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MATHEMATICS TEACHER RESEARCH: PROCESS, PRACTICE
AND THE DEVELOPMENT OF TEACHING

ABSTRACT. This paper discusses a study of teacher researchers undertaking research into self-chosen aspects of their own mathematics teaching at the secondary level. The study used qualitative methods to explore the processes and practices of this research and the issues which it raised. Emergent theory of teachers' research activity fitted strongly with a view of *action* research relating to *critical reflective practice*. The teachers were seen as reflective practitioners, developing knowledge and awareness through enhanced metacognitive activity. Their research was characterised as *evolutionary*, in contrast with established patterns of action research. Despite differences in the substance of each teacher's research, there were commonalities which provide insights into the conducting of research by teachers and its potential for the development of mathematics teaching. The special nature of mathematics in this research, and the role of external researchers in the project, were important considerations.

INTRODUCTION

Hard Questions Lead to Teaching Development

Teacher: If today's activity doesn't get them there, I will try and develop a different activity that *will* get them there.

Researcher: What does *getting them there* look like, and how will you recognise it?

Teacher: All I can say is I wish you wouldn't ask such difficult questions. (Jaworski, 1994, p. 193)

Developments in mathematics teaching occur when teachers address "hard" or "difficult" questions about their teaching and the thinking which motivates their teaching. Such hard questions cause a deep level of probing into the reasons for actions, interactions, activities, decisions, responses – all the elements which contribute to teaching and learning approaches in a mathematics classroom. The questions are hard because they challenge the fabric and philosophy of a teacher's mode of operation. One teacher acknowledged this challenge: "They [the questions] were hard because

they were challenging. They were questions I thought I ought to know the answer to but hadn't clearly articulated. I felt the question was important to me" (Jaworski, 1994, p. 198).

In the research quoted here, teachers acknowledged that what *they* referred to as *hard* questions from the researcher were instrumental in enabling them to delve deeply into their own purposes and become more overtly aware of personal theories motivating their practice. Over time, teachers started to anticipate questions, and to ask their own questions. It was clear that this developing process of questioning led to explicit forms of enquiry on the part of teachers into their teaching. Teachers' increasing metacognitive awareness of classroom decisions and judgements promoted changes in and exploration of classroom activity aimed at students' better understanding of mathematics.

This development is consistent with indications from other research into mathematics classrooms and teaching during the last decade (see, for example, Nolder & Tytherleigh, 1990; Davis et al., 1990; Cobb, Yackel & Wood, 1992; Krainer, 1993; Britt et al., 1993; Wood, 1997). All have indicated the power of teachers themselves exploring the implications of teaching acts for their pupils' developing understanding of mathematics.

The source of such exploration or enquiry lies often with the researchers directing a project rather than with the participating teachers. However central the involvement of the teachers, the motivating agenda lies with the external researchers. It could be a powerful force for teaching development if teachers themselves were to ask the questions and originate their own enquiry. In 1994, at Oxford, The *Mathematics Teacher Enquiry* (MTE) project was set up to explore the possibilities and effects of teachers setting their own agendas for tackling questions related to their teaching. The project was designed to study the effect of teachers' questions on their developing teaching, the processes and practices of such development, and issues arising for the teachers involved. A starting point for the project was the *source* of stimulation for teachers' questions:

In what ways might teachers themselves initiate questions which prove significant in the development of their teaching?

Teacher as Researcher

It was decided to draw on the work of the teacher-as-researcher, or *action research*, movement which had been gaining pace world-wide since the explorations of Lawrence Stenhouse in the 1960s. It seemed clear that undertaking classroom research would involve teachers fundamentally in generating questions which would motivate development of their teaching

(McNiff, 1988; Elliott, 1991; Altricher, Posch & Somekh, 1993, and many others).

The teacher-as-researcher movement had manifested itself in a number of large-scale projects such as the Ford Teaching Project in the UK (e.g., Elliott & Adelman, 1975), many classroom explorations by teachers motivated by the gaining of an academic degree (e.g., Vulliamy & Webb, 1992), and by the formation of an action-research network with its own journal (*International Journal of Educational Action Research*) reporting a wide variety of theoretical perspectives and studies undertaken by teachers. Teacher research seemed most often to be linked to higher degree programmes in universities or to externally directed projects led by university researchers or both. This was no less true in mathematics teaching (e.g., Krainer, 1993; Britt et al., 1993; Hardy, Hanley & Brown, 1994).

In those studies which were motivated by the gaining of a higher degree, research was linked to university studies involving an academic approach to methodology. Thus teachers' research conformed to academic standards and might be seen as a means to an end (gaining a degree) rather than the development of teaching. Nevertheless, as reported by Vulliamy & Webb (1992), outcomes of teacher research in degree courses showed significant influence on the development of teaching. The collaborative nature of many such projects was seen to contribute to their success in teaching development.

Some examples exist of teachers conducting research outside an academic framework but supported by some association or working group (see, for example, ATM, 1987; Zack, Mousely & Breen, 1997). A supportive structure seems important to the development which takes place.

The MTE Project sought to study teachers undertaking research in as naturalistic a setting as was possible, that is, a setting in which teachers were identifying and pursuing their *own* research questions independently of an academic programme. It sought to explore in what ways teachers themselves might motivate, direct and sustain such research. It recognised the importance of collaborative structures, suggested by other programmes, to provide support through collegial interaction.

TEACHER RESEARCH AND REFLECTIVE PRACTICE

Perceptions of Research Formality

Much debate in the action research movement has focused on the relationship between teachers' research in classrooms and established educational

research, particularly in terms of issues of validity and rigour in action research. If the aim is for teachers to ask questions which motivate developing practice and improved learning for pupils, why enter a debate about rigour in the research process? Could teachers' questioning activity be described better in terms of *reflective practice*, than in terms of *research*?

In the early days of the MTE project, the word *enquiry* was used in recognition of the potentially threatening nature of the word *research* for teachers who were unfamiliar with research. Stenhouse (1984) defined research, briefly and succinctly, as "systematic enquiry made public." This elegant and apparently simple definition suggests that enquiry alone is not sufficient. The word *systematic* carries a weight of meaning which might be seen to encompass issues of validity and rigour. In what respects the MTE research was *systematic* became a central issue in analysis of the MTE data.

In a group meeting at the end of the first year of the project, one teacher, Alex, stated the following:

When I started thinking about research, it seemed to me that any teacher is constantly engaged in research. Anything that you do, if you try to learn from it, that's research. It's just that what we're trying to do here is more formalised research.

