TEACHING THROUGH PROBLEM SOLVING: WHAT STUMPS PRESERVICE TEACHERS

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The most important form of mathematical activity in mathematics classes is, and has probably always been, problem solving. The National Council of Teachers of Mathematics (2000) has stated that “problem solving is not only a goal of learning mathematics but also a major means of doing so,” urging educators to build instruction through problem solving. This injunction gives rise to a number of questions: Exactly how should such a program be carried out? What are the basic and most characteristic situations in which teachers must implement it? And last but not least, what skills and knowledge must teachers possess in order to be able to teach through problem solving?

The answer to the last of these questions naturally leads to a discussion of what Shulman (1986) has called “pedagogical content knowledge,” i.e. the mathematical knowledge that a teacher must possess in order to teach mathematics. Attention to pedagogical content knowledge, and consequently the scholarly literature devoted to it, has been considerable (see, for example, Ball et al., 2005; Ball and Bass, 2004; Even, 1993; Even and Tirosh, 1995). It has been argued, in particular, that a crucial concept in this connection is that of “unpacking mathematical knowledge” (Ball and Bass, 2004): teachers must be able to unpack mathematical knowledge for their students, and this is indeed what must be accomplished by them during a lesson, in the course of various mathematical activities and problem solving in particular. At the same time, little is thus far known about the ways in which purely mathematical knowledge—knowledge for oneself—becomes transformed into pedagogical content knowledge—knowledge for others. Note, too, that the goal of teacher preparation is not so much academic knowledge, as the ability to apply it in class (first and foremost, perhaps, in the organization of mathematical activities). In this respect, it appears useful to talk specifically about “pedagogical content skills,” i.e. the special ability to work with mathematical materials in order to solve the pedagogical problems that teachers face. The most important among such pedagogical content skills is the ability to select and to conduct mathematical activities, including the ability to structure instruction in a way that enables students to learn mathematical concepts and rules in the process of solving problems.

Looking at it from another angle, teaching through problems means that teachers must choose or invent their own mathematical problems. In this respect, the analysis of the ways in which activities in mathematics classes might be structured can draw upon the analysis of the ways in which mathematical problems can be posed, how the teacher can work with mathematical examples, and cases of interaction with the class in the event of various mathematical difficulties (Brown and Walter, 1990; Watson and Mason, 2005; Merseth, 2003).

Despite the growing interest in this subject matter, the practice of teaching through problems as it actually takes place in the classroom has not been sufficiently investigated. In particular, it is important to study the degree to which beginning teachers are successful at teaching through problems. The aim of the study undertaken by this author was to identify typical situations in which beginning teachers encountered difficulties in teaching through problems—
ultimately preferring to provide their students with one or another “recipe” instead, for example, or conducting a problem solving session in a way that left no room for any exploration. The study focused on middle and high school teachers. Note that, because the issue of pedagogical content knowledge and skills has received relatively greater scholarly attention in the context of elementary school education, there exists a view—although it is not always explicitly stated—that with regard to middle and high school teachers it is possible to speak only about strictly mathematical knowledge and about very general pedagogical demands. However, it is precisely subject-specific pedagogy at the middle- and high-school level that requires greater study.

The methodology of our study involved the observation of classes conducted by preservice teachers, as well as the analysis of the materials which they compiled for use in teaching, and of their reflective journals, which contained their observations about their own teaching and about the teaching of their fellow preservice teachers. In the analysis of the collected data, the findings were coded and typical situations were identified (Glaser and Strauss, 1967).

We limit ourselves here to a brief analysis of just one episode from a class, in which the teacher was evidently unable to organize fruitful mathematical activity.

Ninth-grade students at a New York school were asked to write down the equation for a straight line passing through points with coordinates (0, 2) and (1, 4). The student who came up to the board carefully constructed the coordinate plane, traced the straight line in question, and pointing out the rise and the run of the graph, found that the slope was equal to 2; he then wrote down the equation $y = 2x + 2$. Another student, Mike, immediately observed that it was not necessary to draw the graph: “You can see right away that 2-0=2, so the slope is 2. And since the y-intercept is also 2, the equation is $y = 2x+2$. You don’t need to graph anything.” The teacher praised Mike for the solution that he had suggested, but nonetheless said that this was not a recommended way of solving problems, since the exam would require something different.

