

BARBARA JAWORSKI

THEORY AND PRACTICE IN MATHEMATICS TEACHING  
DEVELOPMENT: CRITICAL INQUIRY AS A MODE  
OF LEARNING IN TEACHING

**ABSTRACT.** In this paper I address the challenge of developing theory in relation to the practices of mathematics teaching and its development. I do this by exploring a notion of ‘teaching as learning in practice’ through overt use of ‘inquiry’ in mathematics learning, mathematics teaching and the development of practices of teaching in communities involving teachers and educators. The roles and goals of mathematics teachers and educators in such communities are both distinct and deeply intertwined. I see an aim of inquiry in teaching to be the ‘critical alignment’ (Wenger, 1998) of teaching within the communities in which teaching takes place. Inquiry ‘as a tool’ and inquiry ‘as a way of being’ are important concepts in reflexive developmental processes in which inquiry practice leads to better understandings and development of theory.

MY AIMS IN THIS PAPER

I start from the point of view that improvement of mathematics learning in classrooms is fundamentally related to development in teaching, and that teaching develops through a learning process in which teachers and others grow into the practices in which they engage. In order to avoid perpetuation of undesirable practices, with a shift towards *critical alignment* (Wenger, 1998), I propose *inquiry* as a fundamental theoretical principle and position. As teachers and educators we can use inquiry as a tool to enable ourselves and others to engage critically with key questions and issues in practice. Such practice can involve addressing mathematical tasks in classrooms, developing approaches to mathematics teaching or finding ways of working with teachers to promote teaching development. I believe that use of *inquiry as a tool* can lead to developing *inquiry as a way of being* (Jaworski, 2004) when practiced as part of a community, in which members collaborate, as learners (co-learning, Wagner, 1997), to develop their practice. The paper addresses these ideas from perspectives of theory

and practice to offer some ways ahead for mathematics teaching development.

### TEACHING AS LEARNING IN PRACTICE

Mathematics teaching is intended to promote the learning of mathematics. Although this is probably a 'safe' statement – few will wish to challenge it – it is hard to go further without running into matters of interpretation, judgment, philosophy and epistemology. Over the last four decades mathematics education has addressed philosophical and epistemological perspectives with respect to mathematics and to mathematics learning. It has become common to think of mathematics in fallibilistic terms (e.g., Ernest, 1991; Freudental, 1978; Skemp, 1976), to consider learning as a constructive process (e.g., Davis, Maher, & Noddings, 1990; Glasersfeld, 1987) to situate knowledge and learning relative to communities of practice (Lave & Wenger, 1991) and to debate the commensurability of constructivist and sociocultural learning theories (Lerman, 1996; Steffe & Thompson, 2000). Theoretical considerations like the nature of mathematical knowledge, what it means to know mathematics and to come to know it, how knowing in mathematics is related to knowing in social settings more widely,<sup>1</sup> have been deeply considered and seriously debated (e.g., Bauersfeld, 1995; Cobb, 1996; Confrey, 1995; Kieran, Forman, & Sfard, 2001). Looking back over these three decades we might refer to 'big theories',<sup>2</sup> such as constructivism and sociocultural theories, that have been highly influential in addressing mathematical knowledge and the learning of mathematics. The mathematics education discipline has become mature in such theoretical considerations. However, the position of mathematics teaching remains theoretically anomalous and underdeveloped. While theory provides us with lenses for analysing learning (Lerman, 2001), the big theories do not seem to offer clear insights to teaching and ways in which teaching addresses the promotion of mathematics learning.

We might see one of the problems to lie in relationships between learning, teaching and *practice*, for example *the practice of teaching*. Theories help us to analyse, or explain, but they do not provide recipes for action; rarely do they provide direct guidance for practice. We can analyse or explain mathematics learning from theoretical perspectives, but it is naive to assume or postulate theoretically derivative models or methods through which learning is supposed to happen. Research shows that the sociocultural settings in which learning and

teaching take place are too complex for such behavioural association (e.g., Bauersfeld, 1988; Cobb, Confrey, di Sessa, Lehrer, & Schauble, 2003; Potari & Jaworski, 2002). We can, of course, devise models and methods to promote learning, analyse them through theoretical lenses and use theory to explain associated learning outcomes. It seems reasonable that the practice of teaching mathematics can and should draw on our depth of knowledge of mathematical learning, and learning theory, but to theorise teaching is a problem with which most educators are struggling. As a result of such struggles, we have many theoretical frameworks or constructs related to mathematics teaching and its development, but nothing to compare with the big theories of learning; for example, Tom Cooney has written:

But if we are to move beyond collecting interesting stories, theoretical perspectives need to be developed that allow us to see how those stories begin to tell a larger story. That is we should be interested in how local theories about teachers can contribute to a more general theory about teacher education (Cooney, 1994, p. 627).

I will not claim that this paper succeeds in offering a general theory, but I would like, at least, to start a process of moving towards such theory.

One way to draw on theories of learning to explain or characterize teaching is to see teaching as a learning process. Theory cannot show us what teaching *should* involve, but teachers and educators can search for clearer understandings of what teaching *might* involve; thus we learn about teaching with the possibility to develop teaching. I borrow from Jean Lave the phrase “Teaching as Learning, in Practice” (Lave, 1996), in which she draws on social practice theory (Lave & Wenger, 1991) to describe and explain learning *in* practice. For example, like the novice tailor, being drawn into the (community of) practice of tailoring from a (legitimate) peripheral position, practicing alongside old-stagers in the community, perfecting processes and skills, learning the trade, we might see the novice teacher being drawn similarly into the practice of teaching. Here we see *teaching* as a social practice in which teachers are practitioners. The process of legitimate peripheral participation is one of continuous development, not a sudden move from novice to experienced practitioner on the completion of a module or the passing of a test. The process of growth continues throughout practice. Etienne Wenger (1998) has taken these ideas further in conceptualising learning as developing *identity* (identity as a teacher, for example) through participation in a community of practice. Learning is presented as a “process of becoming”. Wenger states, “It is in

that formation of identity that learning can become a source of meaningfulness and of personal and social energy” (p. 215). He speaks of “modes of belonging”, including *engagement*, *imagination* and *alignment* (174, f.f.). We engage with ideas through engagement in communicative practice, develop those ideas through exercising imagination and align ourselves “with respect to a broad and rich picture of the world” (p. 218). It is hard to do justice to these ideas in a short space, but the notion of alignment, as I expand it here, needs a further comment. Alignment within a community of practice results in individual members aligning themselves with conditions or characteristics of the practice. Through the exercise of imagination during engagement, alignment can be a critical process in which the individual questions the purposes and implications of aligning with norms of practice. I refer readers to Wenger for a more thorough discussion of these modes of belonging. For my purposes here I will suggest a form of *critical* alignment in which it is possible for participants to align with aspects of practice while critically questioning roles and purposes as a part of their participation for ongoing regeneration of the practice.