Alex's point of view was that teachers engage in research in their everyday teaching acts as they construct classroom activities, reflect on these activities and feed back into their teaching the outcomes of their reflection. Interpretation of Alex's words suggests that research which is a part of everyday activity is *informal* research, perhaps because its research nature remains tacit and is not recognised overtly by teachers. The research in which the MTE teachers engaged was seen as more formal. Interpreting again, this suggests that Alex saw his activity in the project as more explicitly *research*. A question for the project was how this perceived degree of greater formality compared with the formality of more established research. Alex went on to suggest that the more formalised nature of the MTE research, as he perceived it, hinged on greater *objectivity* derived from getting "layers of evidence" to confirm "gut feelings." I quote here from Alex's words, recorded during a project meeting. Here teachers discussed differences between seeing good teaching as *reflective practice* compared with something that might be regarded as *research*, perhaps as *action research*. There were differences in view, some people clearly believing that reflective practice is much the same as action research. Atkinson (1994), (referring to Stenhouse, op cit.) suggests that action research, despite many differences in its definitions, is not the same as reflective teaching since "however reflective the teaching is, it would not

usually qualify for the term ‘systematic enquiry’ because of the complexities of the task and because of the nature of the very varied demands.”

Reflective Practice

The essence of reflective practice in teaching might be seen as the making explicit of teaching approaches and processes so that they can become the objects of critical scrutiny (e.g. Schön, 1983, 1987; Elbaz, 1987; Mason, 1990; Jaworski, 1994). Through such critical scrutiny, by teachers, teaching develops. Reflective practice may be regarded as a rather thoughtful way of teaching, evaluating what occurs and feeding into future planning without a demand for overt, critical, knowledgeable action. On the other hand, reflective practice might be seen more dynamically as *requiring* an action outcome. Dewey (1933) led the way with the following definition: “Active, persistent and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusions to which it tends constitutes reflective thought” (p. 9). He suggests further that:

reflective thinking, in distinction to other operations to which we apply the name of thought, involves (1) a state of doubt, hesitation, perplexity, mental difficulty, in which thinking originates, and (2) an act of searching, hunting, inquiring, to find material that will resolve the doubt, settle and dispose of the perplexity (p. 12) and “Demand for the solution of a perplexity, is the steadying and guiding factor in the entire process of reflection” (p. 14).

Dewey’s two points here articulate beautifully the theoretical position on which the MTE project was premised, that is, firstly, a recognition of questions to address, identifying some perplexity, making some aspects of teaching problematic; and, secondly, through some processes of enquiry, to seek solutions, or resolutions to, or new ways of understanding, the problems identified.

Kemmis (1985) has highlighted a relationship between *reflection* and *action*:

We are inclined to think of reflection as something quiet and personal. My argument here is that reflection is action-oriented, social and political. Its product is praxis (informed, committed action) the most eloquent and socially significant form of human action. (p. 141)

He advocates *critical* reflection, in which reflection is concerned with thought itself, transcending strictly technical or practical reasoning to “consider how the forms and contents of our thoughts are shaped by the historical situations in which we find ourselves” (p. 141). We reflect from our own ideological standpoints, and these ideologies change as a result of reflection. We make choices which influence our actions and affect our subsequent experience: “In reflection we choose, implicitly or

explicitly, what to take for granted and what to treat as problematic in the relationships between our thought and action and the social order we inhabit” (p. 148). Kemmis associates reflection with what he calls *emancipatory action research*, a form of critical social science increasingly being employed in educational settings including professional development. It involves participants in:

planning action (on the basis of reflection); in implementing these plans in their own action (praxis); in observing or monitoring the processes, conditions and consequences of their action; and evaluating their actions in the light of the evidence they collect about them (returning to reflection) as a basis for replanning and further action. This is the spiral of self-reflection composed of cycles of planning, acting, observing, reflecting, replanning, further action, further observation, and further reflection. (p. 156)

Thus “action-research” in Kemmis’ terms can be seen as a paradigm emerging from the linking of *reflection* and *action*; a way of conceptualising reflective activity which emphasises the resulting action.

Schön (1983, 1987) has interpreted these theoretical positions into a practical domain, highlighting stages of knowing, thinking and reflecting and their relation to action in practice. A progression from *knowing-in-action*, through *reflecting-on-action* to *reflecting-in-action* (e.g., 1987, p. 25 ff.), traces a development of awareness and understanding and an overt growth of knowledge about practice. There is considerable research which shows that experienced teachers operate from a complex knowledge base (e.g., Calderhead, 1987; Brown & McIntyre, 1993). Where this is not well articulated, or remains tacit, it might be described in Schön’s terms as *knowing-in-action*. As teachers start to question aspects of their teaching, or their thinking about teaching, identifying problems and perplexities, they might be seen to move to a position of *reflecting-on-action* in which they start to look critically at events after they have occurred. This stage involves a metacognitive awareness in which knowledge and action are linked. Mason (1990), in his model “the discipline of noticing” points to the importance of overt “noticing” of significant acts or issues, leading to their “marking” in future practice. *Marking* leads to overt recognition of choices in subsequent activity. Evidence in my own research has shown that persistent *reflecting-on-action* leads to *reflecting-in-action* as teachers start to be aware of actions, decisions and judgments as these occur in their teaching or planning for teaching (Jaworski, 1994, p. 197 ff.).

The widespread applicability of reflection-in-action has been questioned by Eraut (1995) who points out that Schön presents little empirical evidence of reflection-in-action, especially where teaching is concerned. The word *action* itself has different meanings for different professions. In teaching, *action* usually refers to action in the classroom where teachers

operate under pressure. Eraut argues that time constraints in teaching limit the scope for reflection-in-action. He argues that there is too little time for considered reflection as part of the teaching act, especially where teachers are responding to or interacting with students. Where a teacher is walking around a classroom of children quietly working on their own, reflection-in-action is more possible but already begins to resemble time *out* of action. Thus Eraut suggests that, in teaching, most reflection is reflection-on-action, or reflection-*for*-action. He suggests that Schön is primarily concerned with reflection-*for*-action, reflection whose purpose is to affect action in current practice.

From the above discussion, a key term is *action*, both in reflection *in*, *on* or *for* action, and in *action* research. The MTE project has looked critically at the action taking place in teachers' research and judged this in terms of the reflective practice of the teacher researchers along with the resulting research evidence for its influence in developing practice. Analysis suggests that the three prepositions highlighted in the above discussion all pertain to the thinking of teachers at different points in their research.

