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Can we "trace" the phenomenon of compartmentalization by using the implicative statistical method of analysis? An application for the concept of function

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Résumé. L'objectif de cette recherche est d' identifier les difficultés qui concernent la conversion d'un mode de représentation d'une fonction à un autre mode(p.e. graphique, symbolique, verbal) et d'examiner le phénomène de compartimentation, c'est-à -dire l'incapacité à coordonner au moins deux modes de représentation du concept. Nous avons proposé deux tests à 183 élèves de quatrième (élèves de 14-15 ans) et à 404 élèves de seconde (élèves de 16-17 ans). Les deux tests concernent les mêmes relations algébriques mais ils proposent différentes conversions et en particulier la représentation- source de conversions. L'emploi de l'analyse statistique implicative sur les tableaux de données montre une absence de ressemblance et d'implication entre les tâches des deux tests pour les deux groupes d'âge. Ces résultats montrent ainsi que les différents types de conversion entre représentations du même contenu mathématique sont traités de manière distincte soulignant l'existence du phénomène de compartimentation.

1 Introduction: Representations and the concept of function

Nowadays the centrality of representations in teaching, learning and doing mathematics seems to become widely acknowledged. There is strong support in the mathematics education community that students can grasp the meaning of mathematical concepts by experiencing multiple mathematical representations (e.g., Sierpinska 1992; Lesh et al. 1987). An important educational objective is for students to use efficiently various forms of representation in communicating with one another. Multiple representations support students' reflection, help students keep track of their ideas and inferences and also facilitate them to organize their work.

In this paper, the term representations in a restricted sense is interpreted as the tools used for representing mathematical ideas such as tables, graphs and equations (Gagatsis et al. 2002). The representational systems are fundamental for conceptual learning and determine, to a significant extent, what is learnt (Cheng 2000). By a translation or conversion process, we mean the psychological process involving the moving from one mode of representation to another (Janvier 1987). The ability to identify and represent the same concept in different representations, and flexibility in moving from one representation to another, are crucial in mathematics learning, as they allow students to see rich connections, and develop deepen understanding of concepts (Even 1998).

A concept of fundamental importance in the learning of mathematics, which is considered to be a major focus of attention for the mathematics education research community is function (e.g., Evangelidou et al. 2004; Gagatsis and Shiakalli 2004; Sfard 1992; Sierpinska 1992; Vinner and Dreyfus 1989). The understanding of functions does not appear to be easy, given the diversity of representations related to this concept (Hitt 1998). Each one of the various representations of the notion of function points out a different aspect of the concept and all these together contribute to a global representation of it, and none of them separately can describe the notion entirely (Kaldrimidou and Ikonomou 1998; Gagatsis and Shiakalli 2004). That is why some students' difficulties in the construction of a concept are linked to the restriction of representations when teaching. Thus, as Sierpinska (1992) pointed out, it is important to provide students with a broad spectrum of ways of representing functions in order to prevent them from identifying any of these with functions.

The standard representational forms of the concept of function are not enough for students to construct the whole meaning and grasp the whole range of its applications. Mathematics instructors, at the secondary level, traditionally have focused their instruction on the use of algebraic representations of functions. Eisenberg and Dreyfus (1991) pointed out that the way knowledge is constructed in schools favors mostly the analytic elaboration of the notion which deteriorates the approach of function from the graphical point of view. A reason for this is that in many cases the iconic (visual) representations can cause cognitive difficulties, because perceptual analysis and synthesis of mathematical information presented implicitly in a diagram often make greater demands on a student that any other aspect of a problem (Aspinwall et al. 1997).

Furthermore, most instructional practices limit the representations of functions to the translation of the algebraic form of a function to its graphic form. Kaldrimidou and Iconomou (1998) showed that teachers and students pay much more attention on algebraic symbols and problems than on pictures and graphs. Sfard (1992), on the other hand, found that students are unable to bridge the algebraic and graphical representations of functions, while Markovitz et al. (1986) observed that translation from graphical to algebraic form was more difficult than vice-versa and that the examples given by the students were limited in the graphical and algebraic form. Sierpinska (1992) maintains that students have difficulties in making the connections between different representations of functions (formulas, graphs, diagrams, and word descriptions), in interpreting graphs and manipulating symbols related to functions.

