GEOMETRIC STRUCTURE OF DANTE’S HELL: FROM PROBLEMATISATION TO CORPOREITY

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Abstract: We try to focus the question about the ratio between classroom practice and research, by starting from the concept of act and, more specifically, from the one of corporeity, intended as human way of being of each act. To put in evidence the role that corporeity could have in teaching and learning of mathematics, we present a form of problematisation, that we experienced in class: we appeal to a Galileo’s text about geometric structure of Dante’s Hell. This text wealthy of echoes, opens the adventure of a mathematical path imbued of culture and history, that favours the approach to argumentation by articulating its rhythm. Our way to understand corporeity opposes the concept of embodiment and just this opposition could give some interesting hints about the question of the ratio between classroom practice and research.

INTRODUCTION

The centrality of activities in teaching and learning of mathematics needs of a restless reflection that should, over and over, orient their typologies, management and aims. Our reflection takes hint from a specific classroom activity and it extends to some considerations about the words “act” and “corporeity”, as human way to be of each act. The role of corporeity in teaching and learning of mathematics reveal itself in many different situations. In (Locatelli), for example, we show a situation in which the gesture of paper folding favours the approach to argumentation by articulating its “rhythm”. In activity we present here, acts, even if of different nature than gestures, permit both to front problems and to percept the “rhythm” of scientific thinking.
GALILEO AND GEOMETRIC STRUCTURE OF DANTE’S HELL

The analysis of geometric structure of the Dante’s Hell was the chosen activity: the reliance in mathematics, that characterized Christian world since VII century, but that at beginning could be considered an empty shell, was filled, at Dante’s time, by contents, thanks to the recover of Classics and to the interaction with Arabian culture (Hoyrup). Dante richly lives the spirit of his time: «he’s a scientific poet, he’s a poet because a scientist, he’s a scientist because a poet… Science is consubstantial to his poetry. His poetic language is available for all adventures of knowledge, included the careful observation of clinical, optical or astronomical phenomena, … included the mathematical structure that transforms itself in proportion and “convenienza” (from Latin “con- venire”, to agree, to enter in consonance) between world and the vision of world» (Sermonti)

Science and mathematics are implicit in Dante’s mind; he uses them in describing the Hell too, «ma si lo lascia nelle sue tenebre offuscato, che ad altri dopo di lui ha dato cagione di affaticarsi gran tempo per esplicare questa sua architettura (but he left it so hidden in darkness that he caused others got tired in explaining its architecture» (Galileo). Geometric contents that interest the structure of Dante’s Hell found an extraordinary interpreter in Galileo. He wrote, in 1588, the Essay Due lezioni all’Accademia Fiorentina circa la figura, sito e grandezza dell’inferno di Dante (Two lessons at Florence Academy about figure, place and size of Dante’s Hell) . In this essay Galileo wanted «avvicinarsi alla mente di Dante (to approach the Dante’s mind)» and, to this purpose, he run step by step through the journey into Hell, to search for giving a geometrical and rational description of the Hell, and calculating measures and proportions of those places. according to “divini versi” (divine lines).

Before Galileo, the greatest interprets of Commedia already fronted the study of its cosmographic implications. Cristoforo Landino (1424-1498) premises to its edition of the poem of 1481 a discourse about Sito, forma et misura dello ’nferno et statura de’ giganti et di Lucifero, (Place, form and measure of Hell and height of giants and Lucifer) in which he mentioned the researches of Antonio Manetti (1423-1497), Florentine mathematician and architect. Girolamo Beniveni (1453-1542) published in 1506, as appendix to his edition of Dante’s poem the essay Dialogo di Antonio Manetti, cittadino fiorentino, circa al sito, forma et misure dello inferno di Dante Alighieri. (Antonio Manetti Fiorentine citizen’s dialogue on place and measures of Dante’s Hell). Alessandro Vellutello, in 1544, in his polemic comment to Commedia dashed the Manetti’s interpretation. In his two lessons, Galileo disproved the interpretation of Vellutello and defended Manetti’s one.

The exceptionality of Galileo’s interpretation resides both in certainty of mathematical knowledge he uses in applying the experimental method to “object” of study, and in «intrinsic power of his language… in his capability to give prominence to concreteness of problems… to use words to serve his thought… to infuse into his prose a fluent course that makes it lucid and ductile» (Spongano).

The Dante’s Hell came in class activity as gifted of the character of reality that poetry gives it: each place of Hell is real because Dante was able to realize an operation of imaginative engineering in which he used science and mathematics to determine forms and dimensions, and poetry to produce a construction overflowing of life. Galileo’s text is an extraordinary scientific guide in a path through the geometric concept involved in that construction.
CLASSROOM ACTIVITY

As example of the activity we made in class, we limit here to quote the part of Galileo’s text that interests the figure of the Hell.