A Gap in Research

In a provocative lecture in 1996, David Hargreaves slated the current position of educational research in the UK in providing a basis for educational development: "In education there is simply not enough evidence of the effects and effectiveness of what teachers do in classrooms to provide an evidence-based corpus of knowledge." He spoke of a gap between researchers and practitioners: "It is this gap . . . which betrays the fatal flaw in educational research. For it is the researchers, not the practitioners who determine the agenda of educational research." He argued that providing a research base for teaching is essential, but that it "will require a radical change both in the kind of research that is done and the way in which it is organised".

The MTE research speaks to both of these issues; not only does it start to bridge Hargreaves' gap, but it provides insights into alternative ways of conceptualising research. The project set out to address overtly an aspect of educational development – that of the development of mathematics teaching; and it involved practitioners, the teachers, directly in the research. In fact, all participants in the research were regarded as *researchers*, distinguishing as necessary the roles of the teacher researchers and university researchers. An important feature of this research was collaboration between these groups, and their joint contribution to the growth of knowledge within the project.

CONCEPTS AND METHODS IN THE MTE PROJECT

Key precepts for the MTE Project, related to the theoretical perspectives expressed above, were therefore:

- (a) collaboration between teacher researchers and university researchers to originate and study the processes and practices of research in classrooms and its contribution to the development of *mathematics* teaching;
- (b) focus on the nature of teachers' classroom research, in particular in what ways it might be regarded as systematic in keeping with Stenhouse's definition, and in what ways it was similar to or different from established research;
- (c) attention to ways in which teacher *research* interfaced with *reflective practice*, keeping in mind the constructs of reflective practice offered by Dewey and Kemmis; and considering in what respects reflection on the part of teachers might fit Schön's construct of reflection-in-action, recognising Eraut's reservations.

Research Questions

The MTE Project addressed four main research questions:

1. How do teachers formulate their research?
2. What is the nature of the evolution of the teachers' research?
3. What is the role of the project in motivating and sustaining teachers' research?
4. In what ways does the research influence mathematics teaching: how is the nature of the subject, i.e. *mathematics*, significant to this research?

Questions 1 and 2 follow directly from the theoretical debate above.

Despite its aims to study teachers' questioning, research or reflective activity in as naturalistic a setting as possible, the MTE study involved an externally directed project. It was necessary to look critically at the ways in which the project itself influenced the activity of the teachers, and this is reflected in Question 3.

The teachers in the MTE study were *mathematics* teachers with developments in teaching aimed at improving the learning of *mathematics* for pupils. Question 4, therefore, includes a consideration of how the teachers' research, or reflective activity, was related to the nature of mathematics itself as well as to the development of mathematics teaching.

The Participants of the Project

The project was seen, initially, as a pilot study designed to test out theoretical perspectives and methodology and provide indications for further research. It was deliberately small in scale and began with six teacher researchers and two university researchers. It was designed to take place during one year, but, due mainly to the enthusiasm of participants, its work extended to two years.

The university researchers were Clare Lee and myself. The six teacher researchers were Adam, Alex, Jeanette, Julie, Nick and Sam (all pseudonyms). Although several of the teachers have now published writing about their involvement in the project under their real names, pseudonyms are maintained here to recognise the interpretive nature of a university researcher's writing about the work of teacher researchers, as well as for consistency with earlier writing (e.g., Jaworski & Lee, 1994).

Of the six teacher-researchers, Adam began work in the project but dropped out after 6 months due to pressures within his school. Jeanette was pregnant when the project began, and started to play an active role only at the beginning of the second year of the project. Alex and Nick worked together in the same school, and undertook a joint enquiry.

The role of the university researchers was to study the processes and practices in the teachers' research. The teachers each originated and conducted research in their own classroom related to their own choice of focus: sometimes a personal interest or concern of the teacher, and sometimes an issue of essential concern within the mathematics department or school as a whole. University researchers visited teachers in their schools, often visiting classrooms and observing lessons. They also organised regular meetings – two in each of the three terms of the school year – at the university for all researchers. Research funds paid for teachers to come out of school to attend meetings, but some schools were sufficiently supportive to waive charges where classroom cover was not required.

Research Methods

Methodology in the project must be seen at two levels: the *local* level, in terms of each teacher's research; and the *global* level in terms of the study of the teachers' research as a whole. The methodology of the teacher researchers, a central study of the project, was closely related to the substance of their research and both substance and methodology *evolved*, as will be discussed below.

At the global level, research has been qualitative and interpretive (Ball, 1990; Brown & McIntyre, 1993; Burgess, 1985; Hammersley, 1990; Jaworski, 1994; Jaworski, in press). Data was collected through

- (a) participant observations by university researchers in mathematics lessons taught by teacher researchers;
- (b) conversations in school between teacher researchers and university researchers;
- (c) group meetings at the university including all researchers;
- (d) reflections of the university researchers.

Data consists of reflective writing, field notes, and audio-recordings – most of which are transcribed wholly or in part. Triangulation was built into data collection with conversations (b) allowing checking and confirming of observations (a), and meetings (c) providing alternative accounts of events and issues to contrast with data resulting in (a) and (b).

Analysis has followed, loosely, the concept of data-grounded theory introduced seminally by Glaser & Straus (1967). It has taken the form of close scrutiny of the data: listening and relistening; reading and re-reading; noting categories, seeking patterns, triangulating evidence from different sources. As a result of discussion of the early data between the university researchers, a rough coding schedule was devised on the basis of which one researcher went on to refine the schedule through analysis of further data. Subsequently, the second researcher repeated this process, comparing results continually with those from the first researcher, until agreement and saturation was reached. This dual approach was necessary for both researchers to become thoroughly absorbed in the data, and was valuable in checking categories, avoiding omissions or duplication, and providing a rigorous validation of results relative to the *total situation and context* of the research. Central in this respect are relationships between, and the differing goals of, participants, as discussed below.

Wherever possible, accounts written by the university researchers were given to the teacher researchers for reading and commenting. Often these accounts formed the basis of discussion at project meetings, which then provided further data for validation purposes.

Relationships between Researchers

A central issue in the MTE Project was the interrelationship, and interdependence, between teachers' research and the study of this research. This could be seen in (a) setting up the project, (b) questioning teachers about their research, and (c) providing supportive structures to sustain research.

Setting up the project. Firstly, and fundamental to this project, is the requirement that there would be teachers undertaking research, that this would be their own choice without external pressures, and that they should be autonomous in conducting their research. In studying the processes

involved, and potential for teacher development more widely, it was important to see how teachers would approach research issues and contend with methodological concerns. Thus, for example, guiding their choice of questions or methods, or providing a course in research methods, would counter these aims. On the other hand, it was unrealistic just to look out for teachers who might be engaged in classroom research. Experience says that such teachers would be hard to find. Thus, volunteers were invited from a network of mathematics teachers associated with the university through a partnership programme in initial teacher education. There were 7 volunteers initially, and 6 embarked on the project.