These kinds of behaviour can be seen as an indication for the existence of compartmentalization. The particular phenomenon reveals a cognitive difficulty that arises from the need to accomplish flexible and competent translation back and forth between different kinds of mathematical representations (Duval 2002). Vinner and Dreyfus (1989) referred to the notion of compartmentalization in a broader sense and not just within the context of representations. Specifically, these researchers suggested that compartmentalization arises when an individual has two divergent, potentially contradictory schemes in her cognitive structure and pointed out that inconsistent behavior is an indication of this phenomenon.

In the present study we are focused on the more restricted meaning of compartmentalization, which refers to representations. Specifically, the present paper aims at identifying the difficulties that arise in the conversion back and forth between different modes of representation (i.e., graphic, symbolic, verbal) of the mathematical concept of function and examining the phenomenon of compartmentalization which may affect in a negative way mathematics learning. Previous empirical studies have not clarified compartmentalization in a comprehensive or systematic way. The existence of this phenomenon in students' behaviour could not be uncovered thoroughly or verified by students' success rates neither by students' protocols in tasks involving different representations. These types of data can just be seen as an indication of the existence of compartmentalization. Thus, we theorize that the implicative statistical method of analysis, which reveals the similarity connections and implicative relations between students' responses in the administered tasks, can be beneficial for identifying the existence of compartmentalization in students' behaviour. Our basic conjecture is that compartmentalization appears when the following conditions appear: First, students deal inconsistently or incoherently with different types of translation of the same mathematical knowledge from one mode of representation to another; and second, success in one mode of representation or type of conversion of a concept does not entail success in another mode of representation or type of conversion of the same concept.

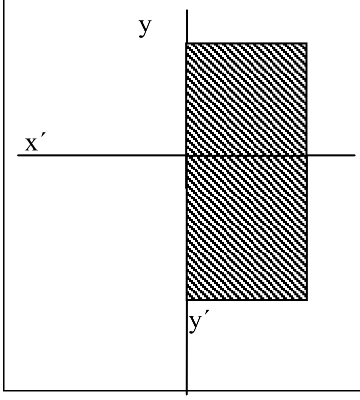
2 Method

2.1 Participants

The sample of the study involved two groups of students in Greece. The first group consisted of 183 students of Grade 9 (14 years of age). The second group involved 404 students of Grade 11 (16 years of age).

2.2 Instrument

Two tests were constructed and administered to the participants. The first test (A) consisted of six tasks in which students were given the graphic representation of an algebraic relation and were asked to translate it to its verbal and symbolic form, respectively. The second test (B) consisted of six tasks (involving the same algebraic relations with test A) in which students were asked to translate each relation from its verbal representation to its graphical and symbolic mode, respectively. For each type of translation, the following types of algebraic relations were examined: $y<0$, $xy>0$, $y>x$, $y=-x$, $y=3/2$, $y=x-2$ based on a relevant research of Raymond Duval (1993).
The former three tasks correspond to regions of points, while the latter three tasks correspond to functions. Each test included an example of an algebraic relation in a graphic, verbal and symbolic form to facilitate students to understand what they were asked to do, as follows:

Graphic representation	**Verbal representation**	**Symbolic representation**
	It represents the region of the points having positive abscissa.	$x>0$

TAB 1- An example of the tasks included in the test

2.3 Data analysis

For the analysis and processing of the collected data based on students' performance in the tasks, Gras's implicative statistical method has been conducted by using a computer software called C.H.I.C. (Classification Hiérarchique Implicative et Cohésitive) (Bodin et al. 2000). Gras's Implicative Statistical Model is a method of analysis that determines the similarity connections and the implicative relations of factors (Gras 1992; Gras, Briand and Peter 1996; Gras et al. 1996; Gras et al. 1997). For this study's needs, a similarity diagram and an implicative diagram were produced from the application of the analysis for each age group of students. The similarity diagram allows for the arrangement of the tasks into groups according to the homogeneity by which they were handled by the students. The implicative diagram contains implicative relations, which indicate whether success to a specific task implies success to another task related to the former one (see the book: Gras et al. 1996).

3 Results

The means and occurrences of each task of the two tests measuring students' ability to translate algebraic relations from one mode of representation to another are shown in Table 2 and Table 3 respectively for each age group.

