Introduction

In order to be consistent with my … and immediately I run into a language difficulty! The possessive pronoun signals a potential problem. Words like principles, beliefs, stance, and approach come-to-mind, but each of these can be challenged as being constructs used by observers to account for observed behaviour, rather than as lived experience. How could I possess principles, beliefs, stances, an approach? They may be associated with my behaviour by others, but my lived experience is not of having these come-to-mind as generating behaviour. Be that as it may, it is my choice (have you observed agency?) to begin with some task-exercises. I shall be suggesting that agency, like so many other constructs, is in the eye of the observer, not lived experience. It is perhaps one of the most interesting constructs precisely because it is so easy to be enculturated into thinking one is experiencing it!

**Task-Exercise 1: Geometrical Arithmetic of Ratios**

![Geometrical Diagram]

Among other relationships in the diagram, \( \frac{a}{b} + \frac{c}{d} = \frac{x}{y} \).

**Comment**

I have struggled so many times to reconstruct a justification that I have no qualms in taking you through some reasoning, confident that when you try to reconstruct it for yourself, you will have to do most of the work for yourself!

In the diagrams shown consider the associated arithmetic calculations:

\[
\frac{a}{b} = \frac{ADB}{CDB} = \frac{ADG}{CDG} = \frac{ADB - ADG}{CDB - CDG} = \frac{AGB}{CGB}
\]

Note that the fourth fraction uses the second and third together with an instance of a general property of ratios: if two ratios are equal, then they are also both equal to the ratio of any non-zero linear combination of their numerators applied also to their denominators:

If \( \frac{a}{b} = \frac{c}{d} \) then \( \frac{a}{b} = \frac{c}{d} = \frac{\lambda a + \mu c}{\lambda b + \mu d} \). The instance \( \lambda = 1 \) and \( \mu = -1 \) was used above.

Similarly,
\[
\frac{c}{d} = \frac{ACE}{CBE} = \frac{AGE}{GBE} = \frac{ACE-AGE}{CBE-GBE} = \frac{ACG}{CGB}
\]
and
\[
\frac{\lambda}{\mu} = \frac{CA+c}{C\mu} = \frac{C\lambda+\lambda B}{C\mu+\mu B}
\]
so
\[
\frac{a}{b} + \frac{c}{d} = \frac{AGB+ACG}{CGB} = \frac{AGB+ACG}{CFG+GFB} = \frac{x}{y}
\]
as required.

**Comment**

It is remarkably difficult both to find and to re-construct the simple reasoning. I stumbled over it once, thought it was trivial, and then three times struggled to reconstruct it. Part of the problem is that there are a multitude of other relationships to trip over. Consequently there are plenty of opportunities to experience shifts in what is attended to, as well as in ways of attending (Mason 2003, 2010). Each step feels like a choice, perhaps an exercise of agency. Whatever it is called, it provides a moment when some action comes-to-mind, although it may or may not turn out to be fruitful mathematically.

There is likely to be a moment of gazing at the whole diagram, and perhaps having-come-to-mind various associations with it. What comes-to-mind may seem like a moment of choice, but close observation reveals that it is not usually a conscious choice, more a waiting for something to come-to-mind (vaguely resonant with Simone Weil 1951). Poincaré’s famous ‘stepping onto the bus’ incident (Poincaré 1956) is a classic instance of this phenomenon.

At some point, attention is drawn to discerning in the diagram the segments corresponding to the ratios being added, as part of seeking a connection to the claimed result, which itself needs to be discerned in the diagram. To justify the conjecture, some action is required in order to transform the sum into a manipulable form. A fruitful approach is to make use of the relationship between areas of triangles with both a common vertex and opposite sides on the same line: ratio of areas is ratio of sides. The other action required concerns the property of equal ratios quoted above. There are thus two forms of shifting between recognising relationships to perceiving properties. One is seeing the area-side ratio as a general property which can be instantiated in several ways in the diagram. It is the shift from specific relationship to general property that enables reasoning on the basis of this property. The other is recognising the possibility of instantiating a general property in a particular case (the ratios result). Finally, there is a third shift, seeing that the particular triangle ABC is not important, only the ratios in which D and E, and hence G, divide the corresponding line segments. This shift to perceiving a property requires becoming aware of what in the original diagram can be changed and yet leave the ratio-sum invariant, as a property instantiated in many forms.

