Research in Digital Technologies and Mathematics Education

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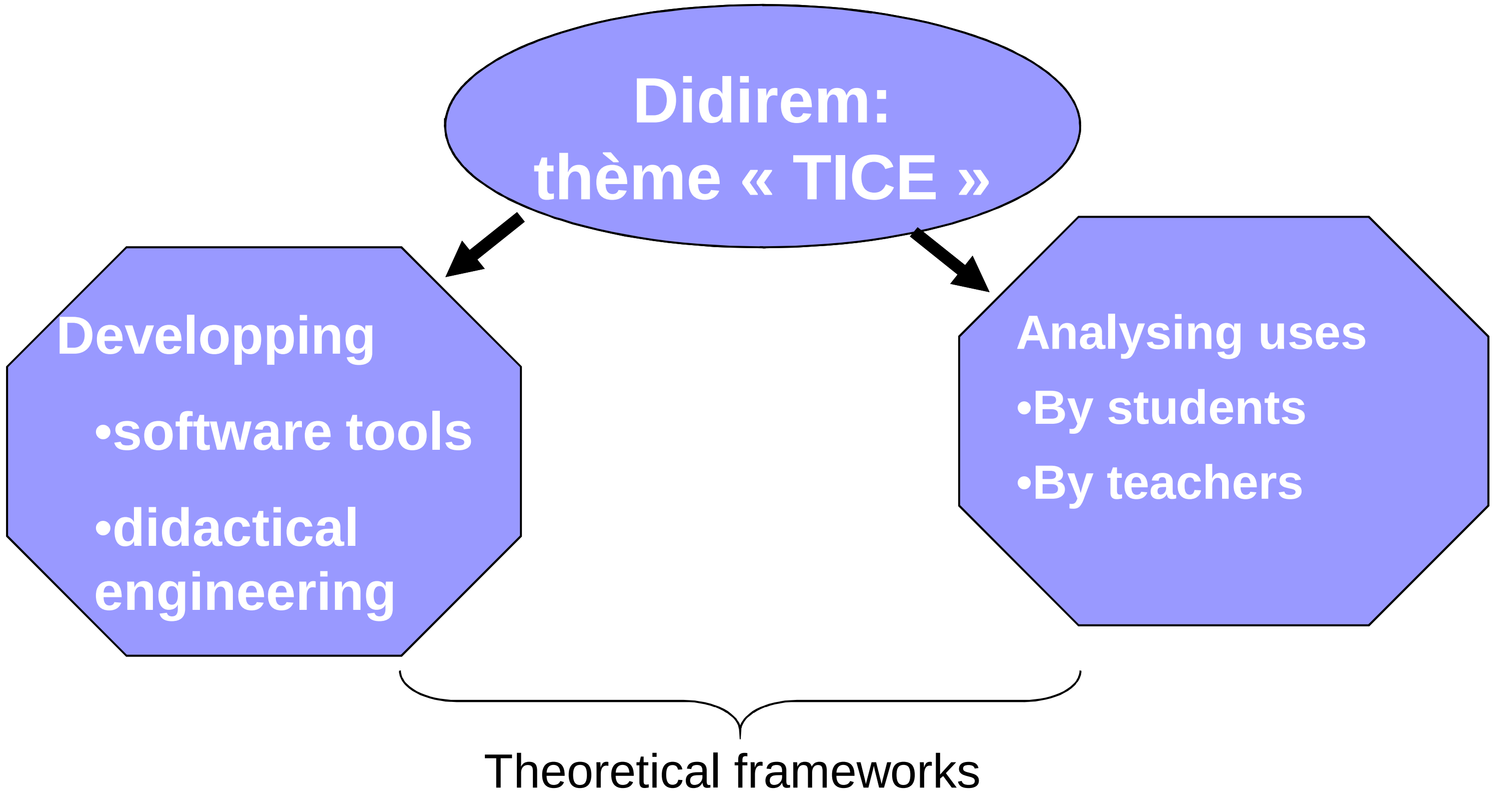
Outline

- Introduction: research in the Didirem team
- Theoretical frameworks supporting research
- The ReMath European project
- Technology in Mathematics Education: Rethinking the terrain (the 17th ICMI study)