If we take these rather abstract ideas into the arena where teaching normally happens, schools and classrooms, then we can start to see both possibilities for explaining school and classroom situations in terms of such perspectives, and problematic consequences for such theoretical application. If the community of practice is the school community in which learning to be a teacher, or developing as a teacher, is seen as a process of becoming a teacher within this community, then Wenger’s vision is one way of seeing the growth of teaching knowledge; learning about teaching. However, looking at any particular school community, would we recognize teachers as aligning themselves critically with respect to a broad and rich picture of the (school) world? What exactly would we see?

Brown and McIntyre (1993) conducted a study of teaching in which they looked at the practices as manifested in normal day to day activity within their selected schools. They characterized teachers and pupils as achieving “normal desirable states” of activity, or being, within classrooms. These normal desirable states were ones in which (tacitly) negotiated goals and practices settled down to achievable routines with respect to which participants could spend their time without aggravation or excessive disruption. Thus the normal desirable state might result in a harmonious classroom, but not necessarily in providing effective learning opportunities for all students.

If we analyse the outcomes of Brown and McIntyre's study in social practice theory terms, then the process of legitimate peripheral participation, for any teacher being drawn into this community, might be seen as one of learning to develop the normal desirable state and maintain it for the well-being of all those involved as 'normally' perceived in the school setting. Thus participation here looks more like a perpetuation of the practice – an alignment that lacks a critical dimension – although I recognize that we might debate the meaning of these terms. I would see critical alignment to include some sense of teachers critiquing and trying to develop, improve or enhance the status quo, alongside enculturation into existing social norms. However, the significance of normal desirable states is just that they *are* desirable within the social practices in which they have developed. It is hard to operate against such practices, or to challenge them in practice. My long experience as a teacher educator has provided many examples of young, imaginative teachers, starting from critical perspectives on teaching, who are drawn into the normal desirable states of school culture in ways that make pursuance of imaginative ideals too difficult to sustain.

Nevertheless, I believe that the notion of teaching as learning in practice is a valuable theoretical position from which to view teaching. I see the term "critical", in "critical alignment", as indicating a key concept for avoiding the perpetuation of un-desirable states. As such, it is a theoretical concept, and it is important to ask: what would (or could) critical alignment look like in practice? Teachers have to practice within school communities, where deep, sociosystemic complexities are involved. What forms would or could critical alignment take with respect to the full range of such complexity and what issues would have to be addressed?

My theme in this paper is to look at teaching as learning in practice with a perspective of critical alignment in mind. The theory that I will suggest here is of teaching as *learning-to-develop-learning* in a mode of critical inquiry in which the normal desirable state shifts to one of co-learning inquiry (Jaworski, 2001). I will suggest that such shifts require a conscious level of awareness of the practice and a desire to achieve more than the practice, at a particular time, seems to make possible. The shift might be seen conceptually as one from learning within a community of practice to one of forming a community of inquiry. Here, critical alignment is achieved through communities of inquiry involving students, teachers and educators, in which all participants are learners and in which learning can be regarded through

learning theory lenses as remarked earlier. The shift itself is of course problematic. This complexity of language and abstract concepts needs some grounding, so I will first offer some examples from practice before coming back to look at these theoretical perspectives in greater detail alongside practical manifestations and problems of practice.

### SOME SNAPSHOTS FROM PRACTICE

Since this JMTE Special Issue is about relationships between theory and practice, it seems important to keep both parts of this in view. However, while we (you the reader and I the writer) can engage with/in theory together through the ideas in this paper, it is almost impossible to engage together with/in practice. At best, I can talk about practice as I see and experience it, and you can summon up images from your own experiences of practice in response. I am going to refer to research in which I have been a participant, in order to summon up images of practice which I/we can then address in theory and theoretically.

For 20 years, since I left school teaching in 1984, I have worked with teachers in school (practice) settings in developmental and research contexts. In this I have been a practitioner, although the nature of my practice has varied according to its particular circumstances and goals. Mainly I have fulfilled the roles of a teacher educator and/or researcher. In fact I might call myself a mathematics-teacher-educator-researcher. Appropriating Wenger's terminology, I believe I have, over the years, had the opportunity to engage with ideas through communicative practice, develop those ideas through exercising imagination and align myself (critically) with respect to a broad and rich picture of educational practice. I regard myself as extremely fortunate in the opportunities that I have experienced for learning and development. I attribute much of this good fortune to the privilege of working with some dedicated and committed teachers and educators with whom I have been able to engage in communicative inquiry. I have learned both to theorise practice and to be aware of issues in practice relative to this theorising.

One of the things I have learned is that teaching is difficult, perhaps even more difficult than teachers (and often educators) themselves realize (Desforges & Cockburn, 1987). To take an example from research, an experienced teacher, Ben, planned an investigative task, involving moving objects on a square grid, for his class of 14 year-olds. He wanted *them* to decide certain parameters of their investigation. Before his lesson, he said that he expected a question of "diagonal

moves to come up”, and, in the lesson, true to his expectations, it did. There was hot debate in the class as to whether diagonal moves should be allowed or not. Then, the following short dialogue occurred between one student and the teacher:

Tony: Why don't you *say* one and tell us to do it?

Ben: Sorry?

Tony: We're going to be here all day – just *say* diagonal or not diagonal.

Ben: That's passing responsibility onto me and not ...

Tony: (interrupts) does it really *matter*? You're the *teacher*, aren't you? (Adapted from Jaworski, 1994, p. 149)

When Ben and I talked about this episode later, Ben referred to *freedom* and *control*, and a *conflict* between them. He said “I'm controlling the direction because I think people should have freedom”, and “I ... gained control and then gave it back again” (p. 150). He recognized that there were complex issues here relating to freedom and control, and how these played out in the learning of the students. The possibility for a student to express himself in the words quoted was indicative of the norms of interaction in Ben's classroom. Lessons often included debate around methods or concepts: sometimes this seemed productive in terms of a class growth of understanding; at others it seemed as if students argued for the sake of arguing. Having found a voice, which their teacher encouraged, they exercised the voice, even when keeping quiet or listening to someone else might have been a more fruitful action. Ben found himself frequently steering participation in the debate rather than contributing to the debate itself.

As a consequence of this willingness to talk and express their views, students in this class were usually willing to talk with me about their experiences in a relatively mature way. So I asked Tony himself about the episode. He replied:

Tony: ... we were going on for quite a long time, wasting time, ... so I just thought that Mr West was the highest authority in the classroom, so ... he might as well tell us.

BJ: Why do you think he didn't?

Tony: Because he likes to give us more freedom and ... (hesitancy) ... so that we could be more independent, so that we could learn for ourselves (p. 151).