**Task-Exercise 2: Constructing an Example**

Write down a number that leaves a remainder of 1 on dividing by 2; and another; and another; express all such numbers as a generality.

Now write down a number that leaves a remainder of 1 on dividing by 2 and whose quotient (the result of the division by 2) itself leaves a remainder of 1 on dividing by 3; and another; and another; express all such numbers as a generality.

Now write down a number that leaves a remainder of 1 on dividing by 2, whose quotient leaves a remainder of 1 on dividing by 3, and for which that quotient leaves a remainder of 1 on dividing by 4.

Why must your number be divisible by 3 and leave a remainder of 1 on dividing by 8?

Construct a sequence like this of your own.
Comment

Being asked to construct three examples leads many people, at least by the third request, to try to be adventurous, and so provokes them to switch from writing down the first object that comes-to-mind to becoming aware of the choices available to them. In other words, the task provides access to becoming aware of a space of possible examples rather than plucking potential examples out of the air. Perhaps then, agency can be exercised. By constructing all possible examples expressed as a generality, you are in a much better position to deal with the second and third parts than if you simply cast around for examples meeting each of the conditions in turn. You are becoming aware of structural relationships as part of your example space. The final construction task requires a perception of the original task not as specific set of relationships, but as an instance of some divisibility properties.

One of the strategies usually provoked by this task is to stop working forwards and start working backwards. Start with an expression for all numbers leaving a remainder of 1 on dividing by 4 (viz 1 + 4n), then multiply by 3 and add 1 to meet the second condition; then multiply by 2 and add 1 to meet the first condition. This gives the space of numbers sought as $1 + 2(1 + 3(1 + 4n))) = 9+24n$. So all such numbers must be divisible by 3, and must leave a remainder of 1 on dividing by 8.

Contact with the space of all possible examples expressed as a generality gives access to a property that must hold for all such numbers: the language of algebra provides access to perceiving properties that hold in general and enables reasoning about all of them at once. Furthermore, working backwards affords access to a general construction for other tasks of this form.

An effective method of ‘teaching’ others to use a heuristic such as ‘working backwards’ or ‘be wise, generalise’, is to provoke them to use it spontaneously, as here. Participating in an action alone is rarely enough however. It is usually necessary, and certainly helpful, to get learners to withdraw from the immediate action and become aware of what they have done, labelling it in some way so that it has a chance of coming-to-mind in the future. Labelling supports coming-to-mind through a combination of metaphoric resonance (something in the situation acts as a structural reminder of the action of working backwards) and metonymic triggering (something in the language or something idiosyncratic, often with a significant affective component, triggers recall of the action of working backwards).

Notice that in the last paragraph there is an implicit generality, using ‘working backwards’ as a placeholder for any other action.

Task-Exercise 3: Effective Affect

Did you find yourself having to struggle to keep going in task-exercises 1 or 2? How does this relate to your propensities and dispositions concerning mathematical tasks?

Write down something you know for sure about teaching and learning mathematics from research;

Write down something you believe about teaching and learning mathematics but for which you don't have research evidence;

What would it take to convince you that our belief was flawed?

Comment

I am using this task to introduce some of the themes or issues that I would like to probe in this session. Were you able to trap the facility with which you were able to distinguish knowledge and belief and their relation to research activity? What were the origins of the construct(s) you used in describing a disposition: research literature, everyday discourse, your own idiosyncratic use of labels?

Reflection

By starting with some shared mathematical thinking, I have illustrated an approach to enquiry:
interrogate my own experience deeply, so as to be sensitised to what others might be
experiencing;
accumulate actions that could come-to-mind appropriately so as to amplify the
possibility of having them come-to-mind in the moment;
construct and refine task-exercises to offer to others in order to see whether they
notice something similar to what I have noticed.

