1. Introduction

Besides being a learning objective, the resolution of problems is a means by which students learn mathematics, helping them acquire "ways of thinking, persistent habits, curiosity and confidence with unknown situations that will be useful outside of the mathematics class" (NCTM, 2007, p. 57).

Problem solving is a favoured activity for students to consolidate, broaden and deepen their mathematical knowledge, but is also essential for learning concepts, representations and procedures (Ponte et al, 2007). Work with problems encourages reasoning and justification, motivates communication, provides the use of different representations, allows the establishment of connections between mathematical content and turns mathematics into a useful discipline for everyday life (Boavida, Paiva, Cebola, Vale & Pimentel, 2008). To many authors, problem solving should be the focal point of all mathematics education as it promotes the development of high-level cognitive functions and ability to relate mathematics with the real world (Fernandes, 1994; NCTM, 2007; Ponte et al, 2007). In this perspective, the NCTM (2007) states that problem solving should be an integral part of mathematics teaching and learning programs should be geared towards enabling students to:

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- analyse and reflect on the process of solving mathematical problems (p. 57).

Good problems, involving concepts and skills that students should develop, are a special resource for learning mathematics (Carvalho, 2008); contributing to stimulate creativity and encouraging students to study mathematics, enabling them to broaden their horizons (Powell et al, 2009). According to Tall (1991), many researchers support that the resolution of problems is the most creative mathematical activity. Students benefit cognitively when solving problems and the gains are evident in the short term and intellectually important over time (Powell et al, 2009).

The troubleshooting championships open new possibilities for mathematics education (Borba, 2009) providing access to interesting and challenging problems, more than those who are solved regularly in the mathematics class (Jones & Simons, 1999; Jacinto, 2008), promoting the development of students' autonomy, since they are allowed to use their own strategies and different representations to communicate their resolutions (Freiman et al, 2009).

2. Research Problem

Many are the students who take part in extracurricular activities for learning mathematics to develop their understanding in this area of knowledge (Kenderov et al, 2009). The activities outside school, such as problem solving championships, as long as challenging, interesting and fun, can be key to a deep understanding and widening of mathematical knowledge, providing opportunities for students to strengthen their mathematical thinking (Losada, Yup, Gjone & Pourkazemi, 2009). They are environments that have the ability to prepare students for testing and deploying their mathematical ideas (Moyer, Niezgoda & Stanley, 2005). These projects originate authentic contexts of learning, looking ahead to develop the knowledge and skills that can be transferred to new problematic situations successfully (Wells, 2007). The importance of investigating the processes that students develop in the business of solving problems within and outside the classroom,
particularly when problems are related to mathematical concepts, is highlighted by several authors
(English, Lesh & Fennewald, 2008; Barbeau & Taylor, 2009).

The main objective of this research project is to study the role of creativity in problem solving by the
participants of the Mathematics championship SUB12 final, through the representations used to
communicate the resolutions, focusing on three main guiding questions:

1. How is creativity manifested in the students’ forms of representation, in the resolution of
mathematical problems of the SUB12?
   1.1. Do the representations used express powerful mathematical ideas?
   1.2. Do the representations reveal specific aspects of the mathematical reasoning?
   1.3. Do the representations include aesthetic elements, relevant to the communication of
       mathematical ideas?
2. How is creativity manifested in the strategies used by the students in the resolution of the SUB12
problems?
   2.1. What characterises the student’s powerful and original strategies?
   2.2. What distinctive elements can be found in creative strategies (mathematical connections,
       establishment of relations, identification of objectives …)?
3. What can be transposed about the creativity shown by the SUB12 students in the resolution of
problems to the process of teaching and learning of mathematics?
   3.1. What do the talented students think about the opportunity to develop creativity within
       mathematics?
   3.2. What do the Mathematics Teachers think about the opportunity to develop creativity within
       mathematics?
   3.3. How does the creativity of talented students relate to the feel of problem solving within
       mathematics?

3. Creativity and Resolution of Problems beyond the classroom

From Silver’s point of view (1997), creativity is an evidence of mathematical knowledge. For
the author, although creativity is often associated with ingenuity or exceptional abilities, it can be
widely promoted in the scholar population in general.

