Mathematics Knowledge for Teaching (MKT): The case of two high school teachers.

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The aim of the research work being reported is to dig deep into understanding of the Mathematics Knowledge for Teaching in High School (MKTH), particularly, with two teachers of an Algebra course at Technological-Scientific and Social-Science studies at high school. Some relevant aspects of theoretical instrument, methodology and data analysis are mentioned in this paper as well as some preliminary results of the study.

Theoretical Instrument

The theoretical instrument for this study is the model of Ball et. al. (2007) for Mathematics Knowledge for Teaching (MKT). Adapting the theoretical framework of Shulman (1986) on the knowledge of teachers, Ball and her colleagues focused on two central domains: Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK). The Curricular Knowledge proposed by Shulman (1986) is part of the PCK.

Within the SMK are Common Content Knowledge (CCK), Specialized Content Knowledge (SCK) and Horizon Content Knowledge (HCK). The PCK consists of Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT) and Knowledge of Content and Curriculum (KCC).

The CCK refers to the mathematics knowledge and skills needed to solve a task given to students (this knowledge is not likely to be unique to teachers), CCK can be shared by others (in high school for example, CCK may be common among teachers and mathematicians, teachers and engineers or teachers and architects) while SCK refers to pure mathematics knowledge specialized to teachers ability to cope with their work (teaching). The HCK is considered as the knowledge of the existing relationships among distinct topics of mathematics and the evolution of learning of a topic during schooling.

The KCS refers to the combined knowledge of students and mathematics (by a teacher); it is a combination of the content knowledge and knowledge of the possible students’ mathematics thinking or actions. The KCT combines the knowledge of mathematics content with the pedagogical principles for teaching a topic. The KCC refers to the knowledge that students should acquire and the place of content in learning (it includes knowledge of materials necessary to teach various contents).

Research Design

Two teachers of last year of high school students (age 17-18 years old) from educational institutions in Huelva, Spain, participated in the study. Both teachers have a college degree in Mathematics and many years of teaching experience (Aly: 13 years, Emi: 21 years). Aly teaches a Technological-Scientific student group whereas Emi teaches a Social-Sciences student group.

Data collection
In line with Kagan (1990), the analysis of teacher’s knowledge cannot be measured or captured by just one but a combination of various instruments. For this reason data are collected through class observations, field notes, questionnaires, remarks (by teachers), lesson planning and interviews.

**Data analysis**

In this research, the practice of the teachers is analyzed using a model of the teaching process by Ribeiro et al. (2008) derived from Schoenfeld (1998, 2000) teaching process model (this latter adapted by Monteiro (2006) for Natural Sciences). Ribeiro (2008) proposed a model for mathematics, to capture in detail and at every moment, the objectives, content knowledge into action and the actions implemented by the teacher.

The identification of the MKTH domains associated with each episode is effected after the class observations have been divided into phenomenologically consistent episodes and sub-episodes. The initial and final events are identified in terms of the aims of the lesson using constant review/comparison methodology separating episodes and sub-episodes. The identification is based on the theoretical instrument for the MKT domains proposed by Ball et al. (2007) but being attentive to obtain new domains too. Data collected with other instruments were analysed through similar procedures. Then, all data from multiple sources were triangulated to support and to buttress the reliability of the research.

**Preliminary results**

One on the initial problems is to differentiate CCK from SCK; making such differentiation is difficult in some cases, as Ball et. al. (2007, p.44) acknowledged. After several discussions in the SIDM, a way was deduced to differentiate the CCK from the SCK: The CCK could be said to be all the knowledge possessed by any professional that enables him/her to successfully solve the tasks assigned to the students by the teacher. SCK is characterized by the use of specific mathematical knowledge for teaching (a background grounded in teacher’s specialized mathematical knowledge to teach students a specific mathematics content). That is, the exclusive use of mathematical knowledge by the teacher is the reason for to characterize it as SCK.

With respect to HCK, the two teachers have mathematical knowledge of the concepts (during the middle and high school levels), however, they acknowledged their lack of knowledge of the evolution of the concepts before or after these levels or outside their own area (mathematics). Both teachers see the need to know more about possible applications of mathematical concept (not only what concepts should be applied but also how they should be applied) covered in this course.

Concerning the KCS, Aly and Emi had a lesson image -usually a mental artifact rather than a written artifact- of how their students will possibly deal with the assignment given to them on the mathematical topic (Algebra). Students’ misconceptions and learning difficulties are explained in the classroom mainly when students ask questions about what they don’t understand. Mostly the teachers spend a lot of time giving long monologues to explain concepts. Several times the teachers ask themselves questions and answer them to point out to their students possible errors they could make. This serves as a precautionary measure to their students.

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1 The Research Seminar in Mathematics Education (SIDM) at the University of Huelva is a seminar for phDr students. PhDr students discuss and argue ideas, the seminar is a space formed to present research protocols, research work in progress and analyzing and unpacking thoughts provided by researchers of Mathematics Education.