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Grade 9 N=183	Test A Graphic→Verbal		Graphic→Symbolic		Test B Verbal→Graphic		Verbal→Symbolic	
	Occurrence	Mean	Occurrence	Mean	Occurrence	Mean	Occurrence	Mean
V1: y<0	139	0.76	102	0.56	132	0.72	109	0.60
V2: xy>0	93	0.51	72	0.39	116	0.63	72	0.39
V3: y>x	67	0.37	46	0.25	83	0.45	91	0.50
V4: y=-x	75	0.41	36	0.20	58	0.32	47	0.26
V5: y=3/2	65	0.36	70	0.38	68	0.37	80	0.44
V6: y=x-2	28	0.15	13	0.07	53	0.29	44	0.24

TAB 2 - Occurrences and means of Grade 9 students for the tasks of Test A and Test B (from the table of occurrences)

Grade 11 N=404	Test A Graphic→Verbal		Graphic→Symbolic		Test B Verbal→Graphic		Verbal→Symbolic	
	Occurrence	Mean	Occurrence	Mean	Occurrence	Mean	Occurrence	Mean
V1: y<0	322	0.80	288	0.71	329	0.81	300	0.74
V2: xy>0	244	0.60	193	0.48	339	0.84	218	0.54
V3: y>x	171	0.42	145	0.36	250	0.62	269	0.67
V4: y=-x	166	0.41	137	0.34	195	0.48	224	0.55
V5: y=3/2	205	0.51	177	0.44	257	0.64	286	0.71
V6: y=x-2	156	0.39	107	0.26	204	0.50	190	0.47

TAB 3 - Occurrences and means of Grade 11 students for the tasks of Test A and Test B (from the table of occurrences)

Even though students in Grade 11 performed better than students in Grade 9 in all of the tasks, some common remarks can be obtained, indicating that similar patterns hold as regards students' abilities to translate algebraic relations from one mode of representation to another. In both grades the first task and the second task of the two tests (y<0 and xy>0) were the easiest tasks, while the sixth task and the fourth task (y=x-2 and y=-x) were the most complex tasks. Furthermore, students of both ages achieved higher outcomes in the conversions starting with verbal representations relative to the conversions starting with graphic representations.
In addition, all the conversions from the graphic mode of representation to the symbolic mode of representation appeared to be more difficult than the conversions from the graphic mode of representation to the verbal mode of representation. Students perceive the latter type of conversion more easily in a level of meta-mathematical expression rather than a level of mathematical expression. In fact, students are asked to describe verbally (in a text) a property perceived by the graph. On the contrary, the conversions from graphical form to the symbolic form entail mastering of algebraic concepts concerning equality or order relations and also using efficiently the algebraic symbolism.
Figure 1 presents the similarity diagram of the tasks of Test A and Test B based on the binary responses of students of Grade 9.

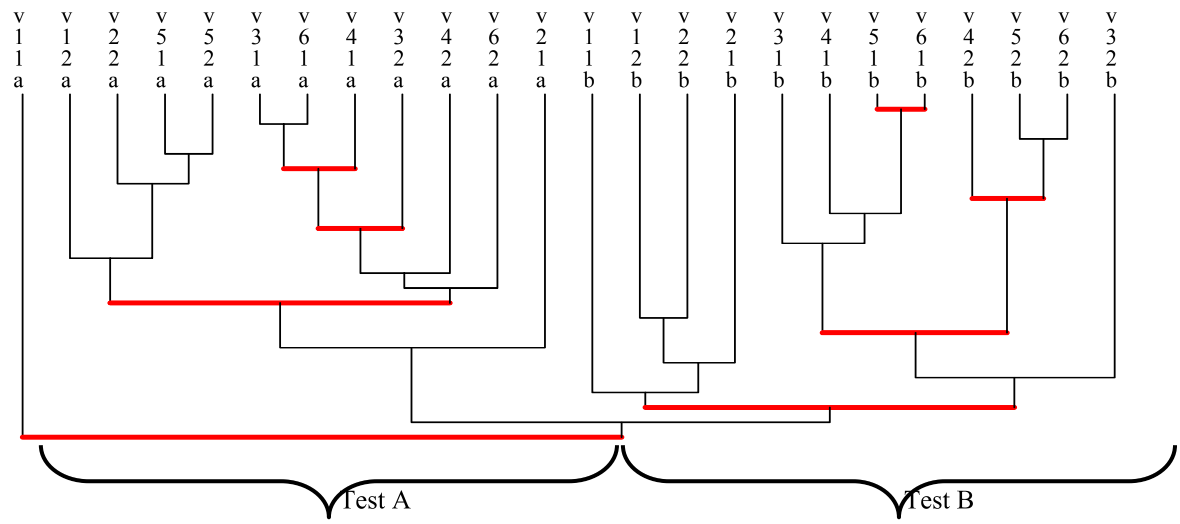

FIG. 1 - Similarity diagram of the tasks of Test A and Test B according to Grade 9 students' responses