This is consistent with the assertion strongly associated with Caleb Gattegno (1970) that
‘only awareness is educable’, because for him, awareness is what enables action. By contrast,
‘only behaviour is trainable’ and ‘only emotion is harnessable’ (Mason 1998).

Whenever I am asked a practical question about teaching or learning, or a theoretical
question about mathematics education, my immediate response is to look for some parallel
version of the phenomenon in my own experience. This makes it possible (awareness
enables action) to explore the phenomenon or issue, to sensitise myself to the phenomenon
and correspondingly appropriate actions I wish to have-come-to-mind. This in turn makes it
possible to speak directly from experience, and therefore to speak to the experience of
others.

My interest has always been in what is possible (in this sense I follow Abraham Maslow
1971) rather than in what currently is the case. Put another way, what interests me is what it
is like to experience something or to encounter a phenomenon, or in other words, what it is
like to be: to be in the process of becoming, whether a mathematical thinker, mathematician,
learner of mathematics, or teacher of mathematics, and how to participate more fully in
responding rather than reacting to mathematical situations. My aim is always to inform my
future practice through responding to situations by having-come-to-mind possible actions in
place of habits and automatic reactions.

Phenomena

Deeply embedded in the origins of what came to be known as phenomenology is the notion
that real observation is a co-emergent transformation of observer and observed, both
experienced as becoming rather than merely statically existing. Indeed most of what we
think we observe is actually ourselves, in the sense that it is our discernments of detail, our
recognition of relationships among entities discerned, our perception of properties being
instantiated to which we may refer for justification for our claims. A phenomenon is already
a generalisation, a likeness to something that has been experienced or discerned previously,
immersed in the historical-cultural milieu of the situation and of our individual pasts. For
early phenomenologists, the phenomena emerged from contact with a core or essence of a
situation, where being moved into becoming, and observer and observed were intertwined.

Extra-Spective and Intra-Spective Enquiry

Most research studies are reports of researchers observing other people. It is extra-spective
because observations are made by an outsider. Even when the observer takes into account
the fact of observing, what is available for study is discerned behaviour spanning a
spectrum from responsive to reactive actions and from well rehearsed to spontaneous
narratives constructed in response to probes.

I prefer to engage in intra-spective observation: developing an inner monitor or witness that
observes ‘from the inside’ (Mason 1992, 1994, 1998a) without judgement. Intra-spective
research depends on the nurturing and growth of an inner observer or witness, captured
nicely by a stanza from the Rg Veda:
Two birds, close yoked companions, both clasp the self-same tree; One eats of the sweet fruit, the other looks on without eating. (trans. Bennett 1964 p108):

One bird engages in activity (eats of the sweet fruit), while the inner witness monitors without passing judgement. It is a form of consciousness or conscious awareness that Norretranders (1998) appears to overlook, and is related to what Ouspensky (1950) and others called self-observation. It is perhaps related to Burke’s notion of cosmic consciousness (Burke 1901 see also Harding 1961). It is certainly related to the notion of becoming. However that may be, Mason (2002) is an attempt to articulate relevant techniques in sufficient detail to make it possible for others to engage in similar intra-spective work for themselves.

A slightly different rendering is that intra-spective research is about harnessing the multitude of selves that for short periods take over the role of ‘driver’ and pretend to be in charge. While this is happening, the energy that usually flows into the frustrations of other selves who feel their power usurped can be transformed into impassively objective observation.

One of the problems of extra-spective research is that at best you can only interpret what you notice, and what you notice tells others as much about you as it does about what you are observing. I refer to this as the education precision principle: the more detailed or precise the observation, the more detailed and precise I can be about the sensitivities of the observer; the ratio of these degrees of precision is, it seems, roughly constant.

One of the problems of intro-spective research is that it is all too easy to fall into the trap of augmenting observation with ‘must have’: “I did this so I must have felt, thought, ...”. This was a problem with the early Gestalt psychologists’ methods. Strategies are required for avoiding being trapped in this way.

**Narratives and Narrators**

For most research, especially but not exclusively extra-spective research, what is analysed is languaged descriptors of a phenomenon, not the phenomenon itself.