It was clear that Tony had insights into Ben's motivation in encouraging the debate but that “wasting time” was a consequence with which Tony was uncomfortable. Some students were engrossed in

the debate, not seeing such meta-concerns. Ben probably agreed with Tony that the particular decision on diagonal moves was not important. But he wanted to encourage student decision-making, and, on this occasion, this was the particular example he chose. He risked losing the cooperation of students' like Tony, and having many others engaged in a fairly unimportant debate. He had his eye on longer term goals of responsibility in decision making, but no local measure of success. The complexity of issues in this small episode is evident. What would we see as being the 'normal desirable states', here, and how might we interpret 'critical alignment'?

In this study, I was an *outsider* researcher (Bassey, 1995),<sup>3</sup> engaging with teachers in a form of *clinical partnership* (Wagner, 1997): I was studying their teaching, taking an ethnographic position. They were cooperating in an interested way, keen to think about issues in teaching, which meant that the questions I asked as a researcher were instrumental in triggering deep reflection. I talked of these questions 'distancing' teachers from their practice, allowing them to look critically at their teaching as if from the outside (possibly promoting some form of *critical alignment* with respect to existing practices, although this was not a term I used then). One teacher, Mike, saw this differently: he said,

[The questions] were hard because they were challenging. They were questions I thought I ought to know the answer to but hadn't clearly articulated ... I'm also not sure your questions *did* 'distance' me from my practice. In fact they really took me deeper *into* it. That was part of the reflective process for *me* (Jaworski, 1994, p. 198).

The concepts here of "distancing" or "deeper" are in my view not in conflict. One way of seeing these concepts, about which I have become more aware since writing the words above, is that the researcher's questions had a distancing effect, enabling the teacher to distance himself from practice to reflect on his practice in ways which took him deeper into the beliefs and personal theories influencing his practice. These words start to offer a characterization of *critical alignment* – a development of awareness of states of practice, a recognition that actions and their consequences are not always easy to rationalize, and a position of inquiring into relationships between action and outcome.

Whatever words we use to describe the effect of the research questions, they resulted often in teachers inquiring into aspects of their own thinking and practice. Such inquiry was not a planned part of this research. Thus, an important learning outcome for me, from this

research, was to recognize that some kind of partnership between researcher and teacher (what Wagner, 1997, calls 'clinical partnership') could be mutually valuable when both engaged in inquiry (or critical reflection) at some level. I was engaging formally in research. I would not call the teachers' practice here *research*, but it certainly deserves the term "inquiry".

Such considerations as I have discussed above led to a new research project in which I invited teachers to get involved in research into self-chosen aspects of their own teaching. The fundamental idea of the project was to create an environment in which teachers, as *insider* researchers (Bassegy, 1995), generated their *own* questions, rather than (or as well as) responding to questions from an outsider researcher. The teachers invited were mentors in our local *Initial Teacher Education* programme; they were experienced and regarded as successful teachers. A colleague and I studied the processes and practices of the teachers' research activity. We called it the *Mathematics Teacher Enquiry* (MTE) Project. The teachers who volunteered (six of them) each engaged in research, or inquiry, in a form of action research. They were reluctant, in the early days, to talk of doing research; research was something outside their experience, and formidable because of this, so we referred to their activity as *inquiry*. However, after 18 months in the project, they became more confident about doing research, in terms of what research meant for them related to their teaching practice (Jaworski, 1998).

One teacher, Julie, underwent a transformation during these months. Initially nervous and reluctant to get involved, she went through several cycles of an inquiry process during which time her (research) questions and (research) focus evolved. It was not a smooth or unproblematic evolution. She had difficulty conceptualizing what she wanted to find out, or what research approach to take. Once she had made a start and had conducted several cycles of inquiry, she felt she was not learning anything really valuable for her teaching. She was not finding out what she wanted to know, but she found it hard to state exactly what it was she wanted to know. At one point she was in a crisis of indecision, in which my colleague and I found it hard to decide how best we should respond. I wrote about the crisis as follows:

Julie became so dissatisfied with her inability to provide insights into the *quality of mathematical talk* that, again, she seemed to come close to abandoning the project. She told me that she was 'stuck', and was clearly unhappy and depressed. However, she did not ask me for suggestions, nor did I offer any. I recognised nonetheless that I had a difficult decision to make in the event that Julie made no further

progress. I discussed with [my colleague, Clare] whether we should decide to make some suggestion to Julie as to potential ongoing activity. We decided to wait until after the next group meeting, where each teacher researcher would report on their research, before making a decision (Jaworski, 1998, p. 18, emphasis added).

For us, this was also a difficult time. We wanted to study how the teacher dealt with the research/inquiry process, and, at that time, wanted to avoid a determining role, although supporting the teacher was certainly part of our agenda. However, serendipity took a hand. I reported that Julie came to our next meeting “(as if) a new person, full of enthusiasm and initiative”. She had talked with Sam, one of the other teachers in the project, and visited his school, and their discussion had given her an idea as to how to move forwards. Her subsequent activity proved successful in rewarding her thinking and providing her with data that she could analyse to address issues of “quality” in classroom talk. My synthesis from analysis of the respective data was expressed as follows:

Julie’s feelings of exhilaration and success came from having experienced the problem and tackled it herself: analytically, we might say this was due to increased metacognitive awareness, providing more power and control. It was clear that Clare and I could not have engineered this fruitful outcome. It seemed crucial that Julie herself would determine what she wanted to find out and that this should be commensurate with her developing awareness of needs and possibilities, and of her own cognitive functioning: what others have called ownership, or *agency* (Burton, 1996; Povey, 1995). As Julie herself attested later in a group meeting, it was more valuable for her to reach these recognitions herself since they carried powerful associated knowledge based on her experience and critical concern (Jaworski, 1998, p. 19).

My purpose in offering these quotations here is to point to another manifestation of relationships between teachers and educators in an inquiry process. Here both we (my colleague and I) and the teachers were all *researchers*. We all engaged in inquiry into aspects of teaching, or inquiry into teaching, or the development of teaching. In addition, we worked collaboratively in a variety of ways, such as project meetings in which all researchers met to discuss issues, and small group or pair discussions in a school environment. In a very real sense, we developed a community of inquiry during the time we worked together. I shall say more shortly about the characteristics of such a community that make it more than a community of practice.

In the paragraphs above, I have mentioned a number of people who have been learner participants in processes of teaching and developing teaching: Ben, Tony, Mike, Julie, Sam, Clare and myself. Each of us was deeply involved in the activities to which I refer, and each contributed to our own and others’ developments in teaching and

thinking about teaching within the communities of which we were a part. The “metacognitive awareness” to which I refer above was a significant outcome from such activity and, for each person, led to personal agency, to feelings of confidence and power in our own professional domains, the ability to inquire into aspects of our practice and to control our own activity and development, at least in part. This does not suggest some cosy self-satisfaction, however. All were aware of the complexity of issues, and the struggle involved in dealing with them. Often we felt uncomfortable, discouraged, disappointed and unachieving; but aspects of community, of commitment to the community kept us going. I believe this fits with notions of ‘critical alignment’: here, *critical alignment* denotes a consciousness, an intention and ability to reflect in ways that question the status quo, yet acknowledge facets of the practices involved that challenge the theoretical ideals behind attempts to effect change.