The failure is obvious here: if the students in this episode did learn anything, then it was not thanks to the teacher, but in spite of him. It is clear that the beginning teacher made a mathematical mistake: although the obtained solution is correct, the solution is mathematically unfounded—it led to the right answer purely by accident. It is significant that the beginning teacher failed to see that such reasoning could not be correct in principle, since it makes no use of the second point. Although the teacher himself knew how to derive the equation that was required, his knowledge was not “unpacked”—he himself was not aware of what it was in the operations that he performed to solve the problem that was important, and why. In the given instance, at least one of the reasons for the failure is clear: insufficient mathematical preparation, which revealed itself in the fact that the teacher had not even anticipated that, in the course of mathematical activity, the students might begin to understand something better. The mathematical activity that he organized (including the way in which he posed the problem and conducted the discussion) could never have been altogether successful, if only because the teacher’s real aim was merely to obtain an answer, in other words, the rote memorization of a certain algorithm.

The question of how the teacher should have acted instead gives rise to significantly deeper considerations. In analyzing the episode described, many preservice teachers noted that the teacher should have involved the students in an investigation of whether the method proposed by Mike was legitimate, i.e. that the teacher should have used this episode as an opening for mathematical activity. Such an approach would indeed have been more fruitful, and it points both to the teachers’ greater mathematical preparedness and to the fact that they set more substantive
goals for themselves in their teaching. Nevertheless, it is clear that it does not by itself guarantee automatic success in the organization of mathematical activities. What kinds of questions should have been posed to the students to make it clear to them (and to Mike himself) why his proposal was erroneous? It would be desirable to include problems that took into account the different levels of mathematical preparedness in the classroom and the individual differences in the students’ thoughts and reasoning; that would be interesting and motivating for the students; that could be developed in future mathematical activities; and much else besides. Teachers must be ready to take into account both the distinctive characteristics of their audience and the distinctive characteristics of the mathematical texts that they produce (select). This, in turn, presupposes the ability to carry out a complicated analysis that takes into account both the psychological-pedagogical aspects of the situation and the purely mathematical ones.

The presentation that I propose to give will draw on the empirical materials that we have collected to lay out a kind of map of the difficulties encountered by preservice teachers in teaching through problems. The three facets listed above—the teachers’ goals (and beliefs), taking into account the distinctive characteristics of the audience, and the pedagogical analysis of mathematical texts—seem to us to be essential for understanding the roots of their difficulties.

Naturally, there is need for further research on the roots of the difficulties of preservice teachers indicated above; however, even the findings discussed here plainly show that one of their causes is inadequate attention to these problems during the course of teacher education. It would not be an exaggeration to say that the beginning teacher is more often than not left alone to face the problem of constructing teaching through problems. Teacher education programs all too frequently limit themselves to conveying to their students a certain body of purely mathematical knowledge, on the one hand, and on the other hand, a set of general pedagogical guidelines. In this way, future teachers are left to their own devices to develop the ability to unpack a mathematical idea by giving schoolchildren problems that they are capable of solving.

Relying on the findings of the study, the presentation will also address the ways in which mathematics teacher education might be improved with a view to overcoming the noted difficulties. While it is not possible to enter into further analysis of the existing literature here, let us note that in order to improve beginning teachers’ skills in organizing mathematical activities (and more broadly, to develop their pedagogical content skills), it is important to expand their subject knowledge, including their knowledge of existing activities, problems, approaches to solving them, and so on. It is natural to suppose that such knowledge will also facilitate the development of the teachers’ own skills (not least by inducing them to formulate new goals in their teaching). At the same time, it seems beneficial to involve preservice teachers directly in activities that require using pedagogical content skills (the most important and natural example of such an activity is teaching). This cannot be accomplished by mechanically increasing the number of hours devoted to preservice teachers’ practice, their students teaching, etc. Preservice teachers must be offered assistance in teaching through problems. Such assistance can come, first and foremost, in the form of scaffolding—through the organization of collaborative activities for preservice teachers and their more experienced colleagues. The forms of such collaborative activities may vary. For instance, they may involve collaboration on the preparation of lessons and activities, followed by the analysis of the way in which these plans were carried out in practice, as in the Japanese model of “lesson studies” (Stigler and Hiebert, 1999); or they may be more modest in their conception and involve nothing more than work on certain materials used in teaching or the analysis of classroom episodes, etc.

The experience of such collaborative work will be discussed in the presentation. In this context, it will be particularly useful to analyze certain examples of courses and classes, typical
of Russian mathematics teacher education, in which the students are explicitly requested to work on resolving various methodological challenges by selecting and writing mathematical assignments. The discussion will also address the experience of conducting a course for American preservice teachers.

REFERENCES