The careful researcher cognizant of the role of observation in constructing the phenomenon observed is careful not to make statements or claims about objects, entities or relations as if they existed independently of the observing (Maturana 1988 p30).

Maturana elaborates this by talking about observational *objectivity-in-parenthesis* and *objectivity-without-parenthesis*. Objectivity-without-parenthesis is a stance in which the world and the phenomena being described are taken for granted. They are assumed to be constituents of ‘reality’, including all the constructs labelled or signified in the discourse being used. This is amplified by using nouns: understanding, agency, ability, and so on. By contrast, objectivity-in-parenthesis is a stance in which the observer is co-constructor of the phenomenon and of the signifieds being referred to. This stance can be amplified by using gerunds: terms lying between verbs and nouns and signalling participation in an action. Thus in place of the first row below, the second row emphasises participation.

<table>
<thead>
<tr>
<th>Knowledge</th>
<th>Belief</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowing</td>
<td>Believing</td>
<td>Acting</td>
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</tbody>
</table>

See later section on Psyche and the chariot metaphor.
An observation by a researcher isolates a moment that strikes the observer who then converts the experience of observing acting (which deserves the gerund) into a construct (noun). This reification creates an object (knowledge, belief, identity, agency, …). Providing a label is an ontological act, bringing into existence and so classifying and pigeon-holing the particular, while at the same time enabling the identification of similarity among phenomena in the future. Turning actions into nouns is so prevalent that we hardly notice it happening.

The role of the in-parenthesis is a Hegelian type of device to bring-to-mind continually the danger of being seduced into accepting a signifier as signifying something extant in the world, rather than signifying an observer construct used to make sense of observed behaviour.

**Acting-as-if**

There is an enormous danger in research whenever the description, the account-of what is observed uses terms that implicitly or explicitly account-for that behaviour. It is all too easy to integrate observation (with or without parentheses) with explanation, justification or excuses. Adjectives, adverbs and value-laden nouns are all give-aways. A careful observer notes that the behaviour they recount (give an account-of) can be explained (accounted-for) accurately only in terms of the person acting-as-if they are knowing, believing, having-come-to-mind, etc. or engaging in some other action (Mason 2002). This suggests a third line in the noun-gerund sequence:

<table>
<thead>
<tr>
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<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowing</td>
<td>Believing</td>
<td>Acting</td>
</tr>
<tr>
<td>Acting-as-if knowing</td>
<td>Acting-as-if believing</td>
<td>Having-come-to-mind</td>
</tr>
</tbody>
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Very often researchers cross the line and make assertions that omit the acting-as-if, being content to assert their own interpretation as ‘what is going on’.

A classic example is belief: do human beings ‘hold beliefs’, or are beliefs ‘things’ that researchers construct to account for the behaviour they observe? The flexibility and power of language leads to a term like belief quickly becoming part of the discourse in which people are immersed so that they integrate the very notion of belief into the narratives they tell themselves. Perhaps the most that can be said is that someone is observed acting-as-if they believe (something) but included in this description is the unjustified assumption that beliefs drive behaviour.

**Principles** are quite similar to beliefs: do I act because of or on the basis of my principles or are my so-called principles my narrative to account-for my behaviour, to weave together a story of my unity, my significance in the world? Do I ‘have an approach’ or ‘maintain a stance’, or are these also constructs used by observers to account-for the aspects of my behaviour that comes to their attention, and which infiltrate my discourse and reinforce my sense that I am in control and make choices?

**Affect** is another tricky domain: do human beings ‘have affect’ such as feelings or dispositions? Do they display behaviour that can be accounted for as-if they possessed these things? Do they experience a flow of energies that activate certain habits? Do they ‘have or experience an identity’, or is this an observer’s description of an experience of affordances and non-affordances? Do students ‘have an attitude’ towards something as general as mathematics, or as specific as ‘multiplication of negative numbers’ or ‘the student t-test’ (Di Martino & Zan 2010)? Are likes and dislikes robust entities (dispositions) or are they sometimes transient (time, place, milieu situated)?