Research in the LDAR



Major projects

Projet LINGOT

Projet CASYOPEE

Projet Européen ReMath

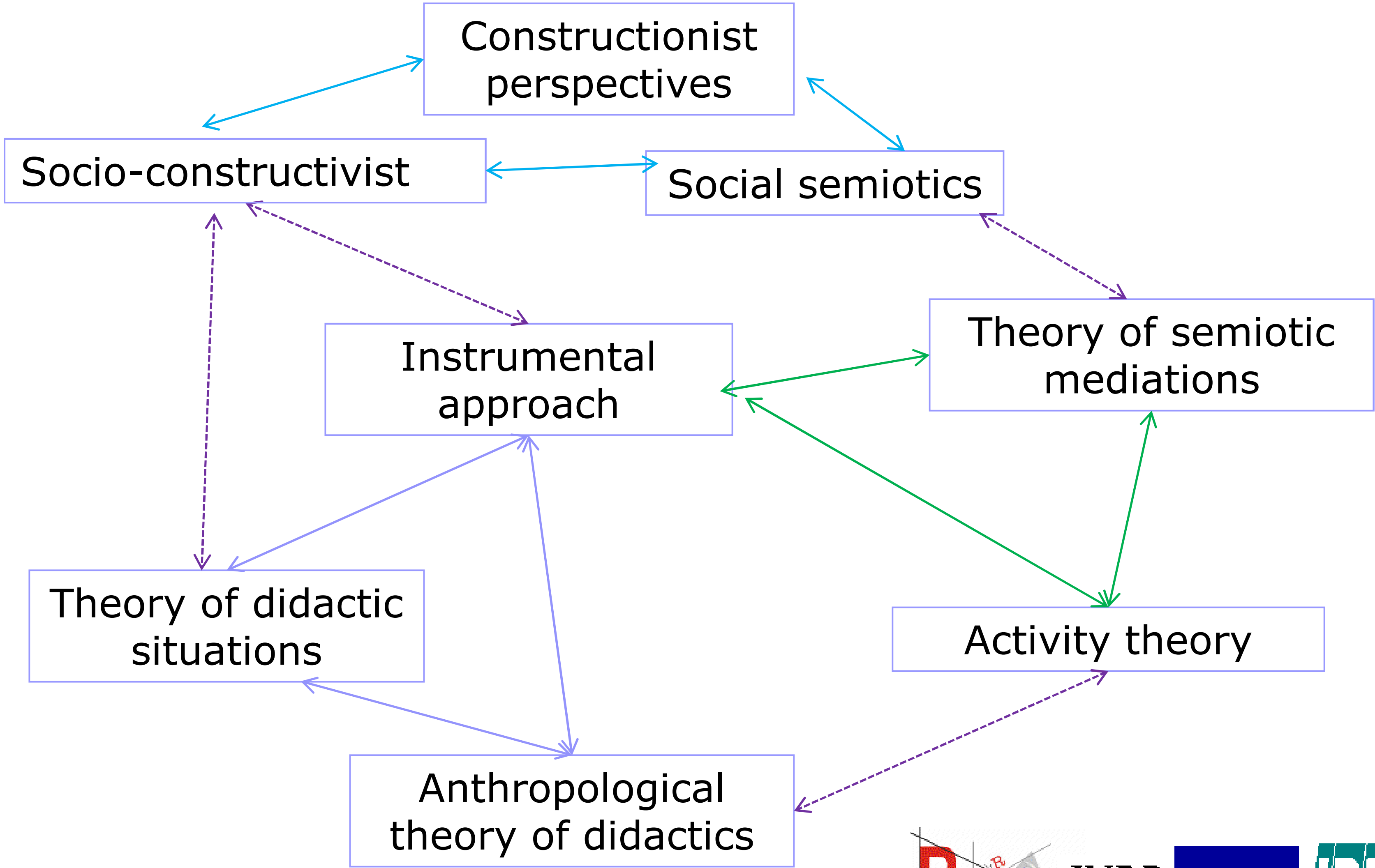
Genèses d'Usages des Technologies par l'Enseignant

Recherches Tableur

Bases d'exercices (Université)

Logiciels Tutoriels Fermés

Theorical frameworks



A meta-study of publications about ICT in maths

Lagrange, J.B., Artigue M., Laborde C., Trouche L., 2003 Technology and Math education: a multidimensional overview of recent research and innovation. In Bishop, Clements, Keitel, Kilpatrick, Leung (eds.) *Second International Handbook of Mathematics Education*, (Kluwer)

Six dimensions

- Epistemological and semiotic
- Cognitive
- Anthropological
- Instrumental
- The situation
- The teacher

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Dimensions	The influence of technology on
Epistemological and semiotic	• the mathematical knowledge taught • the representational structures
“Situations”	• the structure of the situation • students' solving strategies • the didactical contract
“Teacher”	• Teacher‘s beliefs and representations • New teaching situations • Influence of research and pre/in service programs

Dimensions	
Cognitive	The influence of technology on conceptualisation processes
Instrumental	The student’s appropriation of technology as a tool for doing mathematics
Anthropolo-gical	The impact of technology on praxeologies (Chevallard), The role of instrumented techniques

“Cognitive” approaches
1. “reification”

« « Many theoretical and empirical arguments may be employed to show that in mathematics, operational conception precedes the structural.

What is conceived as a process at one level becomes an object at a higher level. »

Sfard, A.,Linchevski, L. (1994) The gains and the pitfalls of reification — The case of algebra. *Educational Studies in Mathematics*, Volume 26, Numbers 2-3

	f
0	0
1	0.5
2	2
3	4.5
4	8
5	12.5
6	18
7	24.5
8	32
9	40.5
10	50
11	60.5
12	72
13	84.5
14	98