The students with mathematical potential show creativity, high concentration, intuition,
originality, stability and flexibility; solve problems differently from other students, enjoy specific
activities since it allows them to create something new and to be autonomous in their approaches to
problems; stand out for their superior capabilities to communicate and explain their symbolic
resolutions (Applebaum, Freiman & Leikin, 2008). They are creative students who demonstrate an
evolved mathematical thinking, able to combine knowledge, imagination and inspiration (Leikin,
Applebaum, Freiman & Leikin, 2008) consider that these students should be offered the opportunity
to show their mathematical knowledge, creativity, curiosity, detail and imagination. Creativity is
evident when students have the opportunity to find and use their own solving methods (Pehkonen,
1997). In that sense, the tasks should be especially challenging (Applebaum & Leikin, 2007;
(2009) believe that the challenging mathematic activities free from routine, research-based and rich
in problem solving, can lead students to discover and realise their talent.

Leikin (2007) defends the resolution of problems as an effective tool to promote and explore
mathematical reasoning, the production of mathematical connections and creativity. For this author,
Problem solving promotes advanced mathematical thinking, since it requires strategic thinking, insight, perseverance, creativity and skill. Meaningful involvement with mathematical problems, over time, allows students to build, find and define effective schemes and strategies important in resolution (Powell et al., 2009). A study by Ching (1997) revealed that by having the opportunity to think for themselves, some students find interesting ways to solve problems. According to the author, creativity and mathematical talent emerge when students are autonomous. On the other hand, Hashimoto (1997) argues that students' mathematical creativity emerges when they are able to combine different ways of thinking about a problem.

4. The resolution of problems and ways of representation

The representations are an important vehicle for learning and communicating, providing vital tools to register, analyse, resolve and communicate mathematical data, problems and ideas (Preston & Garner, 2003).

The representation of mathematical ideas arises from processes occurring in the individuals’ minds (Boavida et al., 2008) and how to represent them is critical to be used and understood (NCTM, 2007; Ponte & Serrazina, 2000). To understand the representations and to be able to represent mathematically, is a skill that increases the ability to think mathematically (NCTM, 2007; Boavida et al., 2008; Ponte and Serrazina, 2000), helping to make learning more meaningful (Clements, 2004).

According to NCTM (2007):
Representations should be treated as essential elements in supporting the students understanding of the concepts and the mathematical relations, communication approaches, arguments, and mathematical knowledge for themselves and others, to identify connections between inter-related mathematical concepts, and application of mathematics to realistic problems through modelling (p. 75).

The representations designate concepts and always refer to something, an object or situation (Ponte & Serrazina, 2000). They constitute themselves through signs that are used when connected to a meaning (Saraiva, 2008; and Matos e Serrazina, 1996) and can take different forms, for example, symbolic, iconic and active (Boavida et al., 2008).

"The symbolic representations consist of the translation of experience in terms of symbolic language" (Boavida et al., 2008, p. 71). The symbols are especially useful for solving problems, since they allow accurate expression of mathematical ideas and in a condensed manner (Ponte, Matos & White, 2008). They are usually used with the objective to facilitate communication regarding the concepts (Matos & Serrazina, 1996).

"The iconic representations are based on visual organisation, in the use of figures, pictures, schemes, diagrams or drawings to illustrate concepts, procedures, or a relation between them" (Boavida et al., 2008, p. 71). Such representations have a very important role in the representation of mathematical ideas. For example, the construction of figures can be a powerful learning experience for students, because they have to make explicit various aspects that are often assumed when the images are pre-established (Clements, 2004).

"The active representations are associated with action. The proper and direct manipulation of objects, (..), contributes to the construction of concepts "(Boavida et al., 2008, p. 71).

Students should be able to use given representations and select from among various representations, which are most useful for a particular situation. It is also desirable to be able to create their own representations (Preston & Garner, 2003). The representations constructed by students are important to the understanding of mathematical ideas (Ponte & Serrazina, 2000), allowing recognition of its modes of interpretation and reasoning used (NCTM, 2007) and can be
used to provoke discussion of mathematical ideas (Clements, 2004). Allowing students to build their representations has advantages in understanding and problem solving, provides the development of own solving methods (NCTM, 2007) and provides a starting point for consideration of alternative representations (Ponte & Serrazina, 2000). The ease of using multiple representations and the flexibility to switch between different representations are critical components in the ability to solve mathematical problems (Heinze, Star, & Verschaffel, 2009). Each form of representation takes place in the used thinking processes that underlies the choice of strategy and tool for its communication (Preston & Garner, 2003). To Heinze, Star and Verschaffel (2009), the use of flexible and adaptable representations is part of a cognitive variability, which allows the solving of problems more quickly and accurately.