Is behaviour an accurate window on feelings? For example, have you ever been sitting down at the end of a day feeling exhausted, then someone you like invites you out and suddenly
you are active and energetic? Interpreting behaviour becomes even more problematic when absence of behaviour is interpreted as inability to act or as lack of knowledge. Anthropologists are fond of the adage

absence of evidence is not evidence of absence

which finds important application in mathematics education. Just because someone is not observed doing something (answering a question, performing an action) it does not follow that they cannot or could not, nor even that it did not come-to-mind. All it means is that they did not act at that time in a manner recognised by the observer. Indeed, even if someone is observed acting in a certain way, it does not follow that they will always act that way in similar circumstances … leading us into the troubled waters of transfer and situatedness (Detterman & Sternberg 1993, Marton 2006) and the lack of robustness of standardised tests when administered several times over a period (Butlin private communication).

Jerome Bruner (1990) suggested that human beings are narrative animals, and it is certainly the case that we tell stories to justify our actions both to ourselves and to others. Note however that even here the construct narrative or story is a product of extra-spective activity, of discourse acquired from the various social milieux in which we are embedded. What someone says in order to account-for their (remembered) behaviour is experienced by others as a narrative; when closely examined, it has many of the elements of a constructed ‘story’, emphasising some aspects and ignoring others, often so as to put the narrator in a good light. The construct of ‘story spinning’ is readily observed in oneself. It requires a little more care and effort to discover for oneself that consciousness as a generator of action is largely an illusion (Norretranders 1998), that consciousness and choice making is a post hoc narrative.

Agency is, as already mentioned, particularly interesting as a construct because it is so attractive to say “I chose to …” when most of the time our sense of having made a choice is an illusion, a post hoc narrative. Close observation reveals that the experience is often more of having being caught up in some action. A way to test this conjecture is to try to trap the moment when you decide to get out of bed in the morning, or when you decide to say something in a seminar or meeting after you have already spoken once. If we are lucky, we may get a taste of participating in a moment of choice, but often this is retrospective narration. The notion of being an agent as a component of an action in which we are participants then starts to make sense, but challenges the usual use of the term agency as initiation of action.

What then can be said of the various distinctions constituting the theoretical frameworks required by editors and thesis examiners? For me they are sets of distinctions proposed by others and expressed in language. Where they arise extra-spectively, there are severe problems about validity and appropriateness as anything more than reflections of the observer’s sensitivities and predispositions. An alternative is to probe one’s own experience so that distinctions are used to discern detail intra-spectively, on the way to provoking others to notice similarly. Language, in the form of labels for phenomena, still plays a role, but retaining the acting-as-if prefix can help guard against the most egregious fictions.

**Historical Background**

For some 40 years I have been involved in writing materials for studying at a distance by teachers and by others who wish to develop their own mathematical thinking, and/or to support others in fostering, sustaining and developing it in others. At first I thought that distance teaching was the antithesis of effective teaching, but I soon learned that there is a great deal to be gained by being forced to think deeply about the minutiae of what is offered to students and why, and also a great deal to be gained by students having a chance to work
for and by themselves, to ponder and reflect in their own time, before coming together to engage with others.

**Psyche**

The title of this paper is intended to resonate with Neville’s *Educating Psche* (1989), and to indicate a commonality with *Psych Education* (Blake 2010) with its roots in the middle east through J. G. Bennett (1956-1966). The Greek strategy of anthropomorphising aspects of human experience, most particularly *Psyche* as a mythological figure, enables people to talk about, think about and discuss detailed aspects of the human psyche as vicarious lived-experience. My intention in using this conceit is to draw attention to the need for wholeness rather than fragmentation, for preserving complexity (Mowatt & Davis 2010). It also illustrates beautifully the importance of language not only for communication with others and as a source of stimulation and action (through metaphoric resonance and metonymic triggering), but also to enable finer and finer distinctions to be made thereby refining sensitivities and sensibilities.

