STUDENTS’ DISCUSSIONS ON THEIR OWN MATHEMATICAL ACTIONS AS A REFLECTIVE MATERIAL FOR LEARNING AND RESEARCH ON MATHEMATICAL ACTIVITIES

Petros Chaviaris & François Kalavasis
University of the Aegean, Greece

THEORETICAL BACKGROUND

The purpose of this study is to investigate the characteristics and the possibilities of a mathematical activity emerged by the organization of students’ metadiscursive reflection. In this contribution we are presenting the evolution of the students’ mathematical discussion, as they observed their own discussions in the video. Our analysis of the situation is oriented in two directions a) the students’ discussion as a type of learning activity in mathematics and b) the correlations between students’ metadiscursive reflection and their self regulation in mathematics as a research activity.

Our approach is based on the assumption that learning mathematics is a complex phenomenon that it can be described in a systemic way, as a subsystem with interactive factors. This subsystem evolves from the process of re-equilibration (deconstruction and reconstruction of representations) of the interactions that they relate the learning subject with the mathematics, with the school system, with the family system, with the social environment, and with himself (self-estimation).

According to the above considerations mathematical activity has to be viewed not through a bipolar model: the mathematical problem and its solution from the student, but as a multipolar cyclic model: the members of a group that try to engage in a mathematical problem, discuss about their contributions, evaluate their activity and redefine their relationships and behaviors. In the first model the obstacles or the difficulties provoke didactical interventions like metacognitive discussions, cooperation etc. In the second one the metacognitive and metadiscursive processes are embodied in the group’s mathematical activity.

From this point of view, we can focus on the study of social interaction among classroom members as they do mathematics, so that their mathematical activity is socially defined. Our main theoretical references are Brousseau’s concept of didactical contract (1989), sociomathematical norms (Yackel &Cobb,1996) and metadiscursive rules(Sfard, 2001). We try to describe in which way models of sociomathematical interaction that take place in the classroom, can influence members’ mathematical activity and how, at the same time, students can optimize their cognitive process. Towards this effort, we have to use fragile concepts as metacognition, reflective and metadiscursive process.

A lot of research has focused on the way that students can become aware of their own thinking and actions during their mathematical activity. Büchel (1990) distinguishes metacognition in two dimensions: the metacognitive (métaconnaissance) and the controlling (contrôle exécutif). Allal and Saada-Robert (1992) named these dimensions metacognitive knowledge (connaissances métacognitives) and metacognitive regulations (régulations métacognitives). Zimmerman (1995) has stressed that the interpretation of students’ self-
regulation has to be treated as a complex interactive process influenced by students’ self-beliefs–system.

Until now, many researchers have pointed out the positive influence of the cooperative metacognitive instruction on the learning of mathematics. The focus of this type of instruction is on the self-addressed questions that concern the cognitive aspects of the mathematical activity like “which is the whole problem?”, “which are the similarities and differences of the new problem with the previous?” “which is the appropriate strategy to solve the problem?” etc. (Schoenfeld, 1992; Mevarech & Kramarski, 1997; Kramarski et al., 2002). The question of how the social interaction is connected with the students’ self-regulation process as they negotiate their mathematical ideas in the classroom needs more investigation.

**METHOD**

This presentation is a part of a broader research program realized in a fifth grade of a typical public school of Athens in 2003-2004 and it lasted six months. Participants were 18 students (9 boys and 9 girls). The students worked in small groups (2 students in a group) 4 times per week during the mathematics teaching that the teacher of the classroom realized. The mathematical topic that they discussed during the research program concerned the concept and the operations of fractions.

Initially, the students were interviewed about their beliefs for their own role, others’ role, the general nature and the goals of mathematical activity. The students worked in groups and their cooperation was videotaped once a week. After a session of cooperation the members of each group participated in a meeting with the researcher. During this meeting, the students observed and discussed on issues concerning their videotaped cooperation. At the end of the program, the members of the group were interviewed about their own role and the others’ role in mathematics.

The data consisted of the videotaped recordings of the small-group students’ work in mathematics, the recordings of the students’ discussions about their own videotaped cooperation and the transcripts of the tape-recorded clinical interviews conducted with each student at the beginning and at the end of the program. The analysis of the data was realized in two levels: the discursive and the meta-discursive one. In the first level, discourse analysis of the small-group students’ engagement in classroom mathematical activities was based on the interactivity analysis that Sfard and Kieran have developed (2001). In the meta-discursive level, the tape-recorded students’ discussions about their own videotaped cooperation were analyzed according to: a) the way that the students assessed their cooperation, b) the critical moments of their interaction and c) the targets they set for their next cooperation.

