USING NEW TECHNOLOGIES TO HELP STUDENTS BUILDING THE MEANING OF THE CONCEPT OF FUNCTION

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ABSTRACT

In this paper we discuss an example of activities, which involved 9th grade Italian pupils, aimed to help students in constructing a meaningful concept of function with the use of new technologies.

INTRODUCTION

The educational tradition in Italy insists on the symbolic aspects of the teaching and learning of the function concept, particularly on algebraic manipulations in order to solve equations and inequalities. So the specific aspects of Calculus are only introduced in the last years of the secondary school. Today, it is easier to trace a diagram or manipulate an algebraic expression by means of software. As a consequence, the didactical praxis, the sequence of arguments and the level at which to introduce them, must be reconsidered. The use of ICT in order to introduce mathematical concepts may change their meanings. According to Chevallard, in fact, the mathematical objects come out from a series of problems and practices useful to solving them (Chevallard, 1992). Since practices depend on the instruments we use, we think that new tools may give rise to new meanings for the mathematical objects (Paola, 2004). We are convinced that the use of DGS and CAS can prepare an ideal ground for the construction of deep cognitive roots for the fundamental concepts of Calculus. In this paper we discuss an example of activities that may help students in constructing a meaningful concept of function (Paola, 2005; 2006).

THE TEACHING EXPERIMENT

The experiment has involved 9th Italian pupils who attend a scientific course with five mathematics classes per week, including the use of mathematical software in computer laboratory. These students are familiar with problem solving activities, as well as with interactions in small groups. During math lessons the students are used to face and solve problems with the employment of a new and still not commercialised (in Italy) Texas Instrument software, TI-nspire, as long as the class takes part to a research project (coordinate by Prof. Ferdinando Arzarello, Turin research group chief) about the use of this software. Here we describe just a part of the resolution process of most capable students. The problem posed to students was the following:

In the Zumbak Republic there are two villages that we call A and B. The first one is 4 km far from the side of a very deep and tight river and the second one is 7 km far from the same side of the river. An international company designs the construction of a pipeline: a straight tube leaving from the village A, catching up a point of the river and leaving from it, again straight on, so to catch up the village B. In this way, water reaches the two villages. The minimum of the total length for the two tubes is the unknown to be searched for.
At the beginning we only ask the students to think individually of the problem for five minutes without using paper and pencil or software. In the next ten minutes they can discuss in couples on strategies to solve the problem only using paper and pencil. Finally, they have eighty minutes to work in small groups also with technological instrument and to prepare a final document. We discuss only the third phase (the work with TI-nspire).

At the beginning the exploration is essentially perceptive. Students move P and see that PA+PB changes (Fig. 1). There is no correlation with a Cartesian Graph, it’s only a drawing. In this process of exploration students realize, sometimes with the teacher suggestion, that point P have to be found on the HK segment, where point H and point K are the projection of the points A and B on the straight line. Some student writes PA + PB; then with the "calculate" tool estimates PA+PB and observes the variation of the number "PA+PB" according to the variation of the position of P (Fig. 1). In this way the students make a conscious and explicit exploration on the numerical register. They understand that PA+PB changes with the variation of P position. The more skilled students with TI-nspire use the tool "activate variable" in order to create the two variable "HP" and "length", that is PA+PB (Fig. 2).

In this phase the students are able to carry out a refined exploration; they realize that there is a functional relationship between the length AP+PB and the length HP. Students can now use the TI-nspire resource “automatic capture of data”. The length of AP+PB and the length of HP are collected in two columns of an electronic sheet, where the first column represents the independent variable and the second one the dependent variable, as shown in Fig. 3. The students collect the data automatically and estimate the first order differences of the independent variable to verify if it varies with a constant step. Later they try to evaluate first and second order differences of the dependent variable "length", not only finding with excellent approximation the position of P which minimizes the length, but also finding information on the monotony and the concavity of the function length = length (HP). Finally they move to the diagram in order to see with their own eyes what they already have imagined (Fig. 4).
CONCLUSIONS

According to us there are intriguing relationships between some TI-nspire resources (in particular we are referring to the variable declaration, to the automatic capture of data and to the symbolic manipulation allowed by the TI-nspire’s spreadsheet) and the possibility of prematurely exposing the students to the symbolic aspects of mathematical objects.

We would like to hint at an episode that is paradigmatic if referred to that aspect. Many studies in mathematics education noted the complexity and the students’ difficulty with the function concept (Bloedy-Vinner, 1995; Gray & Tall, 1994; Akkoc & Tall, 2002; Wilson 1991). An experience, certainly common to many teachers, is students’ mistakes in calculating $f(x+h)$, given functions such as $f(x)=x^2$: in fact also upper-level students write $f(x + h) = x^2 + h$. Besides, many students have difficulties also in using a correct recursive notation to write expressions such as

$$
\begin{align*}
f(1) &= a \\
f(n) &= a + 0.4f(n-1)
\end{align*}
$$

(1) in order to express the evolution of a discrete dynamical system.

In particular students don’t recognize the functional relationship between the independent variable $n$ and the dependent variable $f(n)$. These difficulties were also noticed in students who are able to use electronic spreadsheets in calculating sequences of values as (1) or in expressing it by natural language. Instead, students who generally use TI-nspire at home and during the math classes haven’t any difficulty in understanding and using either recursive notation or functional notation as $f(g(x))$, given $f(x)$ and $g(x)$.

We suppose that TI-nspire may improve students’ understanding in mathematics symbolic aspects, mainly thanks to the following software functions:

a) the automatic capture of data (in a spreadsheet students usually generate numbers starting from a formula, hence, if they have tables of data in the same environment, they immediately think of the generating formula);

b) the possibility of making symbolic calculations with the electronic spreadsheets.

REFERENCES


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