Questioning teachers about their research. The study of the processes of the teachers' research involved university researchers in observing, talking with and asking questions of the teacher researchers. It would have been naïve to ignore the university researchers' potential for influence on the teachers' research activity. However, explicit care had to be taken, by the university researchers, to remain as neutral as possible where teachers' decisions and judgements in the research were concerned. Questions were deliberately restricted to *finding out* about the teachers' thinking, planning, teaching and research. Nevertheless, the impossibility of avoiding influence was recognised as a significant factor in observation and analysis. The questions asked, the language used, the relationships which developed were all central to the environment in which the project grew, and thus objects or issues in its analysis.

Providing supportive structures to sustain research. Part of the study involved observation (by the university researchers) of teachers' responses to and needs in undertaking research. Support in beginning and sustaining research was essential to the conducting of teacher research. The university researchers had to be a major source of support, and ways in which this was achieved were critically scrutinised as part of the research process. Based on evidence of the value of collaborative processes in learning environments (e.g., Slavin, 1983, 1989), regular meetings of all researchers provided alternative forms of support, for example in peer interactions between teacher researchers.

THE NATURE OF TEACHER RESEARCH IN THE MTE PROJECT

This section looks at the forms which teacher research has taken in the MTE Project. It focuses particularly on the status of teacher research relative to expectations of established research, and questions whether the research

might be considered *systematic*. Paradigmatic examples from the research of individual teachers, Julie, Sam, Alex and Nick, are provided to highlight issues and relate theory and practice. The roles of the university researchers are scrutinised and criticised.

Teachers' Formulation of Research

Experienced researchers begin a research project often with a clearly designed rationale and methodology, or at very least with a security in research knowledge and expertise. For teachers who have little or no research knowledge, training or experience, the prospect of engaging in research can seem foreign and daunting. The teachers in the MTE project were volunteers. Thus, to some extent they were willing to address the challenge of engaging in research. Four of them had no experience, and two only minimal previous research involvement. It would have been possible for the university to provide some form of research training, but this would have countered the expressed aims of the project, to study teacher research in a naturalistic setting.

In order to commence research, teachers had to identify some question, interest, issue or concern which their research would address. This identification and subsequent moves towards beginning research were the first hurdle of the project, more problematic for some teachers than others. Analysis of these beginnings indicates a *tentativity* which, although taking different forms, was a characteristic of the research of all the teachers.

When the study began, Jeanette, Sam, Alex and Nick all had starting points in terms of issues or questions to explore. The tentativity for them became evident as they considered approaches to the research, or when they reflected back later on their initial activity. For Adam and Julie, what to focus on initially was more of a problem. Adam did not get beyond this stage, but Julie did. Her initial struggles point towards what might be typical for many teachers in their first forays into research. I therefore look at the issue of tentativity in some detail through Julie's experience, and then contrast this with examples from the other teachers.

Julie's beginnings. Julie volunteered to become part of the project in July, 1994. In September, after the summer break, she was panicking and ready to drop out before the project had really begun. I resisted trying to persuade her to continue, since such persuading would be counter to the aims of the project: it might lead to teacher participation for the wrong reasons (for example, pleasing university colleagues) and set up a vision of fitting into predefined guidelines. My response was to ask her why she had volunteered – what aspects of her practice she had been interested in exploring – and to

try to focus on her concern for her own teaching. My belief, deriving from personal experience, was that putting emphasis on her (familiar) teaching rather than on (unfamiliar) processes of enquiry might prove reassuring in the short term.

Julie did continue. Realising that she did not need, immediately, to specify her research programme removed some of the pressure and reduced her panic. One of her concerns was that the research could not be a substantial “add-on” to her work load. It had to be part of, and relevant to, her day to day work as a teacher. For all the teachers this was an important criterion in two respects. Firstly, time constraints and other pressures on teaching meant that the research had to fit into a heavy schedule. Secondly, the research made sense only if it was firmly related to teaching.

We jointly acknowledged these provisos. Julie invited me to sit in on some of her lessons and talk with her about what she might explore. In her classroom she is a confident teacher, having clear rapport with students and a well defined approach to teaching. There were evident differences of confidence in Julie’s approach to her classroom teaching and to her research activity.

Research notes I made on her lessons, and transcripts of audio recordings of our conversations, trace the progress of Julie’s thinking and enquiry. Initially, as a result of a school inspection which judged the mathematics department to be weak in the *investigational* elements of its mathematics teaching, she wanted to focus on the investigational nature of her teaching.¹ This seemed rather vague, and the vagueness was part of her problem. She could not conceive of what she might actually *do* to commence enquiry.

As we discussed Julie’s lessons, her aims for what students should achieve, and the ethos of her classroom, a clearer focus began to emerge. She was interested in the interactive nature of classroom activity. In particular, she was interested in *mathematical talk*. In some way she wanted to gain access to how such talk affected students’ mathematical understanding. Her own account, (Hall, 1997) provides insights to this emergent focus.

Beginnings for Sam, Alex and Nick. Sam is a highly confident Head of Mathematics Department in his school, a strong mathematician with a deep love for mathematical enquiry and an extrovert approach to teaching. He wants his students, like himself, to engage in mathematical problems and questions, develop their own mathematical arguments, and work towards deep relational understandings (Skemp, 1976).

At the time of beginning research, he had developed a classroom approach which he applied confidently, but he noticed that some classes, or

students, worked “productively” with the approach, while others were more “resistant” – Sam’s own words and constructs. Sam wanted to explore the reasons behind these observations. His classroom approach had involved students working in interactive groups on problems which he set for them. Thus, his first steps in enquiry were to try to gain access to what students did and said in their groups when he was not present, a difficult task to achieve himself. He felt that access to their discussion would be helpful to him in finding out more about how well his approach was working for his students.

Sam’s approach to his research seemed to reflect few of Julie’s worries: He seemed very happy to engage in enquiry and to suggest what the initial stages might involve. He asked various people – myself, other teachers, student teachers – to observe certain groups in his classroom and report the students’ activity and thinking to him after a lesson. His research thinking seemed not to go beyond this initial action. What he would do with the accounts from the observers was left to be decided once some accounts were available.