“… figure… is as a concave surface that we call conic, the vertex of which is in the centre of world, and the base of which is toward the surface of earth…; imagine a straight line that comes from the centre of the greatness of earth (that is also centre of gravity and of Universe) and goes up to Jerusalem, and an arc that from Jerusalem lays over the surface of water and earth, along the twelfth part of its major circumference: this arc will end with an extreme in Jerusalem; from the other extremity, to the centre of earth, draw another straight line, and we’ll have a sector of circle …

Imagine then that, by keeping motionless the line that joins Jerusalem with the centre of earth, the arc and the other line are moved in turn, and that in this motion they go cutting the earth and that they move as they come back where they started; in this way a part of earth similar to a cone will cut; if we imagine to take away ground, in its place a hollow in form of cone will be left; this is the Hell…

About the size, the Hell is deep as the half diameter of earth; and in its opening, that is the circle around Jerusalem, it has the diameter as the one of earth, because the arc that corresponds to the sixth part of the circle subtends a chord equal to same diameter…

If we make the counts according to what Archimedes demonstrated in *De sfera* and *De Cilindro*, we’ll find that the hollow of the Hell is something less than the fourteenth part of the earth… the base that is covered by a huge vault of earth is large as the eighth part of the half diameter, that is 405 15/22 miles.

This text has been proposed to a class of sixteenth years old students; they were studying Dante’s Hell with the teacher of Italian literature, Middle Age and Renaissance and the first element of scientific method with their teachers of History and Physics.

Just in cause of this extent of involvements, the proposed text can be considered besides a text of a geometric problem, a text wealthy of echoes and a way for a mathematical adventure along an imbued path of culture and history. So it has been dealt at two levels: at level of problem and at level of cultural “sign”.

At level of problem, students worked in little groups upon the Galileo’s text: they made explicit numerous questions that arose as reading proceeded in differentiated contents, as, for example, “How long the ray of earth is?”, or “Does great circumference coincide with equator?”, or “What is the name of the angle of cone?”, and so on.

The frequent activities of the big group spaced the ones of small groups, and questions were discussed and, if necessary, suitably framed; unknown terms, as “angle of half opening of cone”, or “spherical cap”, and so on, were suggested by teacher; questions were organised according to topic; Internet too was used by students to answer some questions; at last, a figure of Hell was derived and its correct representation extracted.

The second part of activity concerned problems: students newly went along the text in searching for numerical and geometrical problems, as for example “How the size of central angle, that correspond to a given arc, can be determined?”, or “That area of the spherical cap that covers the Hell, is?”, or, “How to determine the part of terrestrial sphere taken by Hell”, and so on. Necessary forms was drawn by usual teaching aids.

Other types of work were carried out alongside with this one on text as problem: a work of contextualization of Dante’s poem into mathematical knowledge of his time, and a work of
reading of Galilean text at higher level. In this regard, the passage that describes the hellish cone looks as the most interesting:

Imagine then that, by keeping motionless the line that joins Jerusalem with the centre of earth, the arc and the other line are moved in turn, and that in this motion they go cutting the earth and that they move as they come back where they started; in this way a part of earth similar to a cone will cut...

In this text Galileo doesn’t limit himself to describe a cone; he instead guides the reader; “with the power of his words”, in a process that puts in evidence the concreteness of construction; “the arc and the other line are moved in turn, and by this motion they go cutting the earth...”; the object is bodily covered; figure is almost caved by hands; construction of cone becomes an “act”. This text results figurative and wealthy of bursting force, representation of rising experimental method as succession of “acts”: an articulation of thinking by acts that gives to it a primary aspect of scientific nature. The Galileo’s language, by articulating the thinking by acts, dresses itself by the rhythm of scientific process. This is sometimes forgotten in teaching, on behalf of formal and abstract aspects, neglecting so that primary characteristic of science.

The articulation of thinking by acts that is kept in Galileo’s text, finds “corporeity” thanks to Dante’s poetry, to images that poetic description continuously renews and revives.

**ACTS AND CORPOREITY**

Our reflection moves from a kind of activity that emerges from the Galileo’s text about the cone of Hell, and goes on a path of generalisation that involves the body in its entirety, in order to teach and learn mathematics. “Acts and corporeity are its central words.

The word “act” rises strong “echoes” (Boero). In the development of western thinking, it has been engaged with pregnant meanings in different contexts: from the “act” of Aristotle to “biological act” of von Weizsäker, from “linguistic acts” of Austin and Searl, to activism of Dewey, Montessori, Piaget, Bruner and so on. It echoes history, society, culture, praxis.

In our context it encloses the different typologies of activities that interest the teaching and learning of mathematics, as the successions of mental “acts” finalized to create and transform images (for example the way of proceeding of Galileo on the Dante’s Hell), or the concrete operating, or the use of technology, and so on.