Note: The symbolism used for the variables of this diagram (and the diagrams that follow) is explained below.
1. "a" stands for Test A, and "b" stands for Test B
2. The first number after "v" stands for the number of the task in the test
i.e., 1: y<0, 2: xy>0, 3: y>x, 4: y=-x, 5: y=3/2, 6: y=x-2
3. The second number stands for the type of conversion for each test, i.e., for Test A, 1: graphic to verbal representation, 2: graphic to symbolic representation; for Test B, 1: verbal to graphic representation, 2: verbal to symbolic representation.

Two distinct similarity groups of tasks are identified in Figure 1. The first group involves similarity relations among the tasks of Test A, while the second group involves similarity relations among the tasks of Test B. This finding reveals that different types of conversions among representations of the same mathematical content were approached in a completely distinct way. The starting representation of a conversion, i.e., graphic or verbal representation, seems to influence students' performance, even though the tasks involved the same algebraic relations.

The corresponding implicative diagram of Grade 9 students (Figure 2) is in line with the similarity diagram and the above remarks. In particular, one can observe the formation of two groups of implicative relations. The first group involves implicative relations among the tasks of Test B and the second group involves implicative relations among the tasks of Test A. It can be asserted that carrying out a conversion of a function with a particular starting representation did not necessarily imply success in the conversion of the same function with another starting representation. For example, students who accomplished the translation from a graphical representation of an algebraic relation to its verbal representation were not automatically in a position to translate successfully the same algebraic relation from its verbal representation to its graphical form.

Overall, based on the relations included in the similarity and the implicative diagrams for students in Grade 9, it can be inferred that there was a compartmentalization between students' responses at the tasks of the first test and the tasks of the second test, which involved conversions of the same algebraic relations but different starting modes of representation (i.e., graphic and verbal respectively). Students' higher success rates in the tasks of Test B, i.e., conversions starting with graphic representations, relative to the tasks of Test A, i.e., conversions starting with verbal representations, provide further evidence for their inconsistent behaviour in the two types conversions.

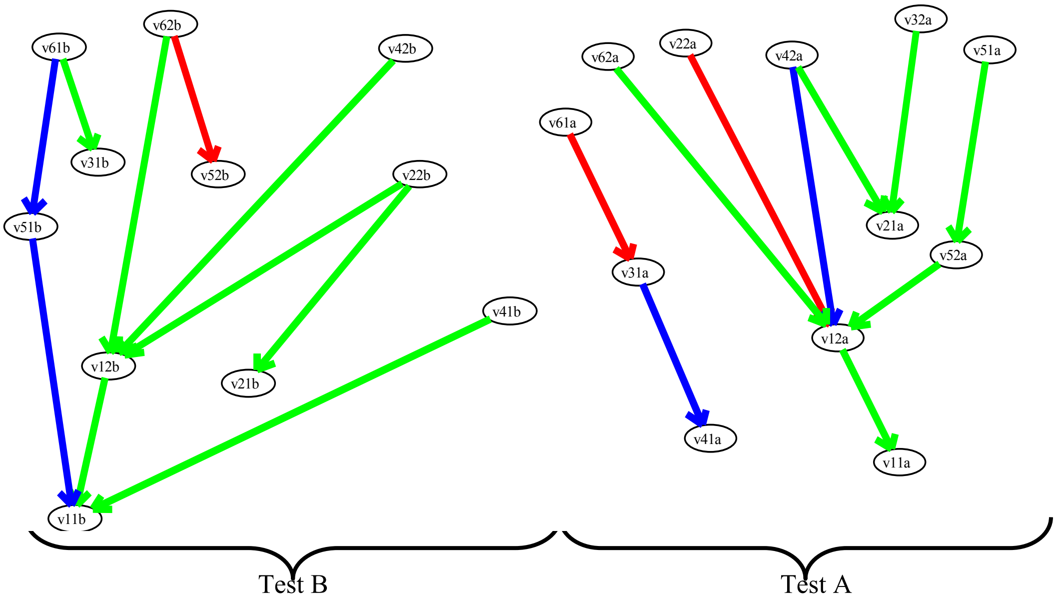

FIG. 2 - Implicative diagram of the tasks of Test A and Test B according to Grade 9 students' responses