**RESULTS**

1. **The students’ profile**

   According to the data from the initial interviews that realized in this class, 8 students answered that they wanted to cooperate with their classmates in mathematics, 4 students answered that they wanted to work alone, and 6 students answered that they wanted to cooperate only with their teacher. Furthermore, the students’ justifications about their preferences revealed the following beliefs that prevented the development of cooperation in mathematics:

   a) mathematical knowledge is acquired with personal effort (e.g. “if someone doesn’t work on their own, they cannot understand mathematics”),
b) different ideas in mathematics cause confusion and create difficulties in understanding (e.g. “everyone can express their opinions and they might quarrel about which one is right as they are unable to make head or tail of it”),

c) classmates’ errors in mathematics negatively influence student’s thinking and prevent their learning (e.g. “my classmate may make an error so I will be confused”) and

d) the exposition of a student’s thinking to his/her classmates does not protect his/her self-image.

2. The evolution of the students’ metadiscursive reflection in mathematics

The study of the ways that the students assessed their interactions in mathematics and set targets for their improvement as well as the investigation of the critical moments of their reflections through the observation of their video-recorded cooperation revealed three levels of development concerning the students’ metadiscursive reflection:

a) Focusing on the achievement of personal goals

At the first level, the assessment of their cooperation mainly concerned the quantitative characteristics of their interaction (how much time they talked to each other). Moreover, their assessment was connected with the result of their personal effort to solve the problem.

The critical moments of their metadiscursive reflection focused on their partner’s actions, which every student considered obstacles for his/her personal mathematical activity. These actions concerned their partner’s suggestion about the way they had to think as well as the partner’s disagreement for a solution.

The group’s members self-regulation was mostly connected with the increase of their interaction time and the avoidance of the partner’s actions.

b) Focusing on partners’ responsibility

At the second level, the students’ metadiscursive reflection began to focus on the qualitative characteristics of their discussion. The students began to assess their cooperation according to the correctness of the ideas offered from their partner for the solution of a problem.

The critical moments concerned: a) the role of their existing beliefs of their partner’s ability in mathematics, b) the failure to investigate different ideas and c) the absence of explanation for a mathematical decision.

The targets that the members of each group set for the improvement of their cooperation concerned concrete actions that were usually addressed to their interlocutor: offering an idea and explanation of the proposed idea from their partner.

c) Focusing on mutual responsibility

The students began to reflect on their own responsibility about the development of their interaction. They assessed their cooperation according to their own efforts to explain their way of thinking to their partner in order to make a common mathematical decision. The critical moments of their reflection were related to the fulfillment of their obligation to handle their different ideas by respecting their interlocutor’s way of thinking. The self-regulation concerned their own personal actions, that is the actions that a student had to do in relation to his/her interlocutor thinking in order to improve their interaction. This fact revealed the mutual responsibility that the students began to develop in order to find common solutions in mathematics.
3. The evolution of the students’ cooperation in mathematics

The following table presents the evolution of students’ intentions as they cooperate in mathematics classroom.

<table>
<thead>
<tr>
<th>Levels of metadiscursive reflection</th>
<th>Intentions</th>
</tr>
</thead>
</table>
| 1. Focusing on the attainment of personal goals | - to present the result of his/her solution  
- to take personal decisions  
- to make a suggestion without justification  
- to accept his/her partners’ solution without investigation |
| 2. Focusing on partners’ responsibility | - to demand a proposal from their partner  
- to assess their partner’s proposal  
- to ask for an explanation from their partner |
| 3. Focusing on mutual responsibility | - to explain his/her proposal without their partner’s request  
- to justify his/her solution  
- to make a common decision for the solution of the problem |

At the end of the program, the students’ beliefs for the role of cooperation in mathematics had begun to change. Most of the students considered that the cooperation with their classmates in mathematics helps their mathematical activity (fourteen students). They considered that their classmates’ different ideas were fruitful to their mathematical discussions. Furthermore, they did not inculpate errors as obstacles for their cooperation in mathematics.

**DISCUSSION**

According to the above results we could discuss on the two components of the students’ metadiscursive activity in mathematics: a) the benefits of the concrete mathematical activity to the improvement of learning and teaching mathematics and b) the possible revelations in the relation between students’ metadiscursive reflection and their self regulation in mathematics.

The changes of students’ behavior in the classroom, like thinking on their partners’ ideas, making a common decision, explaining and justifying their solution to their interlocutor, demonstrate the benefits of this activity for the learning of mathematics. Furthermore, the changes of the relationships between the group members, like discussing about their responsibility of an error, thinking about obstacles in their cooperation and posing targets and concrete actions for its improvement, demonstrate the benefits of this activity for the students participation in teaching mathematics.

The students’ reflection on their mathematical discussions revealed that a lot of factors concerning the students’ social behavior (expectations, intentions, and beliefs) in mathematics, influence their self regulation. The negative consequences of these factors can reduced through students’ metadiscursive reflection.

The designing of mathematical activities that take into account the understanding of the correlations among the psychological, anthropological and social perspectives of the learning and teaching mathematics allows a holistic approach by researchers, teachers and learners.

**REFERENCES**