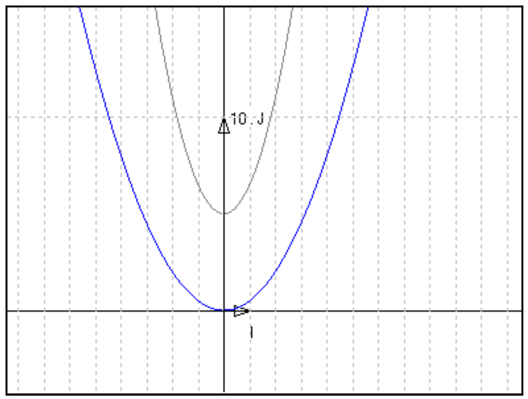
$$f(x) = \frac{x^2}{2}$$

reification

→

encapsulation

$$g(x) = 3 \cdot f(x) + 5$$



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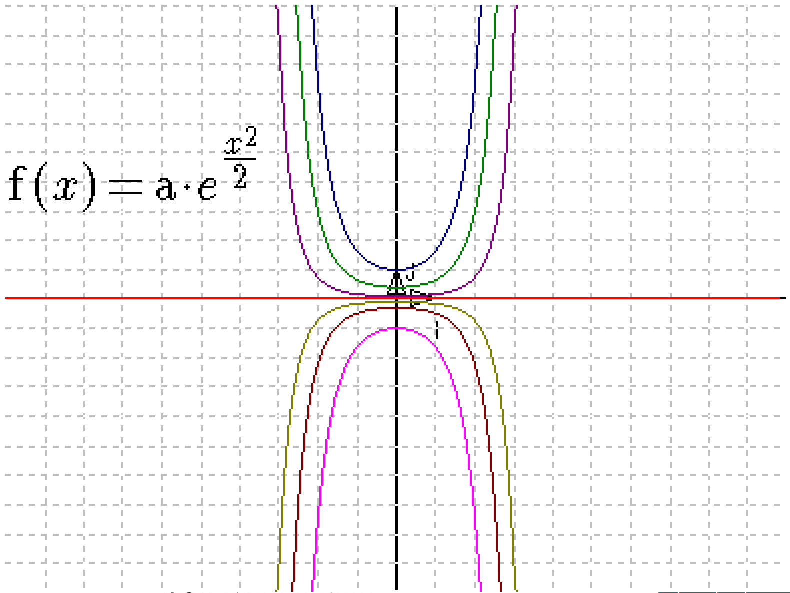
Procepts (Tall)

- An elementary procept is the amalgam of
  - a process,
  - a related concept produced by that process
  - a symbol which represents both the process and the concept.
- A procept consists of a collection of elementary procepts which have the same object.

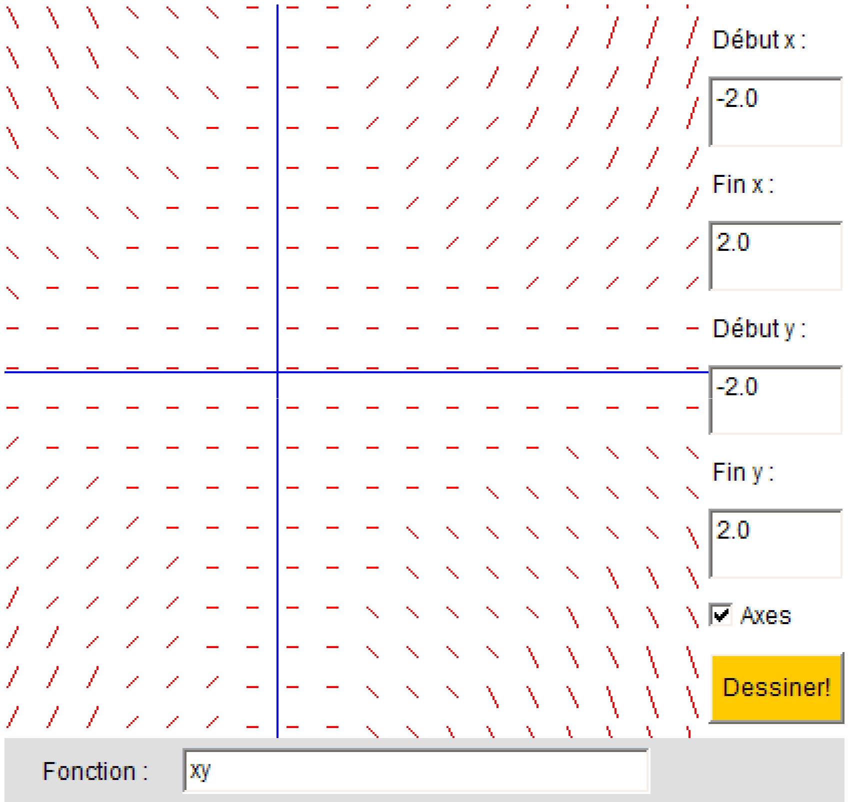
A more flexible approach

- The process involved must not first be given and "encapsulated" before any understanding of the concept can be derived
- Differential equation : y'=x . y



- Processus
  of solving -> Object solution

- software to show a small line whose gradient in (x,y) is x.y

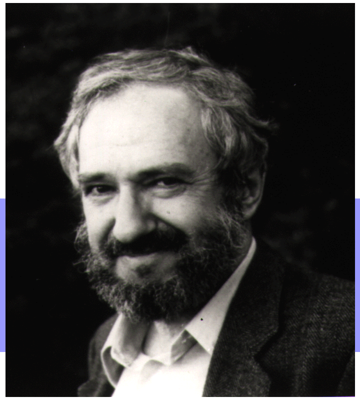


- This builds an *embodied notion of the existence of a unique solution* through every point,
- It provides a skeletal cognitive schema for the solution process *before* it need be filled out with the specific methods of constructing solutions.
- It uses the available power of the brain *to construct the whole theory at a schema level* rather than follow through a *rigid sequence* of strictly mathematical action-process-object.

"Cognitive" approaches
2.The idea of micro-world

- A more or less virtual space for learners
  - freely conceptualise by considering questions and constructing solutions.
- Powerful enough as to evolve
  - from the first vision linked to 'turtle geometry' (Papert 1980)
  - to recent projects like Mathlab (Noss & Hoyles 2006), based on the idea of building new representations.