There are of course many different ways to make distinctions concerning human psyche. For example, Jung (1928) distinguished *sensation, feeling, thinking* and *intuition*, while Western psychology generally acknowledges *enactive, affective* and *cognitive* dimensions (Edinger 1985). The education triad of *student, content, and teacher* is a commonplace (St. Maurice 1991) for thinking about teaching and learning. Seeing human beings as embedded in a social environment adds what Brousseau (1997) called the *milieu* to this triad. Not all teaching and learning is recognised in consciousness (Norretranders 1998), which is why Gattegno (1970) introduced the term *awareness* to refer to ‘that which enables action’, whether conscious or not (Mason 1998).

A picture of the human psyche emerges which can be integrated into a metaphor with ancient roots (e.g. various of the Upanishads: see Radhakrishnan 1953; a similar image of a mansion looked after by servants occurs in Plato’s Republic)

> The human psyche can be thought about as a chariot. The chariot is drawn by horses connected to it by shafts via their harnesses. The driver directs the horses by means of reins according to the owner’s instructions.

The chariot refers to the body, or in psychological terms, to enaction, in other words to physical actions. The chariot is connected to the horses by shafts, which refers to (people acting-as-if they have) habits. The horses refers to sensations, or again in psychological terms, to emotions or affect, the primary interpreter of sensations. The driver is cognition or intellect and is responsible for maintaining the condition of the chariot, harness and horses. The reins refer to mental imagery which is how cognition directs the flow of energy and motive force from the horses. The owner refers to will, which cannot act if the driver is not listening, or if the chariot and horses are not in good condition or are over indulged.

This metaphor informs a framework of distinctions for preparing to teach any (mathematical) topic (Mason & Johnston-Wilder 2006) which elaborates on the notion of *concept-image* (Tall & Vinner 1981) and integrates it with a broader structure of human psyche.

**Research Questions**

The questions that interest me are of the form

- What is it like to …
- What is it helpful to have come-to-mind when …
- How can I sensitise or attune myself to notice …
In addition, I find myself constantly working on mathematics myself, in order to keep alive to the ways of thinking that are of particular interest and relevance to teaching and learning.

Topics I work on include

- the effective use of mental imagery in thinking mathematically,
- the role and nature of attention in mathematics,
- the role of animation and interactive media in mathematics
- how generality is indicated in ancient and modern texts, including e-screens
- promoting mathematical thinking and supporting others who wish to promote it
- task design and classroom interaction

**Validity**

Unlike most other research paradigms, the products of intra-spective research such as that adumbrated in the *Discipline of Noticing* (Mason 2002, for historical roots see Mason in press) are not reports of analysed data collected through observation, interview, questionnaire etc. Rather, the results are task-exercises designed to direct attention to distinctions and related actions that have proved fruitful for others. Thus validity is in the experience of the reader, not in a Popperian third-world of reports in libraries (Popper 1972).

A classic final justification and statement of validation of a thesis based on the discipline of noticing is a statement that validity lies with the reader as to whether they find themselves noticing more sensitively and-or having alternative actions come-to-mind in the midst of preparing or conducting lessons or some other relevant setting. I always recommend to PhD candidates that they obtain some sort of evidence that others have indeed been sensitised in some way by the exercises they offer, but this is an extra meta-level of data collection and involves an analysis that reverts to ordinary extra-spective enquiry, rather than remaining with inter-spectivity of the original enquiry.

**Sample Research**

**What is it like to construct an example of a newly encountered concept?**

The adage that *you really learn a topic only when you find yourself having to teach it* resonates with my own experience. One of the components of preparing to teach a topic is the construction of didactically pertinent examples. Once I became aware of this I was mystified as to why example construction had not been an integral part of my learning. In fact I think I did sometimes construct examples for myself, but rarely if ever under guidance of a set task imposing particularly instructive conditions. Then I realised that all mathematical tasks can be seen as construction tasks: construct something that meets all the constraints of the ‘problem’ (Watson & Mason 2002). However there is a benefit: approaching tasks as construction tasks brings-to-mind actions such as removing some constraints and trying to express the most general objects that meet the remaining constraints.