Alex and Nick were teachers from the same school; Alex, Head of the Mathematics Department, and Nick a teacher in the department. They decided to look, jointly and complementarily, at who was seen to have responsibility for the mathematics which was taught in the school. Alex wanted to explore this in order to inform his decisions as Head of Department, and Nick was interested in his students’ perspectives and how these might inform his own teaching. I quoted earlier Alex’s statement about all teachers engaging in research. He said, “I had the feeling I was already doing research and I wanted to do it in a more formal way and see if it led to the same answers”.

Alex and Nick seemed (a) to know how they wanted to begin, and (b) to engage in a systematic approach to their research. They started by devising a questionnaire for students, trialling it with one or two classes, modifying the questions as a result of initial responses, and administering the questionnaire to a whole year group of the school. This process seemed, at that stage, to fit more established norms of research. It was not until much later in the project that remarks from the two teachers pointed towards its tentative nature. Towards the end of the first year of the project, at a group meeting, Nick reported: “It is becoming more evident as we are doing this research what we are interested in. At the start we were . . . well waving our hands around, but now we seem to be going in definite directions.” He indicated that they had been unsure in the initial stages what tools were right for that stage. So they had begun with trial and error: “We tested the water, without plunging in head first, by using a questionnaire . . . the

questionnaire hit the top of the list at the kick-off because it is a standard tool of research – you know, for Gallup polls and things.” It was clear that, despite their research seeming to be more systematic in its initial planning and selection of research tools, it was tentative in its overall rationale.

What emerged clearly from this early work was the value for teachers of *doing* something in the initial stages on which they could reflect. Reflection led to new thinking and new action. Reflection here seems to be *on* rather than *in* action.

The Evolution of the Teachers' Research

The tentative way in which research began was common to the teachers but reflected differently in their research substance and approach. Their enquiries might seem like individual explorations, with idiosyncratic emphases and emergent issues, from which no overall conclusions can be justified. However, as Delamont & Hamilton (1984) point out:

Despite their diversity, individual classrooms share many characteristics. Through the detailed study of one particular context it is still possible to clarify relationships, pinpoint critical processes and identify common phenomena. Later abstracted summaries and general concepts can be formulated, which may, upon further investigation be found to be germane to a wider variety of settings.

It seems reasonable, from this early work, to suggest certain indications for further consideration.

1. Teachers, lacking knowledge or experience of research processes, while secure in their own knowledge and philosophy of teaching, find conceptualisation of the research process initially problematic.
2. For such teachers, starting by *doing* something related to their interests and concerns is an effective way to begin conceptualising the research process.

It is the second of these indications that I take up in this section. In all cases, *doing* led to further enquiry. The results of the doing needed to be questioned. In some cases, the doing led to unexpected events or identification of issues. Questions were raised which teachers wanted to explore. Activity led to interest, curiosity, questioning and subsequent enquiry.

From this position, subsequent enquiry led to further insights and issues which, in their turn, suggested further questions or enquiry. Analysis of such cycles of activity, insights and issues leads to a conceptualisation of the research process as one of *evolution*: one phase developing dynamically from the previous phase through the activity and reflection of the teachers. The *evolutionary* nature of the research is a quality synthesised

from field work and analysis. It is here that we see the theoretical perspectives underlying this project being manifested in the practical substance of the project. We might ask how consistent is this *evolutionary* nature of research with paradigms of *action research*, and to what extent the research might be seen as “systematic.”

Julie’s evolution. Julie’s enquiry went through a number of stages, each evolving from the previous one, not preplanned. These stages followed a pattern of devising some research task or activity which Julie thought might address her current concerns.

Initially, she wanted to find out more about the mathematical “chat” (Julie’s word) in her classroom. First she asked an observer, myself in some cases, to sit with a group of students in the class, keeping a record of instances of mathematical or non-mathematical chat in the group. The results of this observation provided information about how much of the talk was about mathematics, but she gained little information about the *nature* of the mathematical talk. She realised that its *nature* was important, a characteristic which had not been obvious earlier. This realisation was an outcome of the recording activity.

Subsequently, Julie decided to record, herself, her perceptions of the talk she observed in a lesson. She would record, from memory, rough percentages of different categories of talk (e.g., one student explaining to a group; or a group arguing about a concept), based on her observations. That often percentages added up to more than 100% was not a problem as she was looking for general trends rather than accurate figures.

The data generated through these two modes of enquiry proved to be both valuable and unsatisfactory. They were valuable in focusing Julie’s attention on what it was she wanted to explore, showing trends in the activity of different classes or groups of students, and in allowing Julie to develop awareness of the nature of her enquiry. They were unsatisfactory in that they provided little insight into the *quality* of mathematical talk, the need for which Julie came to see more clearly as its lack became evident. Analysis showed that the early stages of her enquiry sought quantitative data – how *much* of different kinds of chat – and that she moved toward more qualitative data in her search for deeper meaning, eliciting concepts of *nature* and *quality* of chat. This paralleled similar development in the research of Sam, and of Alex and Nick.

Julie became so dissatisfied with her inability to provide insights into the *quality* of mathematical talk that, again, she seemed to come close to abandoning the project. She told me that she was “stuck,” and was clearly unhappy and depressed. However, she did not ask me for suggestions, nor

did I offer any. I recognised nonetheless that I had a difficult decision to make in the event that Julie made no further progress. I discussed with Clare Lee whether we should decide to make some suggestion to Julie as to potential ongoing activity. We decided to wait until after the next group meeting, where each teacher researcher would report on their research, before making a decision.

Excitingly for us, Julie came to the meeting (as if) a new person, full of enthusiasm and initiative. In the intervening time, she had talked with Sam about the use of tape recording to capture elements of student discourse. This had given her the idea of leaving a tape recorder with groups in her class, to record their “mathematical chat,” to which she could listen at her leisure. It would then be possible to work on what *quality* might mean, having the possibility to play and replay students’ words. In her subsequent enquiry, Julie identified significant episodes from the recorded tapes, and her analysis involved identification and justification of the nature of their significance for her.

The word *quality* initially seemed to capture a rather vague property of mathematical chat. However, Julie’s awareness of wanting to focus on quality led to her recognition of aspects of quality on which she could focus. This led to her explicit seeking for examples of these aspects in her data. Analysis suggests that this process could be conceptualised as reflection-in-action. It seems to encapsulate a degree of metacognitive functioning which allows a more knowledgeable approach to further action.

