TEACHER HELP FOR COLLABORATIVE MATHEMATICAL LEVEL RAISING

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Abstract: A field study with 16-year-old students in senior general secondary education was executed with the following research question 'Do students working in pairs on investigation tasks with the computer attain more mathematical level raising when they are supported by a teacher who stimulates their interaction (process help) than when they are supported by a teacher who gives mathematical help (product help)?' Students in both conditions improved, but the two types of help showed no significant difference in level raising. Also, students in both conditions had serious problems with the learning materials, and wanted the teacher to explain and correct more. For students at this level of education, learning with investigation tasks in small groups appears to be very difficult.

INTRODUCTION

Guiding students by teachers in a collaborative learning process means creating optimal conditions in which they can learn from one another. Within the context of a mathematics lesson, the issue is whether the teacher should stay away from the mathematical content of what students are saying and focus on their interactions, or whether the teacher should provide mathematical hints when students ask for help. Generally speaking, should a teacher leave the learning content to the students and keep watch that the collaborative learning process stays under way, or, conversely, should the teachers take part in the collaboration by focusing on the content of what has to be learned?

Collaborative learning is fruitful for students if it gives them the opportunity to sharpen their own thinking (Webb, 1991; Dekker & Elshout-Mohr, 2004). The quality of students' interactions is influenced by several factors, such as the learning materials, the composition of the groups, and the role of the teacher. In this study (Pijls, 2007; Pijls, Dekker and Van Hout-Wolters, in press) we will focus on this last aspect, building on a study performed by Dekker and Elshout-Mohr (2004). They compared the effect on mathematical level raising by a teacher who focused on the students' interactions with the effects by a teacher who paid attention to the mathematical content of the students' work. We operationalize the learning of mathematics as attaining mathematical level raising (cf. Van Hiele, 1986). The collaboration with peers is expected to stimulate reflection and to induce level raising. Dekker and Elshout-Mohr (1998) developed a process model in which they describe the interactions between students that lead to mathematical level raising. They distinguish students' key activities: to show one's work; to explain one's work; to justify one's work and to reconstruct one's work.

Two types of teacher help were compared:

- **product help**: teacher's help focused on the quality of the mathematical product students are working on (mathematical hints).
- **process help**: teacher's help focused on the quality of the interaction between students.
METHOD

We executed this experiment in order to answer the following question:

Do students in grade ten of senior general secondary education who work in pairs on investigation tasks with the computer attain more mathematical level raising when they are supported by a teacher who provides process help than when they are supported by a teacher who provides product help?

This question was answered by a pretest and post-test analysis. We expected the process help to lead to more mathematical level raising, since process help promotes students to perform key activities (to show, explain, justify, and reconstruct one's work), which in turn leads to mathematical level raising. Based on Dekker and Elshout-Mohr's results, we expected product help to disturb the process of mathematical level raising, since it keeps students from executing key activities and hence, denies them an opportunity for mathematical level raising. Two mathematics classes and their teachers from the fourth year of senior general secondary education participated in our experiment. In general, these students do not see themselves as mathematicians. They have chosen a set of subjects with Economics, and Math is obligatory for them. It concerned 52 students (53 minus one student who did not make the post-test). One week before the experimental lessons started, the students made a pretest consisting of 12 open-ended paper-and-pencil questions, with a total maximum score of 46 points. We used the results of the test to assign participants to two condition groups that were comparable with regard to their pretest results and that contained an equal number of students from each class.

At the end of all the lessons the students made a post-test. The open-ended questions in this post-test were very much comparable to the pretest. The learning materials consisted of a computer simulation with accompanying collaborative investigation tasks (to be solved with paper and pencil) on the subject of Routes and Probabilities. Any domain of science or mathematics could have been taken into consideration, but this part of probability theory was chosen because it has both a visual and an abstract component. The computer simulation was composed of simple (gambling) games with the underlying structure of a grid, a mathematical concept for counting possibilities and calculating probabilities. An example of such a game is TIC-TAC, as shown in figure 1.

![TIC-TAC](image.png)

Figure 1. The game TIC-TAC.

Students were asked if they had a winning strategy for the game TIC-TAC, and then they got the investigation task:

‘What is the probability for a ball to end up in the node with 100 points?’.
The idea of this question was that they would realize that the probabilities to end up in a certain box were not equal for all. In the paper-and-pencil task that followed, both students (each drawing on a separate working sheet) had to draw possible paths in the TIC-TAC gameboard from the starting point towards the box with the 100 points. Subsequently the students were asked to describe the paths in terms of RURUUUU (with an R meaning ‘to the right’ and an U ‘upwards’) to compare them with their peer and to formulate what struck them. In the following tasks the concept of ‘counting routes in a grid’ was developed.

During two sessions we prepared the two teacher roles. The ‘process teacher’ provided no content-related help; he stimulated the interaction between students. The initial instruction of process help to all students was:

I will not help you in content, but I want you to discuss a lot, to show your work to one another, to explain to one another, that's what you learn from, and criticize one another, so that the work improves…

An example of process help to students who asked for help:

I want you to decide by yourselves, think about it, be critical towards one another's ideas.