The similarity relations within each group of variables are also of great interest since they can be seen as indications of students' way of understanding of the particular algebraic relations. Specifically, two clearly separated sub-groups are identified within the group of Test A. The first sub-group consists of the tasks 1, 2 and 5, while the second sub-group involves the tasks 3, 4 and 6. Tasks 4 and 6 represent functions, while task 3 corresponds to a region of points involving a function. Students handled tasks 1, 2 and 5 differently form the other tasks of Test A, but coherently between them. Tasks 1 and 2 represent regions of points, while task 5 refers to a constant function, which represents a horizontal line, parallel to the abscise axis. Its graphic form is dissimilar to the graphic form of the other functions of the test. That is perhaps the reason why students dealt differently with this task relative to the other tasks involving functions (3, 4 and 6). The latter tasks were approached consistently by the students, influenced by their functional character. Further evidence for the above distinction of tasks is given by students' performance at these tasks. Specifically, students' success rates in the tasks 1, 2 and 5 were higher than their success rates in the tasks 3, 4 and 6. To sum up, the above remarks indicate that the way students carry out a conversion of an algebraic relation from a graphic representation to another representation is influenced by the conceptual (i.e., type of relation: involving function or not) and the perceptual features (graphic form) of it.

As for the similarity group of Test B, two sub-groups are also distinguished. The first sub-group involves tasks 1 and 2 which correspond to regions of points. Tasks 3, 4, 5 and 6 form the second sub-group probably

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due to the fact that they involve functions. This distinction of tasks is supported by the students' different success rates in the tasks 1 and 2 and the tasks 3, 4, 5 and 6. Students performed clearly better in the former tasks relative to the latter tasks. Moreover, within this sub-group strong similarity relations are identified among the tasks involving the same type of conversion, i.e., verbal to graphic representation, or verbal to symbolic representation. It can be inferred that the way students carry out a conversion of an algebraic relation from a verbal representation to another representation is influenced by the conceptual characteristics of it as well as the target of the conversion.

Although students in Grade 11 achieved higher success rates to the tasks relative to students in Grade 9, congruent results were attained by the application of the implicative statistical analysis on the data concerning the former group of students. From the similarity diagram shown in Figure 3 two distinct similarity groups of tasks are formed. The first group involves similarity relations among the tasks of Test A, while the second group involves similarity relations among the tasks of Test B. This finding reveals that students of Grade 11 dealt inconsistently with conversions starting with graphic representations and conversions starting with verbal representations, although they involved the same mathematical content. The former type of conversion appears to be more difficult than the latter one (see Table 3). Thus, like ninth graders' outcomes, evidence is provided for the existence of compartmentalization in eleventh grade students' behaviour when dealing with different types of conversions of algebraic relations. Similarly to the similarity diagram for ninth grade students, within each similarity group strong connections are identified among tasks involving relations of a functional type (i.e., tasks 3, 4, 5 and 6), indicating students' consistent behaviour when translating mathematical relations involving functions. Tasks 1 and 2 are also associated to each other, indicating the homogeneity by which they were handled by Grade 11 students, since both tasks represent regions of points, not involving function.

