DO PUPILS SOLVE MATHEMATICAL PROBLEMS WITH INSIGHT OR DO THEY ONLY MEMORIZE?

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Abstract: This text describes a part of a project which was carried out within the author’s diploma thesis. It presents its results and interpretation. We focus mainly on questions regarding how last year pupils from primary schools are prepared for grammar school entrance exams. We are also interested in the way they are taught. Do they solve problems (tasks) with insight or do they use only a memorized decision procedure of a particular set of tasks? This text also deals with the question of constructivist approaches to teaching mathematics and project based learning as an educational strategy.

INTRODUCTION AND AIMS

This text describes a part of the project which was carried out within the diploma thesis of the author of this article. The project was focused on two target groups, teachers of mathematics at the primary schools and last year pupils from primary schools (that is Grade 9 in the Czech Republic). Here we would like to focus only on the target group of pupils. We gave the pupils a questionnaire asking for information regarding their free time activities, achieved education of their parents, opinions of mathematics and what is important for pupils in the process of education.

One of the research aims, we would like to present, was focused among others on a question how last year pupils from elementary schools are prepared for grammar school entrance exams. We tried to find out whether they solve the problems (tasks) with insight or use only a memorized decision procedure of a particular set of tasks.

We were also interested in the teacher’s style, that is whether he/she uses transmission techniques and transfers “ready-made information” or whether he/she follows the principles of constructivism.

THEORETICAL BACKGROUND

Constructivism is a philosophy of learning founded on the premise that, by reflecting on our experiences, we construct our own understanding of the world we live in. Each of us generates his/her own "rules" and "mental models", which are used to make sense of our experiences. Learning, therefore, is the process of adjusting our mental models to accommodate new
experiences. Constructivism is often preceded by adjectives, such as radical, social, didactical, etc. What all the different approaches to constructivism have in common is an emphasis on the mathematical activity of pupils: “Learning mathematics requires construction, not passive reception, and to know mathematics requires constructive work with mathematical objects in a mathematical community.” (Davis, Maher & Noddings, Eds., 1990, p. 2). There have been many attempts to use principles of constructivism in teaching (e.g., Jaworski, 1994, a survey of literature is in Confrey & Kazak, 2006). In the Czech mathematics education, principles of didactic constructivism have been formulated (Hejný, Kuřina, 2001, shortened):

1. Mathematics is understood as a specific human activity, not only as a result of activity.
2. An important component of mathematical activity is searching for relationships, solving of the problems, creation of concepts etc.
3. Knowledge is not transferable. It is constructed in our minds.
4. Construction of knowledge arises from experiences.
5. Incentive surrounding is the basis for educational process.
6. Interaction contributes to the development of knowledge construction.
7. It is very important to use different representations and structural development of mathematical word.
8. Communication and cultivation of different mathematical languages are as well highly important.
9. Educational process should be evaluated from three points: understanding of mathematics, gaining the skill, application of mathematics.
10. Knowledge based on reproduction leads to mechanical understanding.

We believe, that project based teaching is one of the ways to fill the above principles of constructivism. It is one of the possible methods to lead pupils to independent and creative work, where the teacher has a role of a coordinator (or facilitator) and pupils are responsible for their learning. They learn mathematics by their own doing, by their own activity.

Projects as an educational strategy have many advantages, for example:

- Pupils have an opportunity to manage their own educational process, which increases the responsibility, own initiative, creativity and motivation.
- The formulation of the problems develops analytical skills
- Pupils can see mathematics from a different perspective
- Projects develop communication and cooperation

However, project teaching is not often used in Czech schools.

**HYPOTHESIS AND RESEARCH QUESTIONS**

Our questions were divided into two groups. The first group dealt with teachers, the second one with pupils. In this piece of work I focused on questions related to pupils, especially on the following:

1. What are free time activities of the pupils? (Do they have something in common with mathematics?)
2. What are their opinions of mathematics and what is important in the process of education for them? (How do they display in pupils' results?)
3. How are last year pupils from elementary schools being prepared for grammar school entrance exams?

On the basis of observation, discussions with pupils and teachers of mathematics and from sample entrance exam tests we formulated the following hypotheses:

1. Pupils prefer the usage of algorithms and facts. They do not develop their logical thinking; they proceed within the frame of acquired algorithms. They poorly connect knowledge from different fields with the knowledge from mathematics, they do not think about problems.

2. In entrance exam tests, tasks dominate which test more or less the level of numeric skills rather than the development of intellectual abilities. Therefore, the teachers are partially obliged to prepare their students to solve the tasks with the plain usage of algorithms and memorized formulas. They have no choice but to drill students and they miss time that could be spent on various and more interesting kinds of work where they could stimulate the development of constructive and creative thinking of their students.

3. Pupils who like/are interested in mathematics, and/or spend their free time by doing activities related to mathematics, and/or depend on their results in mathematics, reach better results.

