques, as indicated by participation, student self-observations, parent-comments about the program.
2. High levels of performance on standardized tests of knowledge.
3. Documentation of predetermined curriculum objectives demonstrated through alternative assessment practices, including observations, individual and small group interviews, and samples of student work.
4. High quality of performance in creative narrative writing with self-chosen mathematical topics.
5. The impact of student action projects on themselves and on their chosen audiences.
6. The experience of teaching in this way changed the ways that the teachers of mathematics view the nature of mathematical activity, including the author and the student teachers.

CONCLUSIONS

The creation of an infrastructure of apprenticeship as a mathematician, rather than a structure of activity, enacts each of Gloria Ladson-Billings’ (1997) six habits of highly effective teachers. When teachers design activities that foster the exchange of mathematical thinking in order to decode and invent mathematical objects, they employ the metaphor of apprenticeship, in which students who come to the classroom already as mathematicians develop their craft in a mathematical studio. Students in such a classroom become intellectual leaders, because their ideas and suggestions to others directly influence the direction of mathematical exchange, and because their contributions become part of the ongoing construction of the mathematical canon within their community. Students who read, write, listen and speak mathematics in such a community are working with a broad conception of mathematical literacy. They employ funds of knowledge in ways that legitimize the mathematics of their everyday lives.

Students and teachers in this project challenge the mathematics education status quo by questioning most of the basic assumptions of mathematics education. And, by working in these ways with students, teachers facilitate all members of the community to enact a politics of mathematics education that promises social transformation of the institution of schooling.

This work is also an extension and critique of Freire’s notion of decoding. When teachers and students are theorized as bringing Funds of Knowledge to communication about mathematical ideas, the mathematics of everyday life and of the classroom changes from static received knowledge to a collection of cultural resources (Appelbaum 1995). As cultural resources, mathematical funds of knowledge are used in the community not only for the decodification of (mathematical) objects, but also for their invention. This has much in common with Deleuze and Guattari’s (1996) idea that philosophy is the construction of concepts. Broadly conceived, then, I claim that mathematics studied as science (c.f., Mason) enables mathematicians within the mathematical community to act as philosophers of mathematics, i.e., as mathematical philosophers.

REFERENCES


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