**Researching Problem Solving from the Inside**

In order to speak directly to the experience of others I find it invaluable not only to be working on mathematical problems myself, but trying to construct tasks that will afford me some experience parallel to that of the people with whom I am working. I attribute the success of Mason, Burton & Stacey (1982, 2nd edition 2010) over such a long period of time to the way in which we were able to speak to the experience of readers through having researched (paid attention to) our own experience (Mason 1992, 1994, 1998a).

**Role of Attention in Teaching and Learning Mathematics**

My attention (*sic*) was drawn to the problem of attention by becoming aware that in almost every workshop I lead, someone wants to know where my accent is from. I began to try to
become aware of what I was attending to and how. It seems clear that if teacher and students are attending to different things, communication is at best impoverished if not impossible. I soon realised that it was not simply what people were attending to, but also how they were attending. This work has been developing since 1982 (see Mason 1982, 1989, 2003, 2010). Of course, my observations were informed by distinctions offered by others, such as those articulated by William James but most particularly by J. G. Bennett (1956-1066, 1964) derived from self-observation and from ancient psychologies from the Middle and Near East.

**Leading Questions**

Let me now briefly address the leading questions for this summer-school:

**How to choose research problems and research questions?**

For me, questions worthy of enquiry arise when I notice some incident, when I am struck by something that someone says or does. Is this an instance of a more general phenomenon? Can I find something similar in my own experience, or can I generate something similar when engaging in exploring, learning or teaching mathematics myself?

For example, observing a lesson recently, I was struck by a novice teacher reviewing an arithmetic task with learners and saying “mine is …”. I actually heard “minus …” at first, and it struck me that some learners might actually hear what I heard and be confused. Are there other potential mis-hearings leading to mis-construals? Can I attune myself so as either not to use such an expression in future, or to improve my diction, or even to exploit it as a method of getting the learners to consider negative numbers?!?

Most of the issues that I explore have to do with trying to sensitise myself so as to have come-to-mind some action other than what has become habitual, so that I can respond freshly rather than react automatically. For example, I found myself teaching five different classes of 16 year olds recently and during my preparation I was aware that there might be some behavioural management problems. I imagined myself stopping a lesson and engaging the learners in meta-questions about the purpose of studying mathematics. I used my mental imagery to prepare some actions, and at one point I became aware of ‘filling out’ those thought-forms exactly as I had imagined. My preparation afforded me an opportunity that might otherwise not have been available: I might have soldiered on against the behavioural opposition. This is an example of how the Discipline of Noticing informs my practice.

**How to refer to and use existing literature and frame one’s own research?**

Existing literature provides labels for distinctions that others have found fruitful, whether it be in large scale or longitudinal studies, or small scale in-depth observation and interview. Unfortunately (and I am as much a contributor as anyone else in this respect) there is a plethora of labels for fine distinctions. Very often it seems that people use different terms for the same thing, and equally often, people use the same term for different things. The latter is especially the case when a term achieves widespread use. For example, zone of proximal development is used in a variety of ways that have little or nothing to do with Vygotsky; concept image is used with a wide variety of nuances as to what is included and what excluded; belief is used differently by philosophers and by social scientists.

If someone else’s distinctions are being used as a theoretical framework then the critical issue is whether your use is a validation of their terms (“look, I can discern their distinctions in my data”), or whether you use those distinctions to draw some further conclusions (“I interpret this behaviour as ... and this means that ...”). The latter is rare, because most frameworks are not theories in the sense of predicting or having consequences. Most often what happens is that data is analysed yet more closely and further more subtle distinctions
are offered which refine the framework of distinctions taken from the literature. In principle this should lead to finer and finer distinctions enabling people to observe more and more closely. Unfortunately what happens in practice is that distinctions simply accumulate. If there are no consequences from making distinctions, there is no point in making them in the first place. That is why the Discipline of Noticing is foremost about sensitizing oneself through discerning distinctions so as to have come-to-mind alternative actions informed by those distinctions.