Julie’s feelings of exhilaration and success came from having experienced the problem and tackled it herself; analytically, we might say this was due to increased metacognitive awareness, providing more power and control. It was clear that Clare and I could not have engineered this fruitful outcome. It seemed crucial that Julie herself would determine what she wanted to find out and that this should be commensurate with her developing awareness of needs and possibilities, and of her own cognitive functioning: what others have called ownership, or *agency* (Povey, 1995; Burton, 1996). As Julie herself attested later in a group meeting, it was more valuable for her to reach these recognitions herself since they carried powerful associated knowledge based on her experience and critical concern.

Sam’s evolution. Sam’s initial activity resulted in reports from colleagues who observed groups of students in his classroom, and reported on their thinking and activity when he was not present. These reports presented, to Sam, cognitive challenge, socially experienced. He acknowledged their salutary nature. For example, observations of groups in his class pointed

to two girls whose activity was quite different to Sam's expectations and, as he saw it, reflected unhappily on his teaching (Jaworski & Lee, 1994). In a number of cases, the perceptions of certain students in his class were revealed as different from his expectations, which led to a sincere concern to find out more about students reactions to his teaching. He made clear that this was not an easy recognition or line of action. It threatened his confident teaching approach and left him uncertain of what alternatives might be possible. However, he recognised also its potential for developing a teaching approach more sensitive to his students' experiences and needs.

As a result of such challenges, Sam's research proceeded in two ways: (a) interviews with pairs or groups of students in two of his classes, either by himself or by me, using an interview schedule which he provided; and (b) audio-recordings of groups of students during a lesson which he could later replay and analyse.

Analysis of data from Sam's teaching shows evidence of reflection *on*, *in* and *for* action. Our conversations after his lessons are one example of his reflection *on* action. These reveal his developing awareness of issues related to his preferred modes of teaching and their effect on students. Evidence of reflection *in* action is provided by episodes in which Sam's sensitivity to students becomes more overt *during* his teaching and he modifies his plans accordingly. An example of this appears in the next section. Reflection *for* action is evidenced in Sam's further research activity, such as the interviewing, in which he attempts to find out more about students' views and learning needs, to inform his future teaching.

Evolution in enquiry. The way I have written about these teachers' research progression makes it appear smoother than in fact it was. Ordinary school pressures acted to distract from and interrupt research. Often little happened for several lessons or even several weeks. Then a teacher might 'lurch' back into action, possibly in response to some idea or stimulus. The visit of one of the university researchers was often the stimulus. The process of lurching was a real and realistic aspect of the research. It was understandable. It fitted well with the circumstances, both of teachers working under pressure, and of teachers in an unfamiliar research situation, being unsure of what to do or where to go next. Its different nature to theorised patterns of action research (McNiff, 1988; Elliott, 1991) was in its lack of a clear direction or of regularity, a well defined process, or a preconceived pattern of action. However, the research included elements of the cycles indicated by Kemmis (1985) "cycles of planning, acting, observing, reflecting, replanning, further action, further observation, and further reflection" (p. 156). These were not neat, regular, well-planned cycles,

and the teachers' research could be accused of being unsystematic. This was one reason for preferring the term *evolutionary*. However, if systematic enquiry is seen as enquiry which *develops* systematically, rather than enquiry which is planned according to a previously defined system, then both Julie and Sam may be seen as involved in systematic enquiry.

It is possible to trace a clear pattern of evolution in these enquiries. The evolution occurred in teacher researchers' research methods as a result of their own developing awareness and cognitive processing. It charts a progression of activity and reflection *on* and *in* action, and *for* future action. The value, and to some extent inevitability, of this evolutionary process in research is in its recognition of the complexities of teaching. Compounding these complexities is the interrelatedness of substance and methodology, which are rarely distinct. The cognitive development of the researcher parallels closely development of the research process and analysis of data (Jaworski, in press). Knowledge grows through experience and cognitive challenge (made overt in the research process) within a social situation (Piaget, 1950; von Glasersfeld, 1995). Such is the metacognitive position to which these teachers were developing in their research activity. The strength of this process can be seen in terms of the teachers' developing knowledge and practice.

The Supportive Nature of the Project

Throughout this section examples have been given of the university researchers' interactions with teacher researchers in the project. As well as a research role, enquiring into the teachers' thinking and action throughout the project, the other main role was a supportive role, enabling teachers to undertake and sustain research. The supportive role was mainly responsive to teachers' declared or perceived needs. For example, had I not encouraged Julie to invite me into her classroom and discuss her teaching, she may have dropped out of the project. On the other hand, Andy did drop out. Perhaps more overt support from the university researchers would have enabled him to continue. Judging the degree and effects of support is very difficult. We avoided making explicit recommendations or providing guidance, which created difficult decision points, as for example when Julie was "stuck."

Support which was overt in the project came through our visits to schools and our organisation of group meetings. Teachers indicated repeatedly the value of the meetings to motivating and sustaining their research. The sharing of experiences and discussing of issues were especially valued. Moreover, there was evidence that visits and meetings were stimuli for research activity as there had to be something to discuss. It is clear that

teachers' research, albeit deriving from their own concerns and questions and taking idiosyncratic forms, was strongly motivated by and sustained through the supportive structures of the project.

THE INFLUENCE OF THE TEACHERS' RESEARCH ON THEIR TEACHING OF MATHEMATICS

The teachers in this project were *mathematics* teachers. The focus of their lessons, and ultimately of their research, was students' learning of mathematics and its improvement. One of the research questions (Question 4) relates to the mathematical nature of the teachers' research:

In what ways does the research influence mathematics teaching:
How is the nature of the subject, i.e. mathematics, significant to this research?

The substance of the research was mathematics teaching, which suggests two interrelated foci, those of pedagogy and of mathematics. Analysis suggests that, for the teachers, issues of substance were more explicitly connected with pedagogy, albeit mathematical pedagogy, rather than with mathematics per se. However, with a more critical scrutiny, it is possible to see mathematical issues underpinning the pedagogical ones. I shall present three examples from mathematics lessons to elaborate this position.

Area of parallelograms. The mathematical task for one of Julie's lessons to her Year 10 class (age 15), involved students in drawing and finding areas of parallelograms by counting squares on graph paper and looking for patterns in their results. Lesson objectives were (a) to develop students' understanding of the concept of *area*, and (b) for students to learn how to find areas of parallelograms.

While the class was working on the task, Julie walked around asking questions to get students to talk to her about their thinking: "How are you getting on?", "Can you see what's happening?" She believed that encouraging students to talk about what they were doing would result in their enhanced understanding of the mathematical concepts. In this lesson, she was surprised at "not having to make more input," meaning not having to provide more explanations, and attributed this to students effective "monitoring" of each other's work. It seemed that, for Julie, such *monitoring* was an important characteristic of valued mathematical talk.