I want now to return more overtly to theory and take further some of the theoretical ideas I have started to introduce above. In particular I will look at notions of “inquiry” and inquiry communities in encouraging critical reflection and promoting critical alignment.

## INQUIRY AS A THEORETICAL BASIS TO PROMOTE CRITICAL ALIGNMENT

### *The Inquiry Movement and Its Origins*

Collaborative inquiry approaches are seen as particularly fruitful for mathematics teaching development in a number of parts of the world as has been demonstrated through their debate in international conferences widely (e.g., Krainer, Goffree, & Berger, 1999; Lin & Cooney, 2001; Wood, Scott Nelson, & Warfield, 2001). In the UK, inquiry (seeking to know through creative exploration) as opposed to discovery (trying to find out what *is*) developed from the work of Stenhouse in the Humanities Research Project and Ford Teaching Project (e.g., Elliott & Adelman, 1975). From such projects, over three decades, teachers across the curriculum started to inquire into classroom processes and practices, establishing the action research movement (Elliott, 1991; McNiff, 1988; Stenhouse, 1984). *Action research* formalizes inquiry approaches to understanding learning and teaching in classrooms.

Meanwhile, in 1982, also in the UK, the *Cockcroft* Report, from a government inquiry into the teaching of mathematics in schools, emphasised the importance of problem solving and investigational work

in a range of teaching approaches that included discussion between teacher and pupils, and pupils themselves, and appropriate practical work (DES, 1982, para. 243 ff). The use of inquiry (or investigation, or exploration) in mathematics teaching had been reflected particularly in a seminal book by Banwell, Saunders, and Tahta (1972) which set the scene for inquiry in UK classrooms by offering a range of activities as 'starting points'. Teachers' writing about their experiences of engaging students in investigational activities may be found in issues of the journal *Mathematics Teaching* (see for example issues 71, 73, 75 in 1975 and 1976). Such writings may be seen as early manifestations of teachers' engagement in classroom research into ways in which mathematical exploration can lead to students' conceptual learning of mathematics. Ruthven (2001) speaks of them as "a form of "popular research" involving careful observation of exemplary cases and systematic reflection on them" (p. 173).

The origins of investigational work in mathematics classrooms can be seen as part of an international movement in mathematics learning and teaching in the 1970s and 1980s that promoted conjecturing classrooms and problem-solving environments in the learning of mathematics (Mason, Burton, & Stacey, 1982; Polya, 1945; Schoenfeld, 1985). Emphasis was largely on processes and heuristics of mathematical problem solving. For example, in the USA, researchers have devised teaching experiments to create and investigate 'inquiry classrooms' (e.g., Davis et al., 1990); in the UK, investigative approaches to learning and teaching mathematics have been studied and characterised (Jaworski, 1994): in Portugal, Ponte (2001) has developed ideas of investigative activity from investigating in mathematics, through investigating teaching of mathematics to considering implications for teacher education. He writes:

Investigations about teaching constitute a powerful framework for professional development, providing a bridge between theory and practice, bringing together what we are learning in mathematics education about mathematics learning and teaching and what we are learning in teacher education about professional development. (Ponte, 2001, p. 69)

Thus we see, internationally, links emerging between inquiry in mathematics and inquiry into the learning and teaching of mathematics and in teacher education.

#### *Locating Inquiry with Respect to Theory in Knowledge Growth*

Collins (1988) speaks of 'Inquiry Teaching' as engaging the student in "using knowledge, so that it does not become 'inert' knowledge like

much of the wisdom received from books and lectures". John Dewey wrote "... no such thing as imposition of truth from without, is possible. All depends upon the activity which the mind itself undergoes in responding to what is presented from without" (Dewey, 1902/1990, p. 209).

The notion of inquiry relates particularly to perspectives in mathematics education dealing with cognition in terms of the active 'construction' of mathematical knowledge, where 'active' implies personal involvement. Inquiry, or investigative methods in mathematics teaching are seen to fit with a constructivist view of knowledge and learning: they demand activity, offer challenges to stimulate mathematical thinking and create opportunities for critical reflection on mathematical understanding (Cobb, Wood, & Yackel, 1990; Glasersfeld, 1984; Jaworski, 1994) leading to development of *conceptual, relational* and *principled* understandings of mathematics (e.g., Skemp, 1976). In other words, through inquiry, learners can go beyond the use and application of algorithms and rules, develop understandings of *general* relationships in mathematics, and deal with problematic aspects of the *abstraction* and *formalism* that is central to mathematics (Nardi, 1996).

Educators and researchers working with teachers have developed constructivist-based models to explain and guide developments in teachers' thinking (e.g., Carpenter, Fennema, Peterson, & Carey, 1988; Cobb et al., 1990; Jaworski, Wood, & Dawson, 1999; Wood et al., 2001). The theory behind such models suggests that teachers will develop *conceptual, relational* and *principled* understandings of *teaching* that enable similar *mathematical* development in classrooms. A *social-constructivist* perspective sees *discussion, negotiation* and *argumentation* in inquiry and investigation practices to underpin knowledge growth in mathematics, in teaching mathematics and in mathematics teacher education (e.g., Cobb & Bowers, 1999; Lampert, 1998; Wood, 1999).

Although these theoretical positions recognise the importance of social interactions in promoting growth of knowledge through inquiry, they are challenged as prioritizing the individual learner rather than attending to the wider picture of learning in classrooms (Lerman, 1996). In other words, inquiry is seen as a tool for individual cognitive development, but not overtly addressing social structures and development more widely. In terms of norms of social settings and sociopolitical issues inherent in such settings, inquiry as a tool seems too limited in its application. Researchers and theorists working in Vygotskian traditions locate learning centrally in a sociocultural environment where individual learning is regarded as derivative of social learning

(Vygotsky, 1978). Seeing learning as a process of social enculturation with the individual internalising knowledge from interactions in the social world (Bartolini-Bussi, 1994; Lerman, 1996) is believed to account better, for example, for pathological social circumstances that result in learners' alienation from schooling (Lerman, 2000; Clare, 2004). This brings us back to 'normal desirable states' and their perpetuation. While inquiry tools might offer developmental possibilities for individuals within social settings, the prevalence of social norms and processes of social enculturation will be more powerful influences on learning than will cognitive stimulus central in constructivist theory.