**How to identify, present and interpret research results?**

Instead of asserting that something is the case, I choose to present task-exercises that may serve to sensitise readers to actions they hadn’t previously had come-to-mind. In Task-Exercise 3 I raised the question of what would have to happen to make you question a firmly held belief. If a researcher reports on the effectiveness of a strategy that is not compatible with the phenomena that I value and the actions that I am accustomed to initiating, am I going to accept their assertion (with analysed data as evidence)? Or am I going to try to replicate their finding in my own situation, in other words, try ‘it’ out for myself?

Unfortunately most research is sufficiently imprecise about what happened in the interactions between teacher, content, student and milieu that it is not only impossible to replicate the study, but even impossible to re-construct sufficiently in order to try out some version for oneself. There are so many factors influencing learning and teaching that either researchers are tempted to simplify unduly (and so lose resonance with readers’ experience) or to retain complexity but lose the phenomenon.

All mathematics education research is plagued by the precision principle: the more specific or precise a researcher is about what they discern, the more specific or precise the researcher is about their own sensitivities and dispositions, and the ratio of this specificity or precision is approximately constant. In other words, observation involves the sensitivities of the observer to discern details in the actions being observed. Interpretation of what is observed takes place through the filter of the interpreter’s predisposition. Hermeneutics are alive and well in the background of all research enquiries in mathematics education!

**How to position theory in research conceptualisation and analysis?**

Theories are either descriptive, predictive or explanatory or some combination. However most theoretical frameworks in mathematics education are principally descriptive, consisting of a collection of labelled distinctions, because they make no predictions and their explanatory power is weak (claiming a cause for what is observed is highly ambitious given the complexities of teaching and learning).

In conceptualising research, what matters most seems to be what it is you want to find out, guided by what you would be able to do with the results that you cannot do now.

**‘Why’ Questions**

For example: why do students make such and such an error?

‘Why’ questions require a mechanism such as cause-and-effect, legitimate-peripheral-participation or socially-enculturated-practices. Chemistry provides other models of mechanisms, including catalysis (actions amplified by the presence of other agents) and reaction (multiple co-present (re)agents leading to multiple outcomes with sensitivities to conditions/milieu).

**‘What’ Questions**

For example: what do students do when asked to construct their own examples?
‘What’ questions lead to description in terms of labelled distinctions. Phenomenography was developed as a study of the range and variety of experiences or actions in a situation. This led to the study of variation offered by teachers from which students might develop conceptual distinctions and increase the effectiveness of teaching. Variation theory informs actions that a teacher can take in order to increase the opportunities for students to learn what is intended.

‘When’ Questions

When is it appropriate to act in such and such a way?

‘When’ questions offer essay topics but miss the point about having-come-to-mind when appropriate. Knowing-to-act in the moment is independent of knowing-about possible actions when not actually in the situation. Most teachers will have experienced the tunnel vision of being in a situation with few possible actions coming-to-mind, yet when thought about before or after the event various possibilities come-to-mind.

‘Who’ Questions

Who benefits from such and such an action or change? Who makes connections between mathematical topics, concepts, techniques etc.? Who constructs meaning?

Often the ‘who’ question reveals an imbalance or an inappropriate attempt to do work for learners that only they can do for themselves. The current zeitgeist in mathematics education is to emphasise peer discussion as the mechanism for meaning construction, yet many hundreds of generations of mathematicians, including some at this summer school, did not have the benefit of peer discussions in classrooms and yet survived quite effectively.

‘Where’ Questions

Where might this technique be applicable? Where (under what circumstances) might this action be appropriate?

There are analogies between appreciating the scope of a mathematical technique and appreciating the scope of a pedagogical strategy.

Conclusion

At the core of mathematical activity is having an appropriate action come-to-mind, and it is also at the core of teaching. In order to support others in having-actions-come-to-mind, it is useful if not vital to have recent parallel experience so as to speak directly to their experience. This requires careful use of language for describing both what you notice happening in yourself, and what you observe others doing. How you interpret what you observe is likely to have a significant influence on the usefulness of what you offer to others from your research. Remembering that all accounts-of behaviour are most accurately interpreted as someone acting-as-if will help retain immediacy, objectivity and relevance of your enquiries.

References