In the students' work, a relationship between the area of a parallelogram and its dimensions had emerged and been discussed by students in their

groups. Julie found this an important outcome of the lesson. She said, "The outcome I wanted has appeared . . . met partially my objectives. Most people are quite happy with the relationship they've discovered. I can only make overall judgement of how successful it's been if next year they can [give evidence of their understanding of areas of parallelograms]." She differentiated between feeling, locally, satisfied with the outcomes of her lesson in terms of students' immediate responses on the topic, and recognising that a much longer term perspective was necessary to make judgements about learning. In other words, students could, at this stage, find areas of parallelograms, but what this meant in terms of broader mathematical relationships might not be obvious until much later.

This was an important lesson in tracing Julie's development of the concept of mathematical talk and its quality in contributing to mathematical learning. For Julie, research issues seemed to lie in the nature of the talk and its relation to students' learning. Issues relating to mathematics were mainly implicit. However, analysis suggests that there may be an issue here of mathematical talk linking the more instrumental nature of *knowing how* to find areas of parallelograms with the conceptual nature of area and relationships between parallelograms and other geometrical shapes. Studies of language in mathematics would focus more closely on the overtly mathematical nature of these levels of understanding (Pimm, 1987; Sierpinska, 1994), and perhaps Julie's developing research sophistication might ultimately take her in such directions.

Planes through a cube and algebraic precision. Sam introduced a lesson to his Year 10 class with the words, "I'm going to introduce you to some real imaginative thinking today." He had in his hand a perspex cube containing some water. There was agreement in the class that the surface of the water was square in shape. He then tilted the cube and asked about the shape of the surface. There was some disagreement, but certain students maintained it was still square. Sam said, "Lots of people are saying "square." Are we agreed that that's square? I want each table to come to an agreement."

I observed a table of girls where one girl kept insisting the tilted surface was square. Another girl placed a book vertically and rested a pencil case, sloping upwards from the table, resting against the book. She indicated that there was a greater distance to the book along the pencil case, than horizontally along the desk. There was discussion about what the shape of the water surface might be. The girls then went on to construct a 3-dimensional model from 5 books and to explore the result of tilting the book in the middle. Discussion and argument revolved around whether the shape would be a rectangle or parallelogram. The girls were deeply engaged

in the problem, struggling with possibilities for the shape in question. They used the books as ready tools to support their thinking and imagery.

After 10 minutes of the above activity, Sam tried to interrupt the class to gain their attention and initiate some sharing of ideas between groups. Despite trying hard, he was largely unsuccessful. Groups were avidly engrossed in their discussions, and did not welcome interruption. The classroom was buzzing with conversation, all of it (as far as I could see) about mathematics. He waited for about another 15 minutes before finally drawing the class together. He said to me afterwards, “There were just too many good things going on . . . if I had stopped them and tried to get them to listen – to someone who might have a superb justification – I didn’t think they were at the stage. . . .” He said, “I was meeting a lot of resistance there. It could be seen as disruptive on their part, but I didn’t interpret it as that.” This is another example of the teacher reflecting *in action*.

The research issue here, for Sam, was in modifying his teaching in sensitivity to students’ needs, recognising that he might once have interpreted their response as *resistance* to his teaching. He recognised students’ needs to spend more time in developing the mathematical concepts involved, and therefore resisted pushing forward to the next stage of the problem as he saw it.

The above analysis related strongly to that of another lesson reported elsewhere (Jaworski, 1996). The object of this lesson was perception by students of generality in the use of brackets in algebraic expressions. Activity was based around exploring the effects of placing operators (+, −, ×, ÷) and pairs of brackets between the three numbers 6, 3, 2 and inspecting the outcomes (e.g., $(6 + 3) + 2$ or $6 + (3 ÷ 2)$). Sam’s research focus was on students who were “productive” or “resistant” to his teaching. He discovered that some students readily moved from their activity in trying special cases, to a generalisation of the process, which was what Sam had hoped for. This group he termed “productive.” However, one group became very cross with Sam, and resisted when he tried to push them beyond their specific cases. They felt some success in discovering that there were 32 distinct cases of placing brackets and operators, which had been one of the questions of the lesson. It seemed, to them, that they had achieved what they had been asked. To be asked, subsequently, to do more, seemed to devalue what they had achieved so far. Thus, not only did this resistant group not perceive the need for generalisation, they felt unhappy with the teacher’s apparent lack of appreciation of their effort and achievement. For Sam this was extremely salutary. His personal focus as a result of the event was how to adapt his teaching so that it would

be more sensitive to the needs of these students. The mathematical issues were somewhat implicit in this focus.

In both lessons, Sam, the mathematician, had in mind the generality behind specific examples of concepts. In the case of the perspex cube, this generality lay in a recognition of the shapes of various planes through a cube. In the case of the expressions, it was about algebraic representations and the need for brackets. Mathematically, it seems essential for students, to appreciate the generalities involved. For those who remain at the level of the particular, their mathematical development is limited. However, pushing students too rapidly towards such generality may result in their losing interest or confidence both in their mathematics and in the teacher's teaching.

Thus a question arises: What tasks, questions, or classroom activities will enable most students to move less problematically to mathematical generality and abstraction? How is this recognisable by a teacher? For Sam, how does it link to his perceptions of students being resistant to his teaching? Such questions might be a part of Sam's further research.

Research Issues and Mathematical Issues

For Julie, substantive issues were about becoming more critically aware of the nature of mathematical talk which she considered to be of value to mathematical learning, and possibly about how to generate and sustain such talk. For Sam, they were about coming to terms with the problematic nature of his preferred style for encouraging mathematical thinking and involvement in his classroom. Although these issues involve *mathematics* learning and teaching, the issues for the teachers seem essentially pedagogical rather than mathematical.

It was notable that, in project meetings, discussions mainly took for granted that the substance of teacher research was *mathematics* teaching. Thus, often, we discussed aims and objectives in the research, research methods and their uses and outcomes, and only referred to *mathematical* learning or teaching to exemplify research issues. It was as if, for the teachers, the mathematical nature of their work was a given, which was implicit and unquestioned. Decisions about what mathematics should be done, what classroom tasks would be appropriate, and what outcomes would be desired, were a normal part of the teaching process, hard to extract as problematically related to the research issues.

Yet further analysis makes clear the essentially mathematical issues which underpin the research issues, as exemplified above. A further phase of this project might valuably focus more explicitly on these mathematical issues.