Papert



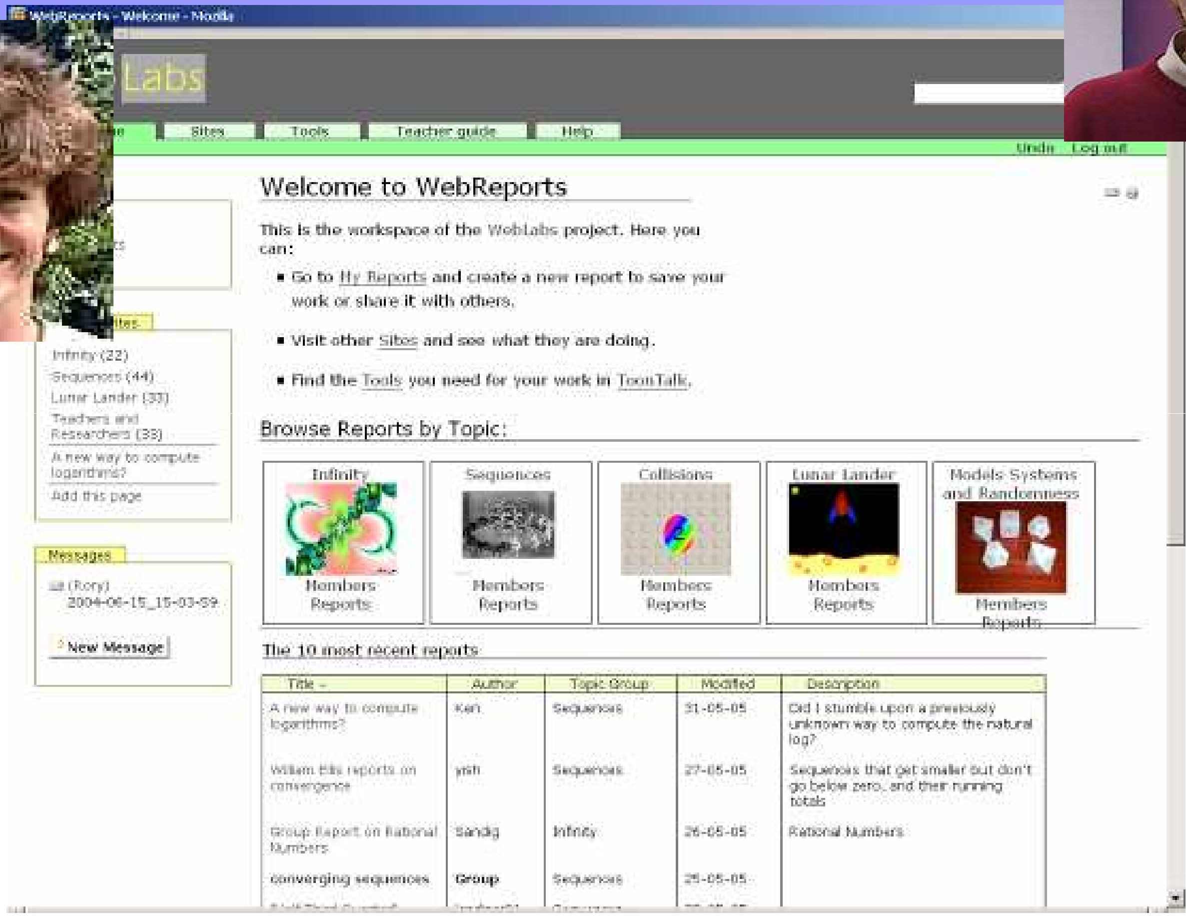
- “*constructionism* shares constructivism's connotation of learning as "building knowledge structures" (and)

then *adds* the idea that this happens especially effectively when learners are engaged in construction for a “public” audience".

Evolutions of the idea of micro-world

- Much Enthousiasm at first
- More realistic visions of possibilities offered par micro-world
- Growing awareness
  - of « situated » knowledge
  - approaches of learning based on
    - building connections (webbing)
    - communicating

Weblabs



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The instrumental approach:

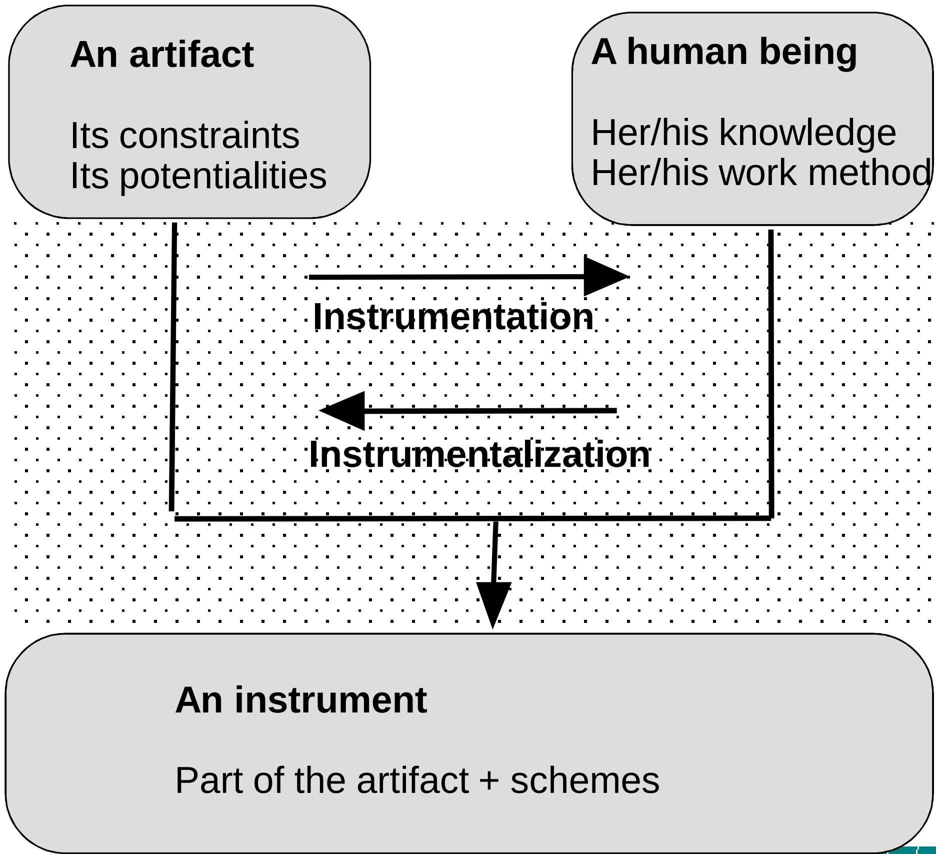
a research about « complex »calculators

$$f(x)=\frac{x^{3}+12\cdot x^{2}+\frac{1}{10}}{x}$$

- To obtain information about properties of the function by zooming
  - What mathematical knowledge?
  - What knowledge about the calculator?