The learning environments that confront students with multiple representations and which enable a fluent and flexible combination of different representations can be effective in helping students understand and learn mathematical concepts by developing a genuine willingness to learn mathematics (Heinze, Star & Verschaffel, 2009).

5. The Research Design

Since the objective of this research is to study the phenomenon in the context in which it happens, we opted essentially for a qualitative methodology. The intention is not to generalise, but to deepen the knowledge of a particular situation in a given context. In this case, it is intended to describe and interpret the creativity within problem solving, outside the classroom, through the representations used by the students to communicate resolutions.

5.1. Empirical Field

The SUB12 is a Problem Solving championship in Mathematics, supported by the Department of Mathematics within the Institute of Science and Technology at the University of Algarve, geared for students in the 5th and 6th grades within the Algarve and Alentejo regions in Portugal. It appeared in the 2005/2006 school year as a complement to mathematical activities developed in the classroom by teachers and it aims to tackle some of the limitations pointed out by teachers, including the difficulty in implementing a regular and solid task in the area of problem solving within the mathematics classroom.

The championship is divided into two distinct phases: the qualifying phase, which consists of 12 problems, held from January until June, and is held remotely via computer and the Internet; the final phase, in which finalists students participate in a tournament at the University of Algarve, consisting of five problems.

5.2. Study Participants

This study will focus on the participants of the Mathematics SUB12 Championship. It is not intended to standardise the results or generalise conclusions, so the selection of participants will be intentional (Ferreira and Carmo, 1998). The method of selection will be determined by scrutinising the resolutions produced by the participants and their interest in solving problems outside the classroom.

5.3. Data Collection

Given the interpretative nature of the research, the methodology to follow, will be based on the collection of qualitative data in order to describe and interpret the phenomenon under study in the most complete and authentic manner possible. It will seek to diversify the sources and data collecting instruments in order to obtain a significant set of data so that the descriptions and interpretations are well founded and valid.
In this sense, the data will be gathered from documents produced by participants of the SUB12, from a diary of notes and reflections of the researcher, complemented by the application of semi-structured interviews.

**5.3.1 Documents produced and written notes**

The documents produced by the participants will enable meaningful data to be obtained on their representations in the resolution of problems, obtained in the final phase of the 2005/2006, 2006/2007, 2007/2008, 2008/2009 and 2009/2010 editions. In the 2009/2010 edition the finalists’ resolutions will be analysed in the final qualifying and in the final, with the aim of confronting the strategies used with and without access to the technologies.

The notes and reflections written will serve to describe what the researcher observes, feels and thinks during the data analysis (Bogdan & Biklen, 1994), especially regarding the resolutions and the processes used by the participants in the championship.

**5.3.2. Interviews**

Interviews will take place in order to obtain more specific data about the participants (Tuckman, 2005).

The interviews will take place after the final stage of the SUB12 championship, for ten of the finalists in attendance and respective teachers, with the intention to collect participant data, orally and through their own words (Bogdan & Biklen, 1994), for example: the participants’ feedback on the resolution of problems; the participants’ motivation to solve mathematics problems; the emotions that the participants feel during the problem solving; the usefulness and importance of solving problems for the participants and the teachers; understanding what teachers think of the participants and about their level of talent and creativity, what do they have of difference, what distinguishes them, what makes them creative, how they react in the classroom, how they work… this way the researcher will be able to develop ideas on how the participants interpret key aspects of the problem under study (Bogdan & Biklen, 1994).

**5.4. Data analysis**

The data analysis will primarily be descriptive and interpretative in order to obtain a characterisation as complete as possible of the phenomenon under investigation and an understanding of it, with the objective to answer the questions asked. The aim is not to establish the relations of cause and effect, but to understand the participants’ views in this study, in a perspective supported by the concepts discussed and analysed within the theoretical framework guiding the study.
6. Bibliography


