A FIRST APPROACH TO THE TEACHING OF SERIES. SOME RESULTS IN CANADA AND UK

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Abstract: This paper gives an overview of an ongoing research project about the teaching and learning of series. At this very first stage some details about previous research results and the analysis of some textbooks will be shown. Although textbooks seem to give a great place to the teaching of numerical series, the approaches that are usually used do not encourage the use of different registers nor the construction of meaning for this concept.

1. INTRODUCTION

Many reports (see Wood, 2001, for instance) state that there is a big gap between Secondary and University practices and contents, which may have produced in the last years the decreasing in the number of students following mathematics-oriented courses. It is, therefore, urgent to develop studies aiming at improving both the comprehensive learning of undergraduate students and teachers’ practices at this level (enriching the teaching of mathematical concepts and not reducing them to their algorithmic part).

One of the complex concepts that students early encounter when they follow post-secondary mathematics-oriented courses is that of numerical series (or just series). A series can be defined as a sum of infinite terms. Although this concept may seem to be quite “artificial” and of few applications, it is essential to understand the modern development of Mathematics (and, as consequence, of many of its applications): the calculation of areas by means of Riemann integrals, the resolution of many differential equations, the spectral analysis with Fourier series, signal treatments too, and so on.

This concept, due to its “mysterious halo” is usually reduced to algorithmic aspects, which later produce many misconceptions to understand the key concept of integral (Bezuidenhout & Olivier, 2000; González-Martín, 2006). In fact, traditional teaching concentrates on studying the different techniques to conclude convergence or divergence and also on the formulae to calculate the sum of convergent series and little emphasis is done on the applications of the concept or on the construction of meaning.

To face this situation, we aim at long term at improving our comprehension about students’ learning of the concept of series in the post-secondary levels; our results will allow us in a future to design a teaching sequence aiming at improving this learning (and to contribute to a better training of future professionals). At this moment, we are just in the previous stage of an epistemological-didactical analysis. These analyses will be taken into account to later explore how students learn this concept and the possible obstacles it can entail. The objectives of this stage are:

- To make a revision of research results concerning the concept of series.
- To explore different textbooks in UK and Québec to have an idea of the different approaches used to introduce series.

* The author would like to thank Guillaume Payette for his valuable help to the development of this stage of the project.
The aim of this paper is to show the most important results of our revision of literature concerning the concept of series and of the approaches that textbooks use to introduce the concept.

2. BACKGROUND
Our starting point comes from the author’s Ph.D. thesis (González-Martín, 2006), which shows that undergraduate students have no visual images associated to the concept of series and also that their difficulties to understand this concept produce some misconceptions with the concept of improper integral. Bezuidenhout & Olivier (2000) have expressed that students lack of appropriate conceptions to understand the expression for the Riemann sums, essential to understand the relations between series and integrals, and our own results seem to support this fact.

In spite of the importance of the concept of series due to its applications and to the fact that it helps to understand other mathematical concepts, many studies have shown the difficulties the students have to understand both the concept of “convergence” (Robert, 1982; Cornu, 1983) and the underlying sums to some mathematical situations (Orton, 1983; Bezuidenhout & Olivier, 2000). In addition to this, as mentioned above, our most recent research proves that some misunderstanding with the concept of series may produce many difficulties to understand the concept of improper integral (González-Martín, 2006). Given the concept’s importance and the difficulties it entails, more research is needed to help us better understand, and support, students’ learning of this concept.

In addition to the latter, much recent research emphasizes the importance of visualisation to better understand mathematical concepts (Eisenberg & Dreyfus, 1991). One of the approaches that research in Mathematics Education has developed to better understand students’ learning of mathematical concepts is that of Duval’s theory of representation registers. Using this approach, a distinction between the algebraic, numerical and graphical registers is made in order to stress the difference between a mathematical object and its various representations (Duval, 1993). As a consequence, it is absolutely necessary to use different representations of a mathematical object in order to understand this object, but these representations are only partial and show different characteristics of this object. As they are only partial, Duval states that, in order to understand a mathematical object, it is necessary to use at least two different representations of it. However, Duval shows how “traditional teaching” (in particular in the post-secondary level) only considers as important the work in the algebraic register, so students do not achieve a good comprehension of mathematical concepts.

Our revision of literature shows that the concept of series has not been exhaustively studied up to now. Many studies deal with the concept of convergence and it was early remarked that the acquisition of the concept of convergence is not made without any problem and remains incomplete to the student during post-secondary teaching (Robert, 1982) and also that the exercises do not let the student construct a correct notion for the convergence of numerical series. Boschet (1983) also pointed out that traditional teaching shows very few examples of graphic representation of convergence (and the existing ones foster static representations) and that sequences are not seen as a particular case of functions. Fay & Webster (1985) also stated that in most calculus textbooks, little of no relation between improper integrals and infinite series is given other than the integral test for the convergence of series. Bagni (2005) suggests the use of historical examples to improve the teaching of infinite series and to overcome the misconception that a “sum of infinitely many addends is infinitely great”; he also encourages the use of visual representations. Finally, Codes (2007) designed an activity to introduce the
basic notions of numerical sequences and before introducing infinite sums as the limit of a type of sequence, based on Oresme’s work, using graphical representations. She states that the use of the computer and graphical representations helped some students and that they used, later on, the computer representations to reason some paper-and-pencil questions.