Instrumentation

- Distinction between tool, artefact, instrument
- Instrumental genesis (Rabardel)
- Interwoven mathematical and instrumental genesis



The instrumental genesis

- Long and complex
- Consequences for teaching

Calculatrices graphiques, la grande illusion.

(Trouche, 1994) Repères IREM

- Not the same for all students

Not the same for all students

Théorique

Connaissances
Interprétation
Pbs généraux

Rationnel

Techniques
Conjectures
vérification

Scolaire

Copié/Collé
Pb syntaxe
Béquille

Bricoleur

Investigation
Machine
Rationalisation

Expérimen-tateur

Comparaison
Confrontation

The anthropological approach

- **The relationship between techniques and conceptualisation**

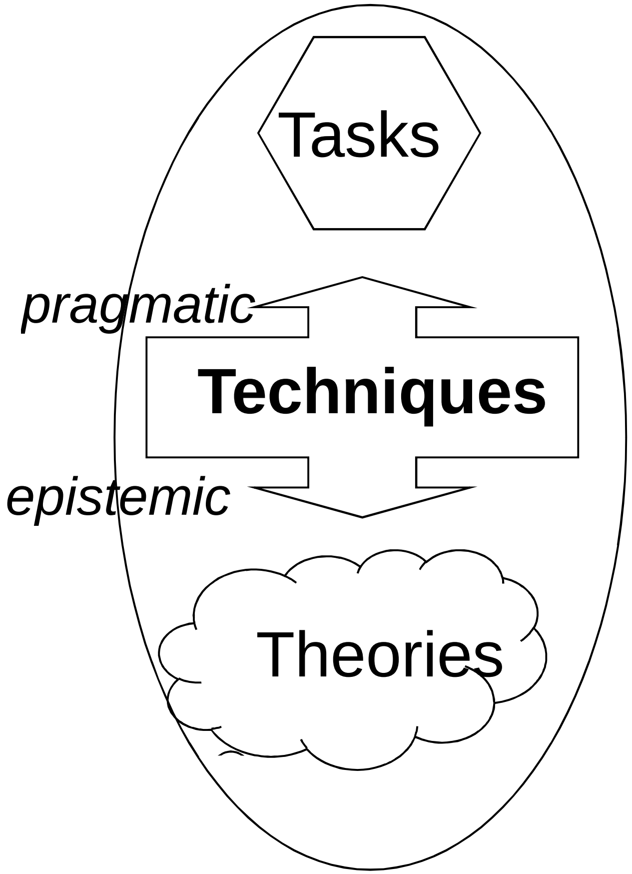
Concepts first, then skills ??

"If mathematics instruction were to *concentrate on meaning and concepts first*, that initial learning would be processed deeply and remembered well. A stable cognitive structure could be formed on which later skill development could build." (Heid 1988, p. 4).

Techniques and concepts

- Not so simple relationship
- Suppressing paper-pencil techniques
  - also suppresses the possibility of reflection on these, useful for conceptualisation,
  - brings difficulties related to teachers' systems of values
- The need for new praxeologies

Praxeologies and Techniques



- a manner of solving a type of task in an institution
- a complex assembly of reasoning and routine.

- a pragmatic value
- an epistemic value

Paper Pencil / Instrumented Techniques

Compare $\sqrt{3}\sqrt{9+4\sqrt{2}}$ et $2*\sqrt{11}$

Paper Pencil Technique	*Instrumented Technique*
1. Compute the squares : $3(9+4\sqrt{2})$ and $4*11$ 2. Compare: $12\sqrt{2}$ and $44-27$ 3. Compute the squares : $144\times 2<17^2$	√(3)*√(9+4*√(2)) 6.631030293 2*√(11) 6.633249581
Pragmatic Value: *Epistemic Value:*	*Pragmatic Value:* *Epistemic Value:*

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Pragmatic Value: weak *Epistemic Value:*	*Pragmatic Value:* strong *Epistemic Value:*

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Pragmatic Value: weak *Epistemic Value:* Properties of squares and radicals Compatibility of order and operations	*Pragmatic Value:* strong *Epistemic Value:*

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Properties of squares and radicals

Compatibility of order and operations

Question: is the equality of approximate values a proof?

For how many digits ?

Is the equality of approximate values a proof? A simple case

$A = a \cdot \sqrt{b}$ $B = \sqrt{c}$ a, b, c positive integers

Suppose that "with the calculator", we find values of A and B smaller than 1000 and "equals" at 10^{-4}. Can we conclude ?

- $A = B$ or $|A^2 - B^2| \geq 1$

$$|A - B| = \frac{|A^2 - B^2|}{A + B} \geq \frac{1}{A+B} \geq 5 \cdot 10^{-4}$$

Ruthven (2002)'s interpretation

"If mathematics instruction were to *concentrate on meaning and concepts first*, that initial learning would be processed deeply and remembered well. A stable cognitive structure could be formed on which later skill development could build." (Heid 1988, p. 4).

- In (Heid's) class, constitution of a quite different system of techniques
- The shift to "reasoning in non algebraic modes of representation [which] characterized concept development in the (Heid's) class"
  - created new types of task,
  - encouraged systematic attention to corresponding techniques
- (Heid's) experimental course
  - exposed students to (…) wider techniques;
  - helped them to develop proficiency in what had become standard tasks,
  - even if they were not officially recognized as such, and had not been framed so algorithmically, taught so directly, or rehearsed so explicitly as those deferred to the final 'skill' phase.

