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BEING MATHEMATICAL WITH & IN FRONT OF LEARNERS:

*Attention, Awareness, and Attitude
as sources of Differences between
Teacher Educators, Teachers & Learners*

Abstract

My aim in this chapter is to probe beneath the surface of the, to me self-evident stance, that teachers cannot 'do' the learning for learners. It is convenient, for the differences I wish to highlight, to use the term *learner* to refer to students of mathematics being taught by teachers who themselves may or may not be influenced by teacher educators. Of course, at any time both teachers and teacher educators may be learning mathematics and more. I shall develop and use the constructs and discourse of attention, awareness and attitude to draw attention (*sic*) to differences between the experiential states of teacher educators, teachers and learners and between the kinds of actions they initiate in their respective communities of practice. The purpose of these distinctions is to highlight the developmental process of what I call *mathematical being*, which is the basis for and which informs *being mathematical* with and in front of others.

Rationale

Despite attempts to the contrary, teachers cannot make learners learn, nor can they do the learning for their learners. Authors of texts may provide tasks, exposition and examples intended to promote learning; and teachers may select tasks and interact with learners working on the tasks. However, none of these can ensure learning, and when conceived of as attempting to provide guarantees, are doomed to failure or at best, partial success. It seems patently clear that what teachers *can* do for learners, indeed perhaps the only thing they can actually do *for* learners, is to direct learners' attention. This they do by a variety of overt and covert means such as pointing, highlighting, and stressing, and by more or less subtle questioning and listening during activity arising from work on tasks. By their presence teachers can indirectly help learners maintain their attention where it might wander or when they might otherwise give up. Through their 'being' they can influence learners, especially when they are being mathematical with and in front of their learners.

When teaching mathematics, attention is directed to mathematical objects, relationships, properties, and reasoning. It can also be directed to manifestation of the presence of pervasive mathematical themes, and to forms of mathematical thinking, in the form of heuristics, use of natural sense-making powers (Mason & Johnston-Wilder 2006) and habits of mind (Cuoco 1996).

When working with teachers, attention can be directed to mathematics, as with learners, but also to choices involving the use of pedagogical strategies

and didactical tactics¹, and how those choices are informed by reference to pedagogical constructs which together comprise distinctions, assumptions, beliefs and theories about teaching and learning. Where the choices being attended to match choices being made, there is consonance between what is practiced and what is 'preached': not only 'walking the talk' but 'talking the walk'.

When working with teacher educators, attention can be drawn not only to mathematics and to mathematical pedagogy, but also to theories and constructs, and to ways of working with both teachers and learners through different modes of interaction consistent with those theories and constructs, thus 'practicing what is being preached'. In each situation, attention may be drawn to actions which are being carried out with lesser or greater awareness, so that in the future the person might become aware of a possibility of choosing actions according to the circumstances (Mason 1998). Overt actions as observed by others, implicitly or explicitly, form practices. By engaging with and in those practices, individuals become members of a community of practice. (Wenger 1988, Lave & Wenger 1991) which may be local rather than global (Winbourne & Watson 1998).

However, nothing can be ensured or guaranteed. It is perfectly possible to display behaviour without generating it from understanding of underlying structure. Thus a learner can carry out procedures without being able to justify, explain or reconstruct them, or, unhappily, without being able to modify them appropriately in changed situations. Similarly a teacher can enact pedagogical strategies and didactic tactics without be able to justify, explain or modify them appropriately, and even without being able to articulate them. Again similarly, a teacher educator can provide scholarly discourse about strategies and tactics without consistency between the theory being expounded and the behaviour being displayed, and without flexible response to changing situations.

Methods

A word about my method of enquiry is in order. This chapter is not based on what is traditionally known as 'empirical studies', in the sense of collecting data through extra-spective observation of other people, followed by analysis consisting of classification. Rather, it draws upon my sometimes systematic observations of my own experience with myself and when working with others. These provide the background for contemplation of that experience, leading to the identification of phenomena, and attempts to find some informative organisation for them. The phenomena consist of similarities perceived by me in my interpretations of my observations of disparate events. Theoretical constructs are then used to organise, characterise, and account for these phenomena (Mason 1991, Hewitt 1994, Mason 2002). As far as this exposition goes, the significant 'data' consist of events and incidents which come to your mind while reading, as resonated or triggered by what is read.