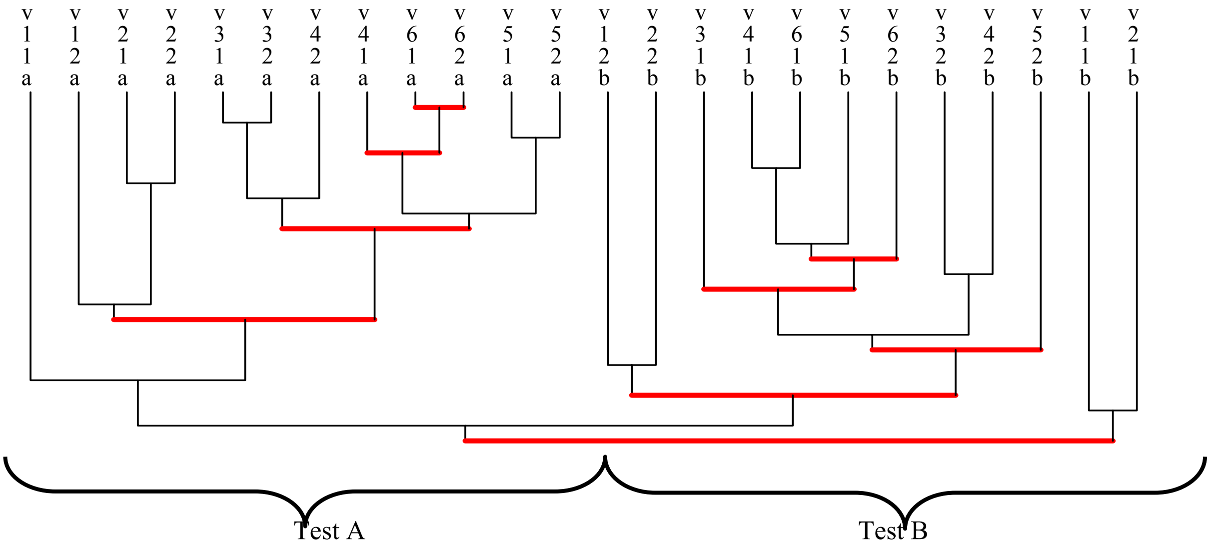

FIG. 3 - Similarity diagram of the tasks of Test A and Test B according to Grade 11 students' responses

The implicative diagram shown in Figure 4 is in accord with the similarity diagram.

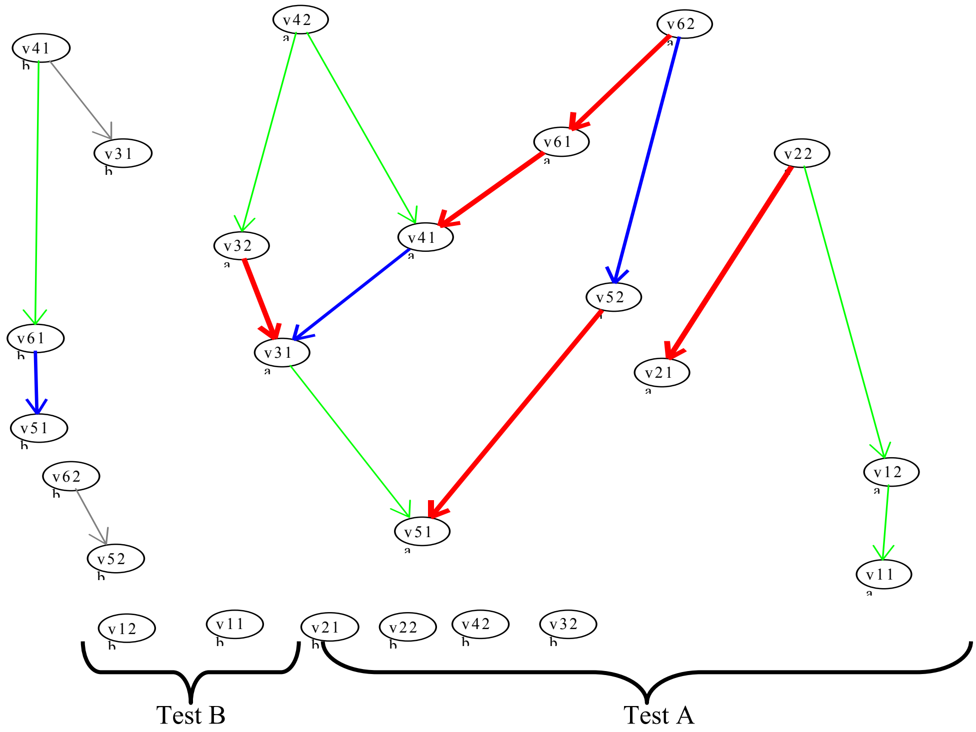

FIG. 4 - Implicative diagram of the tasks of Test A and Test B according to Grade 11 students' responses

Two separate groups of implicative relations are identified. The first group involves tasks of Test B, while the second group involves tasks of Test A. The formation of these relations indicates that success in a conversion starting from a graphic form does not imply success in a conversion of the same mathematical relation starting from a verbal mode of representation. Thus, further support is provided for the phenomenon of compartmentalization. In line with the similarity diagram, direct implicative relations appear among tasks 3-6, which involve functions or algebraic relations with functional character, as well as among tasks 1 and 2, corresponding to regions of points.