A European project
http://remath.cti.gr

- Focus on representations
- Problems related to theoretical fragmentation
- Including a dimension of software developpement (DDA)
- Experimenting in realistic contexts

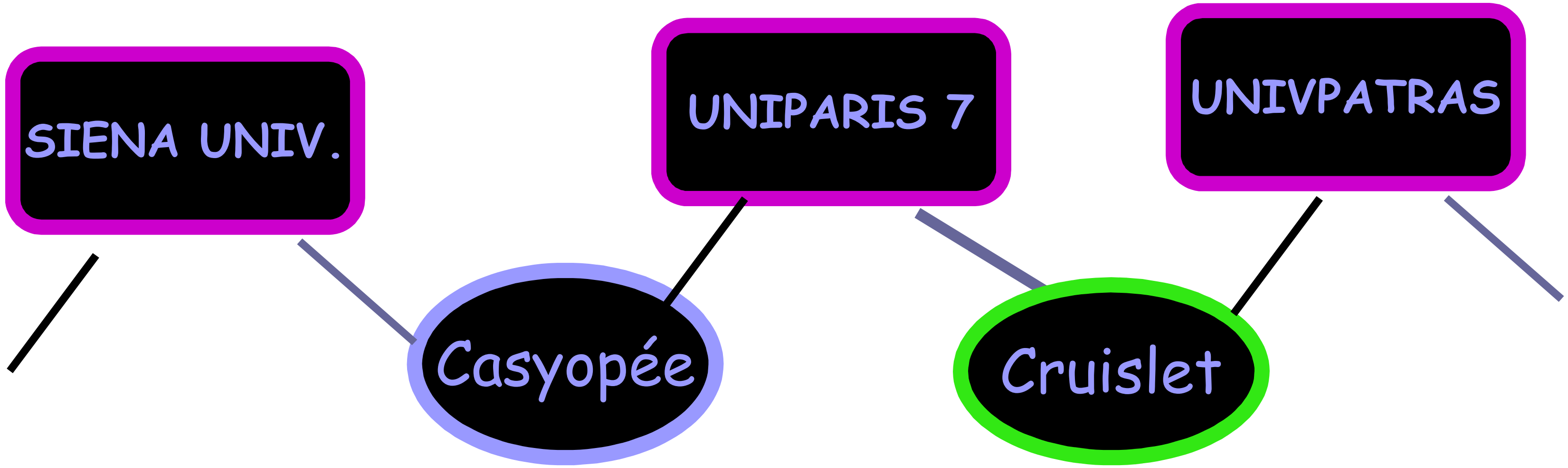
ReMath

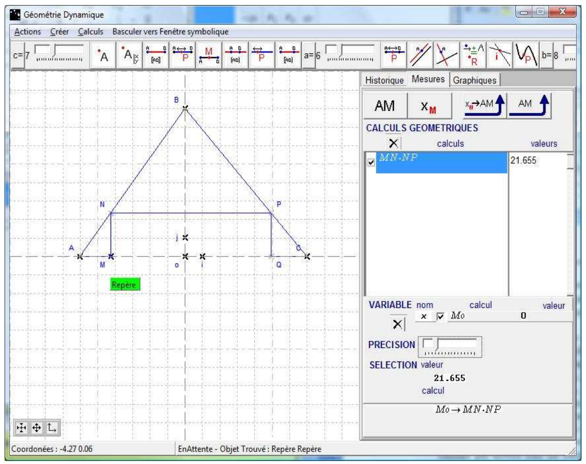
- WP1 : Theoretical integration
- WP2 : Software developpement
- WP3 : Scenarios (pedagogical plans)
- WP4 : (cross) experimentations
- WP5 : Multilingual repository and communication platform (Math.Di.L.S.)

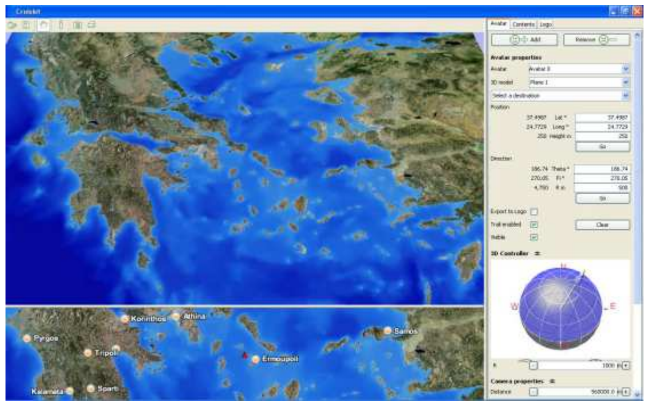
Linking WP1 (theories) and WP2 (developpement)

- In depth analysis of the design of Didactical Digital Artefact
- Precise identification of the role plaid by the theoretical frameworks versus prgamatic or contextual factors.
- Methodology
  - 1. a grid issued of an integrated framework to analyse specifications
  - 2. explicitative interviews about crucial decisions