However, to begin with phenomena would, for the purposes of this chapter, involve me in recording a large number of incidents which would exhaust both

¹ I use *didactic tactics* to refer to topic or concept specific pedagogical actions, reserving *pedagogic strategies* for actions which can be used in several or many different topics.

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the space available and the reader, before getting to any analysis. Therefore I have elected instead to use only a few phenomena to illustrate points from the emerging analysis.

My analysis is based upon three aspects of human experience which I have been pondering for a very long time: the structure and role of attention, the nature of awareness (both conscious and unconscious, explicit and implicit), and the influence of attitude (including intention and all aspects of affect) as manifestation of will. What makes mathematics education so frustrating yet intriguing is that people's behaviour is, despite myths to the contrary, rarely rational (in the sense of logically deduced from beliefs or dispositions; see Bennett 1964, Nørretranders 1998). Nor is behaviour ever simple, so rationalising observed behaviour by recourse to theories is at best an approximation, and attempts to locate cause-and-effect relationships are generally doomed.

The whole of my analysis, what could be called the theoretical framework for this chapter, is based on the ancient metaphor 'human psyche as chariot' which can be found in several of the Upanishads and which was developed by Gurdjieff (1950, p. 1192-1200). Plato (Republic II 488ff) uses the setting of the crew of a ship to portray much the same idea. Among other elements in the extended analogy, there is an owner, a driver, the chariot itself, horses, reins and shafts. The chariot can be taken to refer to the body, whose general state of repair needs looking after if the chariot is to function. In the psyche this includes keeping techniques and skills fluent through rehearsal. The horses can be seen as representing the senses, and more generally as affect including emotions and feelings. These steeds provide the motive energy, but are prone to seeking grass and other delights along the side of the road. They therefore need directing. The driver does this by means of the reins attached to the harnesses, and the response of the chariot is governed by the shafts. The reins correspond to mental imagery, which is the means humans use to guide and direct their energies towards their goals. The shafts correspond to habits manifested through selves which transform motive energy (motivation) from the horses into patterns of behaviour (movement of the chariot). The owner (will) gives the driver general instructions, and the driver (cognition, awareness, though these terms need careful elaboration) provides the direction and guidance needed by the horses. Although I am confident that the western psychological tripartite division of the human psyche into enaction, affect and cognition is based on the Upanishad chariot metaphor, I have been unable to trace the origins of this division in western psychological literature. Study of attention (as the manifestation of will) is a relatively recent phenomenon although introduced by James (1890). Piaget (1954/1981) held similar views of affect as the engine of action.

I have omitted from the title, and from this discussion, specific singling out of the enactive component of the psyche, the behaviour, not because it is unimportant, but because it is enough to work here on the three As. Although it is not, as far as I know, part of the Upanishads to consider the charioteer and owner as members of a social milieu influenced by the ever-evolving practices of the community, there is insufficient space to develop this aspect either. The three aspects of the human psyche being discussed here, attention, awareness and attitude, are human constructs in an attempt to delineate and discern components of an interactive complex which we call generally, 'being human', and specifically, 'being mathematical'. Some authors choose to stress the

social as the source of influence on the individual, and in extreme forms, see the social as the only pertinent entity for educational purposes. Some authors choose to stress the individual as the agent responding to forces from its social and physical environment. Neither extreme is able to account fully for the rich tapestry of experience, in which the individual and the environment are engaged in a mutually evolving autopoiesis or *self-construction* (Maturana & Varela, 1972; see also James, 1890 and Cobb, 1995).

Attention

Over many years it has become clearer and clearer to me that attention is the central concern for and of teachers. It came about because in every workshop I did for some 25 years, someone would ask at the end “where is your accent from?”. I finally woke up to the fact that people were spending time in sessions attending not to what I was saying or asking, but to my accent. This led me to questions such as ‘what are learners attending to?’, with the assumption that if they and the teacher are attending to different things, then there is likely to be miscommunication at best because of little common ground, no matter what is taken-as-shared.