CONCLUSIONS

Conceptualisation of Teachers' Research

The project set out to create an environment in which to study teachers' own questioning of their teaching, and its outcomes related to their developing teaching. While striving for as naturalistic a setting as possible, it had to be recognised that the project itself was a key factor in what was observed. Given this recognition, evidence from the project provides important insights into teachers' questioning and its outcomes. Although the number of teachers was small and self-selecting, certain of the processes, practices and issues were felt to be germane to a wider variety of settings (Delamont & Hamilton, 1984).

Casting the required questioning in the mode of research enquiry was a project decision to provide a basis for questions to be asked. It was successful in this aim. Teachers did ask questions. Initially questions were tentative and ill-formed, but successive cycles of activity led to new and more precise questions. There was evidence that engaging in research led to teachers refining their thinking and making concerns more explicit. As they engaged in research activity, it became clearer what they wanted to know, and moreover what forms of action might lead to this knowing. Metacognitive development was clearly evidenced in the project data.

Analysis of data from observations and discussions in school, and from the project meetings, led to emergent theory regarding the processes in which teachers were engaged. This was seen to fit closely with a rationale of critical reflective practice and action research drawing on the work of Dewey, Kemmis and Schön. Teachers were seen to reflect *on*, *in* and *for* action. Action, a key concept in the research, led thinking initially and became ultimately inseparable from thinking. The teachers' research was described as *evolutionary* to recognise the dialectical relationship between thinking and action as the two developed symbiotically.

In working with a model of action research, it was seen to be important to question teachers' research activity in terms of its commensurability with norms of established research. Although lacking an overt *system*, it was retrospectively possible to see, in the research, a systematic process consisting of cycles of reflective activity through which knowledge grew and was refined. This process lacked regularity and was described as "lurching" from one cycle to the next often influenced by external stimuli of which project visits and meetings were significant.

Growth of knowledge in the project can be seen in terms of teachers own individual learning related to their substantive concerns; in terms of the importance of supportive and collaborative structures in enabling ques-

tioning and enquiry; and in terms of the theorising of the teachers' research process. The first of these is mainly individual and idiosyncratic. Evidence indicates the teachers' growth of understanding in their substantive areas, which led to development of teaching. This development was rather in a way of thinking about teaching than in obviously different practices. However, subtleties in developing practice were revealed which can be justified as closely allied to developing thinking. These are revealed strongly in the writing of the teachers (e.g., Hall, 1997; Edwards, in press).

In making public the outcomes of their research, teachers contribute to research rigour. It is hard to validate the research in terms of substantive outcomes. For example, what Julie learned about mathematical talk must be seen in the context of Julie's classroom and personal teaching perspective. However, in offering the outcomes of her research to other teachers, she shares her own insights and invites responses. Validity rests in the possibility of others seeing and making sense of Julie's work and using it to illuminate their own work and thinking. This is a rather different way of regarding research rigour, but nevertheless important to the use of research as a vehicle for developing teaching. It seems to be an important response to Hargreaves' (1996) "gap" challenge.

In judging the outcomes of the research in terms of developments in teaching, it would be useful to be able to point towards enhanced mathematical experiences for pupils, and to relate these to the special nature of mathematics as a discipline. It must be acknowledged that the project barely began to address these issues. Evidence shows that teachers were all concerned about their pupils' mathematical learning and progress. Their questions and enquiry were aimed at exploring aspects of perceived need in their classroom teaching. Such needs were expressed pedagogically in terms such as pupils' use of language, or of pupils' resistance to modes of teaching. There was little incisive analysis of how such needs related to mathematical learning, or indeed to mathematical objects or structures (e.g., as expressed in Simon, 1995; Steffe & D'Ambrosio, 1995). In the project, teachers' refining of their research questions had not reached a position from which such analysis might emerge. Where one of the project meetings was devoted to a discussion of how mathematics played a role in the research, the discussion revolved around classroom anecdotes and research experiences.

Analysis showed mathematical issues arising from the data, and associated questions to explore: for example, the issue of students' appreciation of mathematical generality, which was outlined earlier. It was tempting, being aware of such issues, to try to draw them to the teachers' attention, but this was resisted as being part of our agenda but not of theirs.

The Influence of the Project on Teachers' Development

The role of the university researchers in this project was primarily to *study* the research of the teachers. To guide or otherwise influence the teachers' research would have altered the nature of this study. However, as one of the reviewers of an earlier draft of this paper pointed out, even though we remained passive and unobtrusive, we might have provided role models, thus catalysing the consolidation of a new community of practitioners. The decision to be minimally interventionist was undoubtedly problematic. By our very presence we intervened in the practice of the teachers. Where we withheld judgments, we sometimes restricted our own learning on issues of interest to ourselves. On the other hand our aims were to explore the teachers' thinking and development, not to pre-empt it.

One reason for withholding judgments lay in the dangerous area of judging the *quality* of teaching and learning. Such judgments needed to come from the teacher researchers, both in terms of good relationships within the project and for the teachers' own development. As researchers it was not appropriate overtly to promote values or to criticise teaching, although, as implicit role models we may have had these effects.

It has been clear that the teachers would have been unlikely to engage with and sustain research without the motivation of the project. However, it was the teachers' choice that the project continued for a second year. They volunteered to come to meetings after school rather than in school time as there was less money available to pay for time out. One reason for this vote of confidence was seen to be the regular project meetings which teachers enjoyed and found valuable in their developing research and teaching. Despite differences in their interests and goals, issues arising for each of teacher researchers were recognisable by the others. Recognition and mutuality led to inspiration and motivation. It is possible to see cognitive and social processes interacting here most favourably. Teachers' independent activity was subjected to social scrutiny in an environment which encouraged and stimulated thought and action.

Issues for Further Research

The value for these teachers of engaging in research to explore questions critical to their teaching was seen overwhelmingly to be a significant influence on teaching development, through the enhanced thinking of the teachers. A new phase of the project is planned to draw on what has been learned from this first phase. Two areas stand out as being important for further research.

1. Some means is required of reducing the influence of the project while still providing a collaborative environment to motivate and sustain

research. A way forward is to locate the research in a group of teachers within their own school environment, working together for mutual support. In this situation the external researchers would be less central to teachers' research activity, and the school itself could benefit from assuming the mantle of responsibility.

2. How might mathematical issues become more overt in the research process? One possibility is to find a school or schools with an interest in making mathematical issues per se the focus of their investigation. Another might be to support one or more teachers over a longer period of individual research, to find out whether their greater refining of questions might lead to a more critically mathematical focus.

We are currently formulating a new phase of the project to address these areas.

NOTE

¹ See Jaworski (1994), Chapter 1, for a discussion of the nature and history of investigational mathematics teaching in UK classrooms.

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