**METHOD**

For the project, which was inspired by TIMSS questionnaires, we used a list of questions on a sample of Grade 9 pupils from elementary school (132 respondents at the age of 14 to 16 years. 46% of questioned pupils were boys and 54% girls, average age – 14.79 years). All questions were closed and pupils chose the answer by ticking one of several offered possibilities (multiple-choice questions). All questions were scaled.

For example:

During lessons in mathematics pupils …

<table>
<thead>
<tr>
<th></th>
<th>I strongly agree</th>
<th>I agree</th>
<th>I do not agree</th>
<th>I strongly disagree</th>
<th>Not specified</th>
</tr>
</thead>
<tbody>
<tr>
<td>do not pay attention to the teacher</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>work and do not disturb</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>work exactly according to the teacher's instructions</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
</tbody>
</table>

Next, we gave the pupils a set of four tasks chosen from grammar school entrance exams from previous years. The types of the tasks were selected in the way to present tasks which appear in grammar school entrance exams most frequently. At the same time we tried to find out tasks which did not focus just on a simple usage of algorithm but in which the pupils have to connect knowledge from different fields with mathematical knowledge.

For example:

A car runs at the speed of 60 kilometres per hour and covers the distance between towns A and B in 2 hours and 15 minutes. How long is the line representing the real covered distance on the map with ratio scale 1: 1 500 000?
The project was divided into several parts. First of all, we asked the Grade 9 pupils to answer the list of questions, and then they worked on the four tasks described above. After we corrected the tests, we asked several students to express their opinion of their solutions, their trains of thought, possible problems which appeared during their work, etc.

RESULTS

From the first part of the project it emerged that it was very important or important for pupils as well as for their parents to have good results in mathematics, Czech and foreign language and sports.

The questions concerning the pupils’ work during the lessons offered us important information. About 70 percent of questioned pupils said that they worked hard in lessons, did not disturb and followed the teacher’s instructions. On one hand it is good that the teacher has an authority, on the other in the classroom it impends the creation of formal atmosphere for work.

The fact that pupils follow the teacher’s instructions precisely can contribute to reality that teaching becomes a transmission of “ready-made information” not creative work that is so important for their own understanding and constructive approaches.

Nearly 100 percent of pupils devote their time to mathematics only to pass entrance exams, 80 percent to get their dream job, 70 percent for their own delight and 50 percent to please their parents.

Most pupils agree with the statement that one needs talent to do mathematics and hard work outside the school, often at home. For more than half of questioned pupils mathematics is not their favourite subject. 67 percent says that they could like mathematics if it was not so difficult. These answers confirm the results published in the report about TIMSS research (Third International Mathematics and Science Study, 1996).

In the second part of our project pupils worked on four tasks chosen from grammar school entrance exams from previous years. As we have already mentioned, the types of the tasks were selected in the way to present most frequent tasks from grammar school entrance exams. At the same time we tried to find out tasks which did not focus just on the usage of algorithms but in which pupils have to connect knowledge from different fields with mathematical knowledge.

After pupils solved the set of tasks and subsequent correction of the tests we asked some pupils to go through the test again and summarize what was the most difficult/easy for them, to clear up their train of thoughts etc.

We would like to present a sample of interview with one of the pupils. (This boy covered nearly all the tasks. He solved successfully the unit conversion and real covered distance.)

T: How did you find out that 1 centimetre on a map is 15 kilometres in reality?

P: Well, I know that I have to cross five zeros.

T: And why just five zeros?

P: Because our teacher told us.

T: Oh well. I have another question. Why did you divide 15 by 135?

P: Because I remembered that I have to divide these two numbers but I was not sure how.

T: What about your result? Wasn’t it a bit strange?
P: I don’t know. But usually there are “nicer” numbers in the results.

Mostly, pupils told us that they could remember some similar problem but they did not remember the whole procedure. It emerged from the discussion that pupils’ knowledge was often only mechanical because they kept in their minds an algorithm or its part but they did not understand it and were not able to use it properly.

Finally, we can say that the class work at the surveyed schools in the Czech Republic is mainly focused on transmission of “ready-made knowledge” and algorithms which are practised on examples. Their knowledge becomes mechanical (formal), they are not able to use it outside the solved examples.

CONCLUSIONS

We cannot make conclusions about the state of teaching in the Czech Republic on the basis of such a small sample of pupils and schools. What we can say, though, is that pupils’ approach to the solution of tasks is often based on looking for a similar problem in their mind and using the algorithm used previously. This is often done without first trying to get an insight into the problem. The interviews with pupils confirmed this and the above extract is an example of such a pupil. We believe that one of the causes of this state is the teachers’ need to prepare pupils for their entrance examinations which mostly test only the proper use of algorithms. The teaching then becomes very formal and drill oriented. In my further research as a PhD student, I will focus on project teaching and how it can contribute to the desirable change of teaching (towards the principles of constructivism).

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