In a special issue of *Educational Studies in Mathematics* (2001: 46), devoted explicitly to contrasting individual and social perspectives, Kieran et al. (2001) emphasise cognitive reorganization as a social process in which inquiry is part of the discursive environment in which communication occurs. Such perspectives emphasise a concept of 'community' and of discourse as a part of community. While such discourse might be of a kind to perpetuate normal desirable states, an inquiry discourse can provide opportunity for critical alignment. It is through *community* that I see some possible rationalisation between individual cognitive development through inquiry and the power of social settings to perpetuate practices.

### *Communities of Inquiry*

The term 'Communities of Inquiry' is used by Gordon Wells (1999) in a discussion of 'dialogic inquiry' rooted in the work of Vygotsky. Wells draws on notions of inquiry as "a willingness to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to make answers to them", and as a means to emphasise "the essential continuity of education (Dewey, 1938, 1956)". This continuity is shown through the use of inquiry by students in classrooms, teachers responsible for their education, and those who are responsible for teachers' initial preparation and continuing professional development (Wells, 1999, p. 122). Wells draws on interpretations of *community* by a number of authors including Rogoff (1994) and Lave and Wenger (1991). He distinguishes communities of inquiry from communities of practice by highlighting the importance of "metaknowing through reflecting on what is being or has been contributed and on the tools and practices involved in the process" (p. 124). His use of "metaknowing" seems to accord with my use of the term "metacognitive awareness" earlier in relation to the members of the MTE community. Wells' research focuses on teachers who are "attempting to develop

such communities of inquiry and simultaneously making their attempts the objects of their own inquiries” (p. 124). This linkage between inquiry as a pedagogical tool, and inquiry into the use of this tool to promote learning is highlighted by Seth Chaiklin, who writes “Social science research has the potential to illuminate and clarify the practices we are studying as well as the possibility to be incorporated into the very practices being investigated” (1993, p. 394). His words emphasise the nature of research, not only as a means to illuminate practice, but as a source of study in investigations of practice: the research itself being part of the practice under investigation. Both Wells and Chaiklin thus point to a community of inquiry as involving a reflexive relationship between a community of practice and its activities in inquiring into and developing practice, although this is my formulation.

I have offered, briefly, these influential (in my view) areas of scholarship to take us a step further in bridging the constructivist-sociocultural divide that the editors of ESM 46 characterise (Kieran et al., 2001). The inquiry stance (see also Cochran Smith & Lytle, 1999) is a form of social positioning taken in a community of teachers in which inquiry has become one of the social norms in practice. We might see critical alignment here to involve shifts from former norms of social practice, familiar in schooling widely, towards newly developed norms of which inquiry has become a fundamental ingredient, or a ‘way of being’ (Jaworski, 2004). However, research shows that this theoretical ideal is unlikely to be manifested in teaching practice where teaching communities have no external stimulus to promote inquiry practices (e.g., Carter & Richards, 1999; Jaworski, 1994).

### *Inquiry and the Development of Critical Alignment*

In a community of inquiry, the novice practitioner is drawn into the community through processes of observation, action, questioning of actions, and inquiry into actions. We might see the combination of such processes to form a model of developmental action. In such a model, Wells (1999) emphasises the importance of *collaboration* between teachers and researchers in investigating ways of improving practice. At the root of this model is the belief in a critical mode of reflective practice in which the roots of social engagement are challenged so that practices are continuously reconceptualised and developed for the benefit of participants and progress in the work of the community.

Such a model is dialectical in its conceptualisation as an individual or social process. It is a social process in the sense that a participant is a member of a community (e.g., of teachers, or of students learning mathematics) with its own practices and dynamics of practice which go through social metamorphoses as inquiry takes place. It is an individual process in that individuals are encouraged to look critically at their own practices and to modify these through their own learning-in-practice. Developments within the community result from rationalisations, implicit and overt, between ongoing practices. In terms of Wenger's (1998) "modes of belonging", for any individual, 'belonging' to an inquiry community is both nurturing and challenging and I suggest these are characteristics of a process of critical alignment.

The notion of *Community of Inquiry* might therefore be seen to draw together elements of (social) constructivism and elements of sociocultural theory: participants grow into and contribute to continual reconstitution of the community through critical reflection; inquiry is developed as one of the norms of practice within the community and individual identity develops through reflective inquiry. This combination can be seen as particularly relevant to the development of teaching through teachers and educators inquiring into their own practices of teaching mathematics as in the snapshots above: for example: Ben grappling with issues of "control", Julie tackling her feelings of lack of achievement; Barbara and Clare seeking ways to interact with Julie. To be sustained, inquiry must be overt to a considerable degree, and it is through individuals and groups making inquiry explicit that critical awareness develops and hence critical alignment. A key question here is where does it all begin? What are the ways in which such communities of inquiry involving teachers can form and be sustained? It is one of the main research questions in current research in which I am working.

### *Teachers and Educators Researching Teaching*

In Mathematics education around the world, there is considerable evidence of projects focusing on mathematics teaching development encouraging models of critically reflective practice resulting in the development of communities of inquiry, of critical awareness within these communities, and development of thinking and practice among teachers and the educator-researchers with whom they collaborate (ATM, 1987; Britt, Irwin, Ellis, & Ritchie, 1993; Brown & Coles (2000); Krainer, 1993; Zack, Mousley, & Breen, 1997). Critical alignment was pursued at many levels developing from mutual inquiry and

creating issues and tensions for both teachers and educators (Jaworski et al., 1999).

A consequence that became clear from many of these projects was that teacher-research was hard to sustain without support or stimulus from externally based colleagues, such as university researchers, or from experienced researchers within a school environment. Thus, this research indicates the importance of communities of inquiry including teachers, educators and researchers. However, this is not to suggest that educators or external researchers bring with them the knowledge to tackle perceived problems in learning and teaching in classrooms. Quite the opposite is often the case. Often, as educators, we do not seem to have clear visions about our activity with teachers to enable the developments about which we speak and write theoretically. Like teachers' activity with students in mathematics classrooms, our activity with teachers in education programmes often stops short of realizing the high ideals we have for learning development. Many of the research reports in JMTE (*The Journal of Mathematics Teacher Education*) indicate that those initiating and/or researching programmes learn significantly from their involvement in those programmes, tackling issues that emerge (see for example, Farmer, Gerretson & Lassak, 2003; Heaton & Mickelson, 2002; McDuffie, 2004). Thus, educators, as well as teachers, are learner practitioners in processes of inquiry leading to tackling the big issues in teaching. Opportunity for tackling issues is a key element of critical alignment in the learning process.