2 In Huelva, Spain, middle and high school levels are usually taught in the same institute, then it is very common that a mathematics teacher teaches in both levels in a school year.

3 Lesson image is […] the written artifact that provides the planned structure of the lesson. A teacher’s lesson image include everything the teacher envisions happening in the lesson -the day’s sequence, the forms of interactions with students, what is flexible and what is not […] and his or her sense of how the discussion will go. (Schoenfeld, 2000, p.250)
The study shows that both teachers recognize the influence of students in their PCK (this fact was one of the main aspects of the research of Park and Oliver 2008, p.279)

Emi: ...students make me change my work strategies at any given time or to improve them (some times it is not a drastic change but certain modifications that help to develop the subject content better).

Aly: Sometimes, questions and comments from students make the teacher realize that some aspects are not as clear as planned so the teacher needs to reconsider the content. Situations like this one help the teacher to discover a better way to explain a concept in the future.

It can also be inferred that the teacher’s experience plays an important role in MKTH domains: The experience gained by teachers through practice, is one of the best clues to choose which models are better to explain a given mathematical content. In this respect Emi commented in the interview:

Emi: Over the years, a teacher can invent new ways or models to represent or display the contents, the graphical forms and mathematical analogies help a lot in allowing students to absorb a given knowledge. Mathematics has an abstract component that must be characterized and symbols and mathematical notation make it possible and enable it to be somehow natural or practical (to the student).

In this sense experience influences the teacher:

- Under the topic of matrices to emphasize that there are properties of operations with real numbers that do not hold with or apply to matrices. If the teacher has already known the most common errors from students, she/he can take preventive measures. For example, students sometimes assume that all the properties that are true about real numbers also hold true to matrices, without taking into account that there are some properties of real numbers which do not hold with matrices, eg. the commutative property of the product of real numbers do not hold with the product of matrices.

- Under linear programming, using tables to organize problem data, and follow a very specific pattern (data, variable definitions, table of distribution, the objective function, constraints, feasible region, analytical solution) (Emi is of the opinion that according to her experience, students could address these contents without too much difficulty if they follow the stated sequence pattern in solving the problems).

With respect to the KCC, it can be said that materials of teachers’ curriculum are mainly three; the program that is expressed in the education law under the Ministry of Education of Spain, the textbook proposed by publishers which are subsequently chosen by consensus by the Mathematics Department Teachers. Finally, the topics proposed for university admission exam (teachers prepare their last year high school students for admission examination into University).

Teachers’ knowledge of how to order or sequence the topics to be taught, was initially acquired when they were high school students and later by their own teaching experience; this background help them to decide the most appropriate sequence of teaching or explaining the concepts while at the same time they remain conscious of the fact that they have to follow the subject syllabus.

The six dimensions of the MKT are crucial for teaching mathematics, but the SCK is one domain that may be of greater interest to the researcher (Ball et. al. 2007, p34), but difficult to measure or detect.
Table 1 shows briefly some characterizations of SCK:

<table>
<thead>
<tr>
<th>Mathematical knowledge that allows her to interpret and assess the validity of students' questions</th>
<th>Aly</th>
<th>Emi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 1: Knowing that if the value of a determinant is expressed as a quadratic equation, then the properties of real numbers can be applied to the equation. That is, change the sign of the first term (-a^2) and that of all the terms of the equation. Aly explained to the students that the value of the determinant does not change sign, but the quadratic equation itself (by applying field properties of real numbers). E1 asked the question: &quot;If the value of a determinant was, in particular, equals to 0, do we have to match that to 0? Can we change signs of the (a^2)?&quot; The expression of the determinant of the matrix is (-a^2+3a-2), the equation would be expressed as (-a^2+3a-2=0) (Aly, class 14, analysis p9)</td>
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<td>Mathematical knowledge that allows her to distinguish the importance of a specific mathematical aspect for teaching.</td>
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<td>Example 1: Knowing that the role of &quot;properly define the variables&quot; is very important for solving linear programming problems, because part of the success or failure in solving the problem depends on how the variables are defined. That is, the mathematics knowledge that comes from the teacher’s reflection about the importance of defining well the variables. (Emi, class 13, analysis p6)</td>
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<td>Example 2: Knowing that matrix multiplication is a binary</td>
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\(^4\) It is notable lack of evidence of SCK in the teachers, the examples shown in this table are possible contexts to SCK still under study.
operation, and therefore in case you have to multiply three matrices, place brackets or just taking in account the need to make the first product of two matrices, ie., to know there is no way of multiplying three matrices simultaneously. We consider that as a descriptor of SCK because it is the teacher, unlike other professionals, who points out this fact to students, the teacher is who emphasizes in her/his teaching that you cannot multiply three matrices simultaneously.

(Emi, class 4, analysis p5)

A Math teacher is the only professional who needs a specialized mathematical knowledge about the content she/he teaches (different from knowledge about Math as would be needed by an engineer or an architect), in order to lead the learning process succesfully.

This research is still in progress and in search of facts to back up the identity and consistency of the MKTH domains.

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