Comparing Figure 4 which corresponds to Grade 11 students to the corresponding Figure 2, which concerns Grade 9 students, we observe the following:

1. The implications between the variables concerning functions are stronger in the case of Grade 11 students in relation to Grade 9 students, which are expected, as the former students receive systematically instruction on functions.

2. In the case of Figure 4 (Grade 11) there are not implications between functions (v6, v5, v4) and region of points (v1, v2). On the contrary, in Figure 2, success in tasks involving region of points is a presupposition for succeeding in tasks involving functions.

Thus, the application of the implicative statistical analysis reveals the difference of the epistemological character between knowledge of Grade 11 students and Grade 9 students on functions. This difference can not be uncovered by the corresponding similarity diagrams.

4 Discussion

The aim of the present study was to identify and investigate the phenomenon of compartmentalization as regards the understanding of the function concept by using the implicative statistical method of analysis. Despite eleventh grade students' higher performance relative to ninth grade students' performance in the conversions of algebraic relations from one mode of representation to another, there was a congruent structure of the similarity and implicative diagrams for both age groups, indicating that the phenomenon of compartmentalization exists irrespective of students' age. For both age groups, success in one type of conversion of an algebraic relation did not necessarily imply success in another mode of conversion for the same relation. Lack of implications or connections among different types of conversion (i.e., with different starting representation) of the same mathematical content is the main feature of the phenomenon of compartmentalization and indicates that students of both age groups did not construct the whole meaning of the concept of function and did not grasp the whole range of its applications. As Even (1998) supports, the ability to identify and represent the same concept in different representations, and flexibility in moving from one representation to another allow students to see rich relationships, and develop deepen understanding of the concept. This inconsistent behavior can be also seen as an indication of students' conception that different representations of the same concept are completely distinct and autonomous mathematical objects and not just different ways of expressing the meaning of a particular notion. In other words, students may have confused the mathematical object of a function or other algebraic relation with its semiotic representation (Duval 1993; 2000).

The differences among students' scores in the various conversions from one representation to another, referring to the same algebraic relation or function provide support to the above findings as well as the different cognitive demands and distinctive characteristics of different modes of representation. For example, students' greater difficulty in carrying out a conversion of a relation starting from a graphic mode of representation rather than a verbal mode of representation, which was observed in the study of Markovitz et al. (1986) as well, may be due to the fact that perceptual analysis and synthesis of mathematical information presented implicitly in a diagram often make greater demands on a student that any other aspect of a problem (Aspinwall et al. 1997). The graphic register functions effectively only under the conventions of a different mathematical culture. Due to students' poor knowledge of this new culture, graphic modes of representation are difficult to decode and work out. It can be asserted that the support offered by mathematical meta-language is more fundamental than the aid given by the graphic register for carrying out a translation from one mode of representation to another.

Besides the effect of the type of conversion (modes of representation) another significant factor that had an impact on students' achievement is the type of mathematical relation, involved in the conversion. As shown by the similarity diagrams and students' success rates, algebraic relations of the same conceptual character (i.e., relations involving a clear connection between x and y, or relations having x and y independent from one another) were approached consistently by the students.

Conclusively, the phenomenon of compartmentalization illustrated that the expected semantic congruence does not appear neither in the case of Grade 9 students nor in the case of Grade 11 students, despite the fact that the latter group of students attended a more systematic and rigorous teaching of algebra. That is because learning can be accomplished through "de-compartmentalization" (Duval 2002). Therefore, the use of multiple representations in mathematics learning and in this case learning of functions, the connection, coordination and comparison with each other and the conversion from one mode of representation to another should not be left to chance, but should be taught and learned systematically, so that students develop the skills of representing and handling flexibly mathematical knowledge in various forms.

Références

Aspinwall, L., Shaw, K. L. & Presmeg, N. C. (1997). Uncontrollable mental imagery: Graphical connections between a function and its derivative. Educational Studies in Mathematics, 33, 301-317.

Bodin, A., Coutourier, R., & Gras, R. (2000). CHIC : Classification Hiérarchique Implicative et Cohésive-Version sous Windows – CHIC 1.2. Rennes : Association pour la Recherche en Didactique des Mathématiques.

Cheng, P.C.H. (2000). Unlocking conceptual learning in mathematics and science with effective representational systems. Computers and Education, 33, 109-130.

Duval, R. (1993). Registres de Représentation Sémiotique et Fonctionnement Cognitif de la Pensée, Annales de Didactique et de Sciences Cognitives, 5, 37-65.