### INQUIRY COMMUNITIES AND THEIR CONTRIBUTION TO LEARNING AND DEVELOPMENT

My references above, in the snapshots and the sections on theory, have addressed three forms of inquiry practice:

- *Inquiry in mathematics*: Pupils in schools learning mathematics through exploration in tasks and problems in classrooms; teachers using inquiry as a tool to promote pupils' learning of mathematics;
- *Inquiry in teaching mathematics*: Teachers using inquiry to explore the design and implementation of tasks, problems and activity in classrooms; educators using inquiry as a tool to enable teachers to develop teaching;
- *Inquiry in research which results in developing the teaching of mathematics*: Teachers and educators researching the processes of using inquiry in mathematics and in the teaching of mathematics.

Teachers-as-learners and educators-as-learners engage with each other to differing degrees in all of these forms of inquiry practice as set out above. Inquiry is a tool that promotes critical alignment with modes of practice and corresponding development of practice. In communities of inquiry, we all engage with inquiry as a tool to develop meta-knowing, a form of critical awareness that manifests itself in inquiry as a way of being. Inquiry as a way of being becomes a norm of the community of practice. A shift from 'community of practice' to 'community of inquiry' provides a perspective in which reflective development of practice by practitioners, individually or in groups, can be seen to result in a developing community. Individuals (educators, teachers or students) are encouraged to look critically at their own practices and to modify these through their own learning-in-practice. Developments within the community result from rationalisations, implicit and overt, between ongoing practices. Participants grow into and contribute to continual reconstitution of the community through critical reflection resulting in critical alignment; inquiry develops as one of the norms of practice and individual identity develops through reflective inquiry. In a community of inquiry, inquiry is more than the practice of a community of practice: teachers develop inquiry approaches to their practice and together use inquiry approaches to develop their practice. This indicates a reflexive relationship between inquiry (or research) and development (where development implies learning and deeper knowing).

In constructivist terms, inquiry can be seen to stimulate accommodation of meanings central to individual growth. In sociocultural terms it is a way of acting together that is inclusive of the distributed ways of knowing in a community, including a wide range of sociopolitical factors. When different communities interact in a mode of inquiry, meta-knowing that results through inquiry processes allows understandings that cross community barriers. The permeating power of inquiry overrides differences in roles and goals of practitioners in different communities.

A criticism by one of the reviewers of an earlier version of this paper was that it all sounds too cosy, "too good to be true". I therefore need to emphasise that, in a community of inquiry, rather than providing solutions to problematic issues in practice, critical inquiry or critical alignment creates the clarity and strength of purpose to recognize issues, and, moreover, to tackle the issues consciously and collaboratively. It does not remove the issues, nor does it remove tension or discomfort. I come back to the snapshots to look again at some of these theoretical ideas in practice.

Ben had worked hard to encourage his class to voice their questions and perspectives. However, such voicing could result in an unhelpful cacophony in which certain groups were disadvantaged. He found himself often focusing his own input towards form and process of interaction rather than the mathematics of interaction (e.g., Jaworski, 1994, p. 177). Yet the mathematics needed his attention too. Sometimes he found himself drawn into an instructional sequence in which *he* offered the needed mathematics, and this seemed to go against his constructivist principles of enabling students to construct through investigative activity (Jaworski, 1995). The presence of a researcher in his classroom, and a spirit of mutual inquiry, enabled such issues or tensions to be aired and addressed, and, in some cases, new practices to be introduced and explored – critical alignment. The statement quoted from Mike who felt that the researcher's questions took him deeper into his practice, indicated, not that they offered him answers to difficult problems of his practice, but they enabled him to address those problems in a constructive way – supporting critical alignment.

In the MTE project, the meetings between all researchers allowed teachers to share their issues from school and learn from each other's approaches to classroom inquiry. The visit of Julie to Sam's school offered the opportunity to gain access to alternative ways of exploring what she valued in her own classroom. As a result, she was able to initiate a new form of inquiry to learn more about her focus. In considering how best to 'help' Julie to sustain inquiry in her classroom, it was the opportunity created through development of the MTE inquiry community that brought some resolution. For Julie, and for her educator colleagues, community knowledge and support enabled the addressing of issues and opened up possibilities for further development.

In a follow-up to the MTE project with teachers Sam and Jeanette who chose to focus on the Teaching Triad (Jaworski, 1994) as a tool for developing their teaching, researchers (Potari & Jaworski, 2002) noted that Jeanette, a teacher sincerely committed to enabling students' self-esteem, acted seemingly in contradiction to her espoused aims. As we discussed such acts in our research team – two teacher-researchers and two educator-researchers – we came to see the range of (often conflicting) forces with which the teacher had to work at any time. Our community made it possible to address conflicting forces in a critical way such that the teachers gained confidence in continuing with theoretical goals (such as fostering self-esteem) despite the social forces which seemed to act against them).

The research project I am working in currently, *Learning Communities in Mathematics*, is designed to create inquiry communities between teachers and didacticicians<sup>4</sup> to promote inquiry at all the levels described above. Our major aim is to try to put into practice, and simultaneously to research, the ideas expressed in this article. Within the community of the entire project, consisting of 12 didacticicians and 40 teachers in eight schools, are various identifiable smaller communities such as that of didacticicians at the college, or the teachers in each school. Analysis of data from meetings of the didacticicians, to conceptualise our roles in workshops and schools, highlights tensions, the addressing of which affords considerable learning opportunity about the practices of creating inquiry communities with teachers. An outcome of such conceptualization, in an ongoing process of critical alignment, has been our own creation of an inquiry community among the didacticicians (Cestari, Daland, Eriksen, & Jaworski, 2005). We recognize ourselves, singly and together, enacting inquiry as a way of being. Analysis of activities and interactions in our work together highlights characteristics of the nature of inquiry, of community, and of inquiry community that will be the substance of another paper. We recognize correspondingly that we seem a long way yet from establishing inquiry communities with our teacher colleagues and we are addressing this as a central issue in the project. This is a part of our critical alignment with the social conditions in which we work.

In referring here to examples from practice at many levels, I have drawn attention to ways in which inquiry communities – consisting of two or more people working together to embody an inquiry approach to learning and teaching, developing inquiry as a way of being within the community – acknowledge and address issues and tensions within their practice. Such acknowledging and addressing (usually) does not remove the issues and tensions, but allows a critical appraisal of what is needed, and possible with respect to social forces and constraints. This is what I see as being the basis of critical alignment. For a community of practice to become an inquiry community, critical alignment is both a goal and an outcome of activity. It results in learning which is the root of teaching development.

I see as important in the material of this article the reflexivity of theory and practice in what is essentially a developmental process involving inquiry as a way of being both for individuals in the projects and for the particular communities. We are trying to address directly the sociopolitical forces that face all participants and crucially affect critical alignment. The theories presented here, concerning critical align-

ment within a community of inquiry, have been and still are developing with our research. The challenge here is to keep readdressing theory as we learn more from practice, and to look at our theory as critically as all the elements of practice. What is needed now is to explore more overtly creation of inquiry communities and their contribution to development of mathematics teaching. In our current project, we aim, centrally, to develop practices that relate to the theories explained here, to study these practices, and in the processes involved learn more about how theory and practice are related through inquiry.