The cross experiment



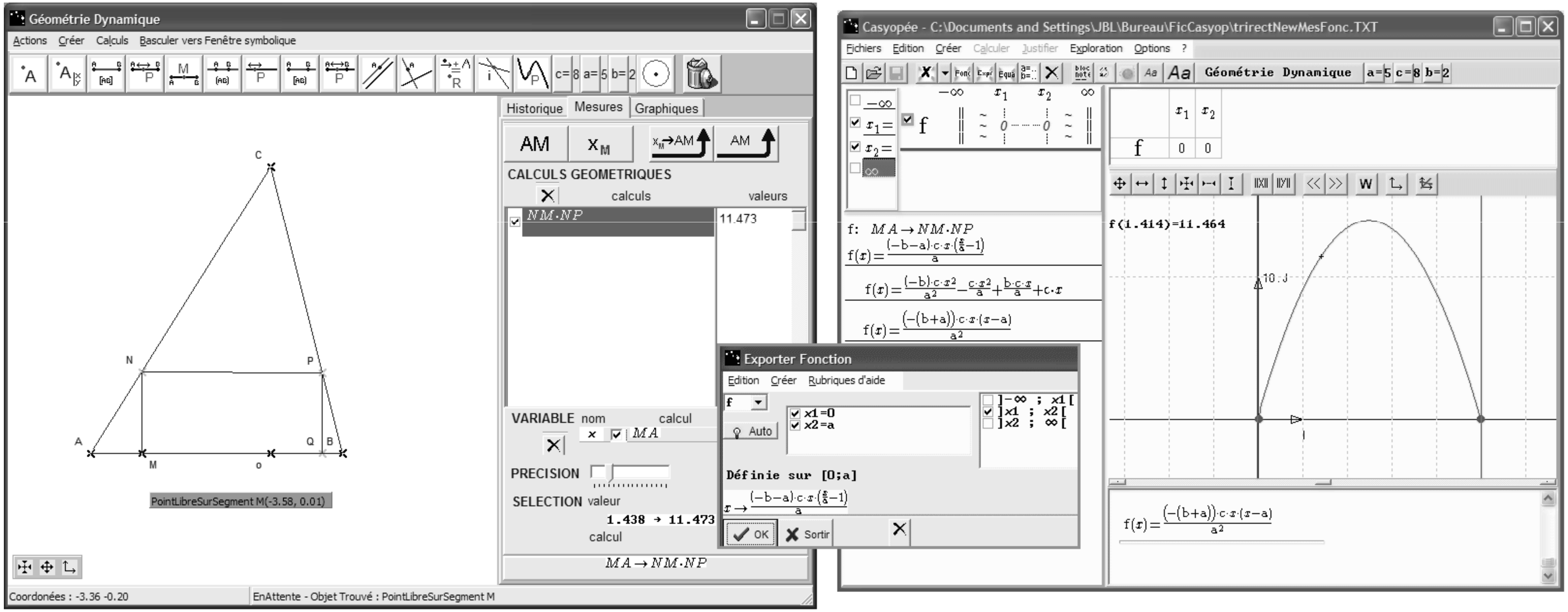




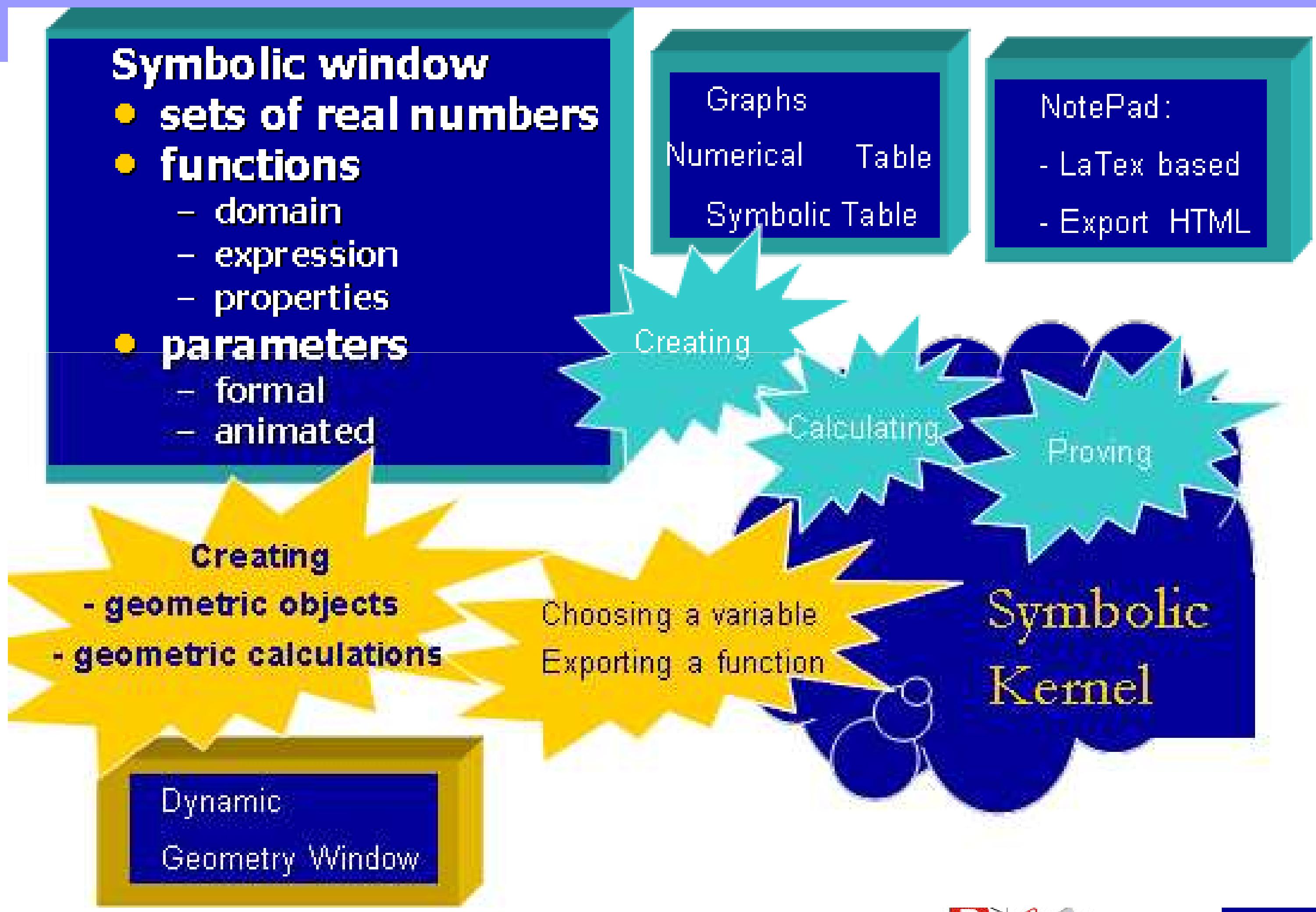
Some Results

- **Software Development**
  - Clarification
    - of decisions in the specifications
    - of the relationship between software and didactical functionalities
- Cross experiments
  - Better understanding
    - of the influence of theoretical frames upon the pedagogical plan, the classroom students' and teachers' activity and upon the analysis
    - of the influence of the cultural context.