Duval, R. (2002). The cognitive analysis of problems of comprehension in the learning of mathematics. Mediterranean Journal for Research in Mathematics Education, 1 (2), 1-16.

Eisenberg, T., & Dreyfus, T. (1991). On the reluctance to visualize in mathematics. In W. Zimmermann & S. Cunningham (Eds.), Visualization in Teaching and Learning Mathematics (pp. 9-24). United States: Mathematical Association of America.

Evangelidou, A., Spyrou, P., Elia, I., & Gagatsis, A. (2004). University students' conceptions of function. In M. Johnsen Høines & A. Berit Fuglestad (Eds.), Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education (Vol. 2, pp. 351-358). Bergen, Norway: Bergen University College.

Even, R. (1998). Factors involved in linking representations of functions. The Journal of Mathematical Behavior, 17(1), 105-121.

Gagatsis, A., & Shiakalli, M. (2004). Ability to translate from one representation of the concept of function to another and mathematical problem solving. Educational Psychology, 24 (5), 645-657.

Gagatsis, A., Elia, I., & Mougi, A. (2002). The nature of multiple representations in developing mathematical relations. Scientia Paedagogica Experimentalis, 39 (1), 9-24.

Gras R. (1992) 'Data analysis : a method for the processing of didactic questions ', Research in Didactic of Mathematics, Selected papers for ICME 7, La Pensée Sauvage, Grenoble.

Gras R., Briand H., Peter P. (1996) 'Structuration sets with implication intensity' in E. Diday, Y. Lechevallier, O. Opitz (Eds) Proceeding of the International Conference on Ordinal and Symbolic Data Analysis - OSDA 95, Springer, Paris.

Gras R. and als (1996) 'L'implication statistique ', Collection associée à ' Recherches en Didactique des Mathématiques', La Pensée Sauvage, Grenoble.

Gras R., Peter P., Philippe J. (1997) ' Implicative statistical analysis ', Proceedings of IFCS 96, Springer-Verlag, Tokyo.

Hitt, F. (1998). Difficulties in the articulation of different representations linked to the concept of function. The Journal of Mathematical Behavior, 17(1), 123-134.

Janvier, C. (1987). Representation and understanding: The notion of function as an example. In C. Janvier (Ed.), Problems of Representation in the Teaching and Learning of Mathematics (pp. 67-71). Hillsdale, NJ: Lawrence Erlbaum.

Kaldrimidou, M., & Ikonomou, A. (1998). Factors involved in the learning of mathematics: The case of graphic representations of functions. In H. Stenbring, M.G. Bartolini Bussi & A. Sierpinska (Eds.), Language and Communication in the Mathematics Classroom (pp. 271-288). Reston, Va: NCTM.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), Problems of Representation in the Teaching and Learning of Mathematics (pp. 33-40). Hillsdale, NJ: Lawrence Erlbaum.

Markovitz, Z., Eylon, B. and Bruckheimer, M. (1986). Functions today and yesterday. For the Learning of Mathematics, 6 (2) 18-28.

Sfard, A. (1992). Operational origins of mathematical objects and the quandary of reification - The case of function. In E. Dubinsky & G. Harel (Eds.), The Concept of Function: Aspects of Epistemology and Pedagogy (pp. 59-84). United States: The Mathematical Association of America.

Sierpinska, A. (1992). On understanding the notion of function. In E. Dubinsky & G. Harel (Eds.), The Concept of Function. Aspects of Epistemology and Pedagogy (pp. 25-28). United States: The Mathematical Association of America.

Vinner, S., & Dreyfus, T. (1989). Images and definitions for the concept of function. Journal for Research in Mathematics Education, 20 (4), 356-266.

Summary

The present paper aims at identifying the difficulties that arise in the conversion from one mode of representation of function to another (i.e., graphic, symbolic, verbal) and examining the phenomenon of compartmentalization, i.e. deficiency in the coordination of at least two modes of representation of the concept. Two tests were administered to 183 students of Grade 9 and 404 students of Grade 11. Both tests consisted of the same algebraic relations, but differed in the types of conversions that they involved and more specifically their starting mode of representation. The application of the implicative statistical method of analysis to the data showed lack of connections and implications among the tasks of the two tests for both age groups. This finding reveals that different types of conversions among representations of the same mathematical content were approached in a distinct way, indicating the existence of the phenomenon of compartmentalization.