### NOTES

<sup>1</sup> One of my reviewers asked here “is mathematics a social setting?”. I would argue yes, in so far as mathematics has developed over millennia through human thought and interaction which are characteristics of social settings. However, I appreciate that there are deep epistemological questions here beyond the scope of this paper.

<sup>2</sup> I would count theories such as Rationalism, Behaviourism, Constructivism or Sociocultural Theory as “big” theories of learning, often with many manifestations or subdivisions (see, e.g., Richardson, 1985).

<sup>3</sup> According to Bassey “The term ‘insider’ means a practitioner engaging in research on some aspect of his or her *own* practice, while an ‘outsider’ is a researcher from the outside” (p. 6)

<sup>4</sup> We use the term ‘didactician’ instead of the term ‘educator’ to acknowledge that teachers are also educators, and that together we can form communities in which both teachers and didacticians are researchers in relation to their own goals in practice.

### REFERENCES

- Association of Teachers of Mathematics (1987). *Teacher is/as researcher*. Derby: ATM.
- Banwell, C. S., Saunders, K. D. & Tahta, D. S. (1972). *Starting points*. Oxford: Oxford University Press.
- Bassey, M. (1995). *Creating education through research*. Edinburgh: British Educational Research Association.
- Bartolini-Bussi, M. G. (1994). Theoretical and empirical approaches to classroom interaction. In R. Biehler, R. W. Scholtz, R. Strässer, & B. Winkelmann (Eds.), *The didactics of mathematics as a scientific discipline* (pp. 121–132). Dordrecht: Kluwer.
- Bauersfeld, H. (1995). The structuring of the structures: development and function of mathematizing as a social practice. In L. P. Steffe, & J. Gale (Eds.), *Constructivism in education*. Hillsdale, NJ: Lawrence Erlbaum Associates.

- Bauersfeld, H. (1988). Interaction, construction, and knowledge: Alternative perspectives for mathematics education. In D. A. Grouws, & T. J. Cooney (Eds.), *Perspectives on research on effective mathematics teaching* Reston, VA: National Council of Teachers of Mathematics.
- Britt, M. S., Irwin, K. C., Ellis, J. & Ritchie, G. (1993). *Teachers raising achievement in mathematics: Final report to the Ministry of Education*. Auckland, NZ: Centre for Mathematics Education, Auckland College of Education.
- Brown, L. & Coles, A. (2000). Complex decision making in the classroom: The teacher as intuitive practitioner. In T. Atkinson, & G. Claxton (Eds.), *The intuitive practitioner* (pp. 165–181). Buckingham: Open University Press.
- Brown, S. & Carter, R. & Richards, J. (1999). Dilemmas of constructivist mathematics teaching: Instances from classroom practice. In B. Jaworski, T. Wood, & S. Dawson (Eds.), *Mathematics teacher education: Critical international perspectives* London: Falmer Press.
- Carpenter, T., Fennema, E., Peterson, P. & Carey, D. (1988). Teachers' pedagogical content knowledge of students' problem-solving in elementary arithmetic. *Journal for Research in Mathematics Education*, 19, 385–401.
- Cestari, M. L., Daland, E., Eriksen, S., & Jaworski, B. (2005, Feb.) Working in a developmental research paradigm: The role of didactician/researcher working with teachers to promote inquiry practices in developing mathematics learning and teaching. Paper presented at CERME4 Working Group 11, European Society for Research in Mathematics Education, Sant Feliu, Spain.
- Chaiklin, S. (1993). Understanding the social scientific practice of understanding practice. In S. Chaiklin, & J. Lave (Eds.), *Understanding practice: Perspectives on activity and context* Cambridge: Cambridge University Press.
- Clare, J. (2004) Ministers and unruly pupils 'causing collapse of schools'. *The Daily Telegraph*, Thursday, May 27th, 2004.
- Cobb, P. (1996). Where is the mind? A coordination of sociocultural and cognitive constructivist perspectives. In C. Twomey Fosnot (Ed.), *Constructivism: Theory, perspectives, and practice* NY: Teachers College Press.
- Cobb, P. & Bowers, J. (1999). Cognitive and situated learning perspectives in theory and practice. *Educational Researcher*, 28(2), 4–15.
- Cobb, P., Confrey, J., diSessa, A., Lehrer, R. & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, 32(1), 9–13.
- Cobb, P., Wood, T., & Yackel, E. (1990). 'Classrooms as learning environments for teachers and researchers.' In R. Davis, C. Maher & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics: Journal for Research in Mathematics Education, Monograph No. 4* (pp. 125–146). Reston, VA: National Council of Teachers of Mathematics.
- Cochran Smith, M. & Lytle, S. L. (1999). Relationships of knowledge and practice: Teacher learning in communities. In A. Iran-Nejad, & P. D. Pearson (Eds.), *Review of research in education* (pp. 249–305). Washington: American Educational Research Association.
- Collins, A. (1988). Different goals of inquiry teaching. *Questioning Exchange*, 2(1), 39–45.
- Confrey, J. (1995). How compatible are radical constructivism, sociocultural approaches, and social constructivism?. In L. P. Steffe, & J. Gale (Eds.), *Constructivism in education* Hillsdale, NJ: Lawrence Erlbaum Associates.