3. METHODOLOGY

As stated above, we are currently organising and developing our research. At this moment, we are developing a revision of bibliography to create a database or research results concerning the teaching and learning of the concept of series. This database will help us to be aware of the different approaches that researchers have used to approach this concept and of their results.

We are also making a revision of textbooks used both in Quebec and in UK to analyse the approaches that the authors privilege (and the representation registers that come into play) and how the concept of series is introduced in both countries. The questions we wanted to answer with this revision are: do textbooks take into account some of the research results?, is there an evolution in the way they present the contents about series?, and what kind of representations are privileged?

For the analysis of textbooks we created an analysis grid and chose four textbooks: A: Charron & Parent (1993); B: Anton (1996), which is a translation of an English version (Calculus with Analytic Geometry); C: Bradley et al (1999), translation of a textbook from Prentice-Hall; D: Charron & Parent (2004).

All these textbooks have appeared in the programs of cégeps¹ in Montréal and cover an interval of 10 years, which could help us in this first approach. In terms of space used for numerical series (this is, the number of pages used for numerical series divided by the total number of pages), the results are:

<table>
<thead>
<tr>
<th></th>
<th>A: 9,59%</th>
<th>B: 14,63%</th>
<th>C: 17,42%</th>
<th>D: 17,05%</th>
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<tbody>
<tr>
<td>A</td>
<td>9,59%</td>
<td>B: 14,63%</td>
<td>C: 17,42%</td>
<td>D: 17,05%</td>
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<tr>
<td>B</td>
<td>2 graphs – 1 picture</td>
<td>0,081/page</td>
<td>2 graphs – 5 pictures</td>
<td>0,112/page</td>
</tr>
<tr>
<td>C</td>
<td>52,25 pages</td>
<td>7 graphs – 1 picture</td>
<td>0,268/page</td>
<td>11 graphs – 3 pictures</td>
</tr>
<tr>
<td>D</td>
<td>74 pages</td>
<td>11 graphs – 3 pictures</td>
<td>0,189/page</td>
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and the number of functional graphs and pictures that appear in the section of series is:

<table>
<thead>
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<tbody>
<tr>
<td>A</td>
<td>37 pages</td>
<td>2 graphs – 1 picture</td>
<td>0,081/page</td>
<td>2 graphs – 5 pictures</td>
</tr>
<tr>
<td>B</td>
<td>62,5 pages</td>
<td>7 graphs – 1 picture</td>
<td>0,268/page</td>
<td>11 graphs – 3 pictures</td>
</tr>
<tr>
<td>C</td>
<td>52,25 pages</td>
<td>11 graphs – 3 pictures</td>
<td>0,189/page</td>
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<tr>
<td>D</td>
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More data concerning the contents in the section for series are:

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<tbody>
<tr>
<td>Introduction</td>
<td>Sequences are functions. An infinite sum may be useful to calculate areas [the integrals are studied before]</td>
<td>Series have many applications. The one to write decimal numbers is shown.</td>
<td>Adding infinite terms is a paradox. Many applications: contamination, writing decimals, medicaments in blood.</td>
<td>Purely mathematical. No applications.</td>
</tr>
<tr>
<td>Applications</td>
<td>4 problems</td>
<td>2 problems</td>
<td>More than 10 problems</td>
<td>2 problems</td>
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¹ Centres for pre-university teaching, as they are called in Québec.
4. DISCUSSION
First of all, we can say that there are not many research papers focusing the concept of numerical series and the ways in which students learn it, if we consider the vast amount of research papers existing in mathematics education.

In the research papers we have found, we can state that the results are quite different and that there is no convergence in the approaches used. This may be one of the reasons why we do not find that the textbooks take into account the recommendations from research. This fact seems to coincide with the remark made by Artigue (2001), who points out that the small impact of research on the post-secondary level might be due, among other reasons, to the fact that up to now many efforts have been concentrated on a few of the domains taught at this level. It is necessary, therefore, to diversify these domains of study and more and unified research on the concept of series may have an impact in teaching.

Regarding the textbooks, it seems that they give a relatively great space (more than 10% and sometimes up to 17%!) to the concept of series, but the approaches used tend to be “traditional”. We find in general a great emphasis on the introduction of algorithms and techniques, but the construction of meaning of the concept and the applications are not always clear features of the textbooks. Even if it seems that the textbooks use more graphs, they are not really integrated in the introduction of the concepts, nor historical examples that foster visualisation. This kind of approach might favour a traditional teaching on the part of the teachers.

Our next steps will continue in the revision of research results and the attempt to find a common point in them, in addition to the analysis of more textbooks. One of our hypotheses is that, in general, textbooks do not take into account the results from research, nor the recommendation to use history or to integrate the use of the graphic register. Our exhaustive analysis of more textbooks will help us to verify this hypothesis.

REFERENCES


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