Casyopée's interface



Casyopée' architecture



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Pedagogical plans

Approaching functions with Casyopee (familiar)

- associated functions
  - introduction
  - targeted functions
  - different expressions of a function
- functions and geometry: variables and equations
  - Introduction (to divide a triangle in pieces of fixed area)
  - Application (to divide a rectangle in pieces of fixed area)
- function and geometry:optimisation

Modelling in Casyopee (alien)

- Didactical cycle 1
  - familiarization
  - Problem of Optimization 1
  - Discussion 1
  - Problem of Optimization 2
  - Discussion 2
- Didactic cycle 2
  - Problem of Optimization 1 - revised
  - discussion on parametrization

Franco-Italian cross experiment

- The Didirem team : several theoretical frames.
- Attention to students' instrumental genesis
- Compatibility with institutional demand
- Process of learning designed through a careful choice of mathematical tasks, with an adidactical potential
- But the teacher's actions and role escapes the PP's design

Maracci M., Cazes C., Vandebrouck F., Mariotti M-A. (2009) Casyopée in the classroom: two different theory-driven pedagogical approaches, Proceedings of CERME 6

- The Unisi team has mainly structured its pedagogical plan according to the Theory of Semiotic Mediation
- The teacher plays a crucial role throughout the whole pedagogical plan, especially for
  - fostering the evolution of students' personal meanings towards the targeted mathematical meanings
  - facilitating the students' consciousness-raising of those mathematical meanings

The 17th ICMI Study
Technology revisited

http://icmistudy17.didirem.math.jussieu.fr

ICMI Studies

- Past Studies
  - 15. The Professional Education and Development of Teachers of Mathematics. (New ICMI Study Series 11)
  - 16. Challenging Mathematics in and Beyond the Classroom. (New ICMI Study Series 12)
  - 17. Digital Technologies and Mathematics Teaching and Learning: Rethinking the Terrain. (New ICMI Study Series 13)
- Studies currently under way.
  - 18. Statistics Education in School Mathematics: Challenges for Teaching and Teacher Education.
  - 19. Proof and proving in mathematics education.

Key dates

- 1985: First ICMI study: the influence of computers and informatics on mathematics and its teaching
- Cornu, B. & Ralston, A. (eds). 1992: The Influence of Computers and Informatics on Mathematics and its Teaching. 2nd edition, UNESCO
- July 2002: decision by the ICMI Executive Committee (EC) to launch an ICMI Study, the 17th, to be called "Technology revisited"

Key dates

- April 2004: First IPC meeting. London. The discussion document
- March 2006: Second IPC meeting. Paris. Selection of contributions
- 3-8 December 2006: Study Conference hosted by the Hanoi Institute of Technology Vietnam
- 2009: publication of the book

Goals

- Taking into account changes since the first ICMI Study
  - major developments in digital technologies
  - an increasing number and range of studies around the use of digital technologies in mathematics education,
- Broadening the focus of concern, in particular to consider the situation of developing countries

Goals

- Addressing diversity
  - in available software and hardware
  - in curricula organisations
  - in culture
- Seeking a balance between :
  - reflecting on actual uses,
  - providing visions for future of technology in Math Education

The Study Conference

ĐẠI HỌC
BÁCH KHOA

- Africa 3; Asia 9; Australia, New Zealand 11; Central and South America 8; Europe and Russia 52; Middle Orient 9; USA and Canada 22
- Plenaries, Working groups, Panels

Local teacher parallel workhop attended by 44 Vietnamese teachers, 3 teachers from Cambodia and 2 teachers from Thailand

ICMI study : The book

- Chapter 1: Executive summary
- Section 1: Design of Learning Environments and Curricula
- Section 2: Learning and assessing mathematics with and through digital technologies
- Section 3: Teachers and Technology
- Section 4: Implementation of technology-rich mathematics curricula: Issues of access and equity
- Section 5: Future directions

A personal view of evolutions

Meta Study (IHB 2003)	ICMI Study 17th
- Over representation of studies about the student and the computer	Still existing but new dimensions like: collaboration with technology 'assessment' with technology closer to the classroom and its daily functioning.
- Fragmentation of theories	Towards reconciling and synthesizing. New broader approaches of cognition with the computer (instrumentation, semiotic mediation…)
- Inexistence of 'dimensions' - Situations-Design - Teacher	Three sections about important questions, • Considering the design of tools *and* curricula as major issue for Math Education • A first successful approach of questions related to access & equity' • More research about the teacher • Tools to deal with teacher education

Still few studies about actual uses by 'ordinary teachers'
An ever evolving field with new questions especially about using the Internet

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Conclusion: Research in Technology and Math Education

- A very active domain with varied aspects
  - Quick evolving technologies
  - Institutional and social demand
  - Strong ideological pressure
  - Many interesting theoretical aspects
  - Many interdisciplinary collaborations
- Interesting phenomena
  - Slow diffusion of technologies into educational systems
  - Distance potentialities/uses/

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