The ‘product teacher’ only provided content-related help when he was asked for it and he gives no hints for the interaction. The initial instruction of product help to all students was:

You will work on these tasks by yourselves; I am here to assist you.

Two examples of product help to students who asked for help:

Do you understand the picture?
Yes, 20 % of this, yes, both lines end up in the same box.

The utterances of both teachers during the experimental lessons were audio-taped and transcribed in order to check whether the help of the teachers was executed correctly. Furthermore it would give us the opportunity to analyze the help of the teacher and the reaction of the students. Did they perform key activities?

RESULTS

In general, the students of both conditions experienced the help of the teacher as very different from the normal situation. Although they were used to working independently, they expected the teacher to help them by providing explanations. In the process group, the teacher did not do this at all. With regard to a difficult task in the learning materials, many students arrived at an impasse in their learning process. This task was hard for them and many students were discouraged. The process teacher encouraged them to trust their own thinking. In the product group, the teacher always asked students to show their work before he gave a mathematical hint. For some students, this was new, since they expected the teacher to give explanations when they told him: 'I don't understand.' During the second lesson, many students in the product condition arrived at a difficult task in which they had to reflect on the previous tasks and use the information in a more abstract way. This was hard for them and they asked the teacher for help. He could not help them all at a time and some of them had to wait for a long time. This made the students feel that the teacher did not provide enough help.

We analyzed the results of the pre-test and the post-test in order to determine whether the students attained mathematical level raising. Each of the tests consisted of 13 questions, with a
maximum score of 46 points. As we see in Table 1, the two condition groups were comparable on pre-test results (that is how they were constructed) and the mean score of 9.31 points shows that the students had sufficient prior knowledge at their disposal.

Table 1.

<table>
<thead>
<tr>
<th>Condition</th>
<th>N</th>
<th>Pretest Mean</th>
<th>SD</th>
<th>Post-test Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process help</td>
<td>26</td>
<td>9.38</td>
<td>4.92</td>
<td>13.46</td>
<td>8.80</td>
</tr>
<tr>
<td>Product help</td>
<td>26</td>
<td>9.23</td>
<td>4.35</td>
<td>15.12</td>
<td>7.10</td>
</tr>
<tr>
<td>Total</td>
<td>52</td>
<td>9.31</td>
<td>4.60</td>
<td>14.29</td>
<td>8.00</td>
</tr>
</tbody>
</table>

The difference between pre- and post-test in both conditions showed that on average all students' learning results improved (t-test for difference between pre- and post-test: \( t = 6.367, \ df = 51, p < .001 \)). We compared the results of the post-test for the process and the product group with a t-test. The test showed that the results of the two groups did not differ significantly (\( t = .75, \ df = 50, p > .05 \)). This was confirmed by a covariance analysis with the pre-test as a covariate, which showed no significant difference in the two means of the post-test between the two conditions (\( F(1,49) = 1.43, p > .05 \)).

At the end of all the lessons we asked the students whether they had any hints to improve the lessons. In the process groups seven out of 13 pairs answered that they wanted to have more explanations. One of these pairs mentioned: ‘We have learned nothing, because we couldn’t ask anything and we only learn if we have made some mistakes and correct them and then make the test.’ Although another pair, consisting of two high-level students, said: ‘We have learned better to think for ourselves.’ In the product group, eight out of the 13 pairs answered that they wanted to have more explanations. One of them said: ‘We did not learn a lot, yes, we learned to cooperate and to consider the problem in a different way and to look for solutions. We did not get enough explanation and help.’

CONCLUSIONS AND DISCUSSION

We investigated what kind of teacher help was most fruitful for students who worked collaboratively with especially developed learning materials and computer simulation. We compared help from the teacher that kept away from the mathematical content and stimulated the interaction process between students with help that addressed the mathematical content of the students' work. It was questioned which kind of help would most lead to mathematical level raising. It turned out that there was no difference in level raising between both conditions. This result differed from the findings of Dekker and Elshout-Mohr (2004). They conducted a similar experiment with a different mathematics task and different type of education, namely pre-university. In their study, teacher help that stimulated the interaction between students and kept away from the mathematical content led to more mathematical level raising for the students. In both studies, students got the same kind of instruction and students were not used to this kind of teaching practices. Students in pre-university education easily grasped the opportunity to solve the mathematical tasks with one another (Dekker & Elshout-Mohr, 2004), while in this study the students in senior general secondary education wanted the teacher to explain the mathematics to
them. It is clear that students in this type of education are strongly dependent on the teacher’s explanations and the teacher’s external regulation, while the learning materials presuppose an investigative approach from the students.

When the students were struggling with a difficult task, both the process teacher and the product teacher wanted to provide more explanations than their role allowed them to. The process teacher wanted to give some mathematical hints and the product teacher wanted to insert some whole-class instructions. Nevertheless, the condition-group in which the teacher gave explanations did not perform significantly better than the one in which the teacher did not explain the content matter.

Concerning the possibility to combine process help and product help, we are afraid that if both types of help are offered, students will relapse into their habitual behaviour of leaning on the teacher for explanations. Especially since these students do not see themselves as full-fledged mathematicians. So, we think that a combination of both types of help should be considered very carefully.

REFERENCES


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