The term “corporeity” has the grounding in the continental, western, European culture and takes shape as an attempt to overcome dualism between mind and body. In this strain it acquires the meaning of human way of being of each act: thanks to their fundamental characteristics, interactivity, acts shape corporeity: each act opens a world; corporeity, as set of acts, put worlds into relation.

These ways of thinking about acts and corporeity, give some hints also to teaching and learning of mathematics. The word “act” would translate the belief that different types of activities can constitute different “semiotic registers” of teaching and learning of mathematics. But this word permits a wider reading too: when thought as interactivity, it leads to a teaching as management of different corporeity, of different interacting worlds: the world of teacher, the world of each student, the word of class and the world of taught object itself; worlds that teacher must handle, challenge, coordinate, in order to arrive to development of knowledge and formation of personality.
CLASS ACTIVITY AND RESEARCH - CORPOREITY AND EMBODIMENT

We believe that many teachers that engage themselves in didactic research, feel disappointed of “monological” choices to propose in class; then they try out new typologies of acts in order to find diversified ways of creating interaction, to exploit different potentialities, to propose and manage acts that helps a more fully involvement of different corporeity. But, usually, they find difficulties in interacting with research.

What the difficulties of this relation are? What their origin is?
We are not able to give wide and articulate answers to these complex questions. More simply, we try to propose a limited reflection about the term “corporeity”, seen in contraposition with the term “embodiment” or “embodied mind” (Lakoff – Nunez); from this contraposition we try to find suggestions for a discussion on the posed questions.

The term “corporeity”, as human way to be of each act, makes manifest as teacher ought to rely on himself when he manages the real class and the individual students; it is an involving term just owing to growing complexity that teacher has to coordinate; growing complexity that, on one side, makes the meaning of the term more and more shifty, on the other side enhances a demand of more elaborated resolutions.

The term “embodiment” stays now at the centre of attention (and perhaps of the fashion), in the field of didactic research and not only. Just in cause of its cognitivist origin, that term tell us that mind and body are not separated and that concepts are basically embodied; they are grounded on perceptive and motor experience and neuronally represented, that is they are built by cerebral mechanism (Lakoff, Nunez). The great part of conceptualization of world is metaphoric, and metaphoric concepts by which our knowledge and reasoning express themselves, are organized in maps and frames, like cartographies that allow understanding abstract concepts by metaphoric correspondences and emotions.

Many are the opposition between the considered terms: the conception of metaphor as cognitive activity that accompanies the term “embodiment” loses symbolic dimension that characterizes it when joined with term “corporeity”. (Ricoeur) “Embodiment” declines any link with history and contains a substantial deafness to culturally and socially variable aspects; it identifies word and concepts without any mediation with entire experiential content. “Corporeity” is deeply loaded of sense of history, of humanistic calls strictly connected with a temporary cultural context. The idea of embodiment reduces body to a system that, even if complex, is amenable to a “mechanistic” organization that answers to stimuli and to image schemas that work the cognitive operations; corporeity instead, just because it’s open to multiplex novelties of acts, opens on a wide, restless horizon, where reasoning is not secluded but lives with others and it’s made to be communicated, that is placed in common, in the sphere of cultural codes and traditions.

CONCLUSION

Activity we made in classroom about geometric structure of Dante’s Hell, constitutes a concrete teaching experience that tries to go beyond traditional didactic proposals: notwithstanding it starts from usual forms of problematisation, it opens in a wider reflection about the concept of act. Two are the aspects about acts that have been pointed out: acts shape the corporeity and acts can become the tool to articulate the rhythm of argumentation and (then) of scientific thinking. In these processes that compete to acts, interactivity is a fundamental characteristic: interactivity among worlds and among corporeity generates the complexity of didactic practice. This complexity finds difficulty in meeting the attempt of simplification that, sometimes, characterizes
didactic research. We believe that this difficulty is adequately represented by the distance between the meanings of terms “corporeity” and “embodiment”: the term “embodiment” seems to offer certainties and finds limited “convenienza” (Sermonti) with the practice in class. In cause of its demands of continuous social mediation, this latter better reconcile with the term “corporeity”.

REFERENCES

Boero P.: 1999 *Argomentazione e dimostrazione: una relazione complessa, produttiva e inevitabile nella matematica e nella didattica della matematica*. International Newsletter on the Teaching and Learning of Mathematical Proof. Luglio – Agosto


Galileo Galilei: 1588, *Due lezioni all’Accademia Fiorentina circa la figura, sito e grandezza dell’inferno di Dante*. In “Scritti Letterari” a cura di Alberto Chiari. Le Monnier, Firenze (1970)


Sermonti V.: 2003, *Con Dante, oltre le “due culture”*. Il Sole 24 Ore. 19 Ottobre