- Cooney, T. (1994). Teacher education as an exercise in adaptation. In D. B. Aichele, & A. F. Coxford (Eds.), *Professional development for teachers of mathematics: 1994 Yearbook* Reston, VA: National Council of Teacher of Mathematics.
- Davis, R. B., Maher, C. A., & Noddings, N. (Eds.), (1990). Constructivist Views on the Learning and Teaching of Mathematics. *Journal for Research in Mathematics Education*, Monograph Number 4. Reston, Virginia: National Council of Teachers of Mathematics.
- DES (Department of Education and Science) (1982). *Mathematics Counts: Report of the committee of inquiry into the teaching of mathematics in schools under the chairmanship of W. H. Cockcroft*. London: HMSO.
- Desforges, C. & Cockburn, A. (1987). *Understanding the mathematics teacher*. Lewes: Falmer Press.
- Dewey, J. (1902/1956/1990). *The school and society and the child and the curriculum*. London: The University of Chicago Press.
- Dewey, J. (1938). *Education and experience*. New York: Collier Macmillan.
- Elliott, J. (1991). *Action research for educational change*. Milton Keynes: Open University Press.
- Elliott, J. & Adelman, C. (1975). *The language and logic of informal teaching. The Ford Teaching Project, Unit 1, Patterns of Teaching*. Norwich: University of East Anglia.
- Ernest, P. (1991). *The philosophy of mathematics education*. London: Falmer Press.
- Farmer, J. D., Gerretson, H. & Lassak, M. (2003). What teachers take from professional development: Cases and implications. *Journal of Mathematics Teacher Education*, 6(4), 331–360.
- Freudental, H. (1978). *Weeding and sowing*. Dordrecht: D. Reidel.
- Glaserfeld, E.von (1984). An introduction to radical constructivism. In P. Watzlavik (Ed.), *The invented reality* London: W. W. Naughton & Co.
- Glaserfeld von, E. (1987). Learning as a constructive activity. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* Hillslade, NJ: Erlbaum.
- Heaton, R. M. & Mickelson, W. T. (2002). The learning and teaching of statistical investigation in teaching and teacher education. *Journal of Mathematics Teacher Education*, 5(1), 35–59.
- Jaworski, B. (1994). *Investigating mathematics teaching: A constructivist enquiry*. London: Falmer Press.
- Jaworski, B. (1995). The critical nature of teaching decisions. *Mathematics Teaching*, 150.
- Jaworski, B. (1998). Mathematics teacher research: Process practice and the development of teaching. *Journal of Mathematics Teacher Education*, 1(1), 3–31.
- Jaworski, B. (2001). Developing mathematics teaching: Teachers, teacher-educators, and researchers as co-learners. In F.-L. Lin, & T. J. Cooney (Eds.), *Making sense of mathematics teacher education* (pp. 295–320). Dordrecht, The Netherlands: Kluwer.
- Jaworski, B. (2004). Grappling with complexity: Co-learning in inquiry communities in mathematics teaching development. *Proceedings of the 28th PME Conference*. Bergen, Norway: Bergen University College.
- Jaworski, B., Wood, T. & Dawson, S. (1999). *Mathematics teacher education: Critical international perspectives*. London: Falmer Press.

- Kieran, C., Forman, E. & Sfard, A. (2001). Bridging the individual and the social: Discursive approaches to research in mathematics education. A PME Special Issue. *Educational Studies in Mathematics*, 46, 1–3.
- Krainer, K. (1993). Understanding students' understanding: On the importance of co-operation between teachers and researchers. In P. Boero (Ed.), *Proceedings of the 3rd Bratislava International Symposium on Mathematical Teacher Education*. Bratislava: Comenius University.
- Krainer, K., Goffree, F., & Berger, P. (Eds.) (1999). European research in mathematics education, I.III: On research in mathematics teacher education. *Proceedings of the first conference of the european society in mathematics education*, vol III. Osnabrück.
- Lampert, M. (1998). What can research on teacher education tell us about improving quality in mathematics education?. *Teaching and Teacher Education*, 4(2), 157–170.
- Lave, J. (1996). Teaching as learning, in practice. *Mind Culture and Activity*, 3(3), 149–164.
- Lave, J. & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, MA: Cambridge University Press.
- Lerman, S. (1996). Intersubjectivity in mathematics learning: A challenge to the radical constructivist paradigm?. *Journal for Research in Mathematics Education*, 27(2), 133–150.
- Lerman, S. (2000). Some problems of socio-cultural research in mathematics teaching and learning. *Nordisk Matematisk Didaktik (NOMAD)*, 8(3), 55–72.
- Lerman, S. (2001). Cultural and discursive psychology: A sociocultural approach to studying the teaching and learning of mathematics. *Educational Studies in Mathematics*, 46, 87–113.
- Lin, F.-L. & Cooney, T. (2001). *Making sense of mathematics teacher education*. The Netherlands: Kluwer.
- Mason, J., Burton, L. & Stacey, K. (1982). *Thinking mathematically*. London: Addison-Wesley.
- Mathematical Association (1991). *Develop your teaching*. Cheltenham, UK: Stanley Thornes .
- McDuffie, A. R. (2004). Mathematics teaching as a deliberate practice: An investigation of elementary pre-service teachers' reflective thinking during student teaching. *Journal of Mathematics Teacher Education*, 7(1), 33–61.
- McNiff, J. (1988). *Action research: Principles and practice*. London: Macmillan.
- Nardi, E. (1996). The Novice mathematician's encounter with mathematical abstraction: Tensions in concept-image construction and formulisation. Unpublished D. Phil. Thesis. Oxford: University of Oxford.
- Polya, G. (1945). *How to solve it*. New Jersey: Princeton University Press.
- Ponte da, J. P. (2001). Investigating mathematics and learning to teach mathematics. In F.-L. Lin, & T. J. Cooney (Eds.), *Making sense of mathematics teacher education*. The Netherlands: Kluwer Academic Publishers.
- Potari, D. & Jaworski, B. (2002). Tackling complexity in mathematics teaching development: Using the teaching triad as a tool for reflection and analysis. *Journal of Mathematics Teacher Education*, 5(4), 351–380.
- Rogoff, B. (1994). Developing understanding of the idea of communities of learners. *Mind, Culture and Activity*, 1(4), 209–229.
- Ruthven, K. (2001). Mathematics teacher education and educational research: Developing “practical theorising” in initial teacher education. In F. L. Lin, &

- T. Cooney (Eds.), *Making sense of mathematics teacher education* (pp. 165–183). The Netherlands: Kluwer.
- Schoenfeld, A. H. (1985). *Mathematical problem solving*. New York: Academic Press.
- Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20–26.
- Steffe, L. P. & Thompson, P. W. (2000). Interaction or intersubjectivity? A reply to Lerman. *Journal for Research in Mathematics Education*, 31(2), 191–209.
- Stenhouse, L. (1984). Evaluating curriculum evaluation. In C. Adelman (Ed.), *The politics and ethics of evaluation*. London: Croom Helm.
- Vygotsky, L. S. (1978). *Mind in Society. The development of the higher psychological processes*. London: Harvard University Press.
- Wagner, J. (1997). The unavoidable intervention of educational research: A framework for reconsidering research-practitioner cooperation. *Educational Researcher*, 26(7), 13–22.
- Wells, G. (1999). *Dialogic inquiry: Towards a sociocultural practice and theory of education*. Cambridge: Cambridge University Press.
- Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. Cambridge: Cambridge University Press.
- Wood, T. (1999). Approaching teacher development: Practice into theory. In B. Jaworski, T. Wood, & S. Dawson (Eds.), *Mathematics teacher education: Critical international perspectives* (pp. 163–179). London: Falmer Press.
- Wood, T., Scott Nelson, B. & Warfield, J. (2001). *Beyond classical pedagogy: Teaching elementary school mathematics*. Mahwah, NJ: Lawrence Erlbaum.
- Zack, V., Mousley, J., & Breen, C. (Eds.), (1997). *Developing practice: Teachers' inquiry and educational change in classrooms*. Geelong, Australia: Centre for Studies in Mathematics, Science and Environmental Education, Deakin University.

*Agder University College*  
*Serviceboks 422, 4604, Kristiansand,*  
*Norway*  
*E-mail: barbara.jaworski@hia.no*

Barbara Jaworski