Attention is the manifestation of will, of intention. It is not a thing, but its influence can be inferred, though certainly not observed, in others. To a significant but not complete extent, it *is* observation: it is the medium through which observation takes place. In a certain sense ‘you are your attention’, or ‘you are where your attention is’: thus we have habits of speech such as “give me your attention”, “thank you for your attention”, and the more sarcastic “are you with us?”. The military command “attention!” is intended to startle people into a heightened state of wakefulness. These uses all signal the centrality of attention in human experience. Of significance with attention is not only what is attended to, but the nature or structure of that attention, as will be developed shortly. How people attend and how they indicate what they are attending to and in what ways, is culturally based, but directed by personal propensities and interests.

In order to be sensitive to what learners are attending to, it is essential to be aware oneself not only of what is being attended to, but also how. Without this it is impossible to make sense of what learners are experiencing or reporting about that experience. What is more, attention is not an all-or-nothing experience: it has both a macro and a micro structure.

Macro structure of attention

Drawing on a variety of ancient sources, on my own experience, and on the results of exercises tried out with others, I began my studies with the macro-structure of attention (Mason 1998). It is common experience that you can attend to more than one thing at once; it may be rapid serial succession as in a serial-computer operating system, and it may be in parallel; that is a matter for neuroscience to discern. The experience itself often seems like effectively parallel awareness. For example, on the one hand, while reading this text you can form an image of where you are likely to partake of your next meal, and you may also be able to imagine the walls around you painted in stripes or polka dots of different colours. You can recite a poem, nursery rhyme, or other familiar word sequence while at the same time revisiting mentally a holiday or other pleasant context. On the other hand, many people find that

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having a radio or television playing makes it difficult to think, especially when trying to read signs in an unfamiliar city while driving. Attention therefore can have multiple loci or be multiply directed.

A more subtle aspect of location is the experience of where attention is based. With a little work it is possible to become aware, for example, of being slightly outside yourself, perhaps looking from the side, or from a bit behind, or from a bit in front, or from inside the back or front of your head. Sometimes people speak of 'being up front' or 'laid back', frozen metaphors which capture something of the location of attention. Mostly we are unaware of the location of the 'search beam' which is our attention (Harding, 1961).

Attention can be expansive, like a broad beam of light, taking in a wide variety of impressions, and it can be narrow, as if confined like binoculars or a microscope. Independently of its breadth it can be sharply focused, especially on something specific or confined, but it can also be fuzzy, indistinct and encompassing: there can be a general 'sense' of what is being attended to without concern about details. Thus at the macro level, attention can vary in multiplicity, locus, focus and sharpness.

Learners' macro attention is usually caught up by their current action. Often they lose track of overall goals. Short-term memory and cognitive load are terms which have been used to describe the same phenomena. The same applies to teachers and to educators: immediate action may absorb so much attention that there is little if any left over for keeping track of what is happening, even for noticing sufficiently sharply so that moments become salient and so can be re-entered later during reflection. Thus the experience may be all of a blur.

When Bruner (1986, p. 75-76) talks about a teacher 'being consciousness for two' he is describing ways in which a teacher's macro attention can keep in mind aspects of a situation which a learner lets go of because of the demands made on their attention by a local task. Thus someone who has not internalised addition or multiplication facts needs support of an external consciousness when exploring number patterns so that sufficient attention can be focused on calculation without losing track of the purposes of the calculation. By contrast, someone who needs little or no attention to carry out mental arithmetic does not need an external 'consciousness for two' to keep track and so can function more independently.

Micro structure of attention

Beyond these macro qualities, attention also has micro qualities, which are conveniently described in terms of how you are attending rather than to what you are attending. Informed by Eastern thinking (Bennett, 1962) I find it useful to distinguish five different ways of attending, which I call the structure of attention. These are holding wholes, discerning details, recognising relationships, perceiving properties and reasoning on the basis of agreed properties. There is a remarkable alignment of these ways of attending with descriptions of levels of geometric thinking (van Hiele, 1986), with the SOLO taxonomy (Biggs & Collis, 1982) and with the 'onion' model of understanding (Pirie & Kieren, 1994), but there is no space here to elaborate on these connections. The principle differences are that I see the different structures of attention, the different ways of attending, as following one another in rapid

succession and in no particular order, rather than as sufficiently stable to be classified as ‘levels’ or ‘layers’ of reasoning, understanding or thinking.

Take for example the expression $\frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3}-\sqrt{2})(5+2\sqrt{6})}=1$. A first impression

might be that it is very complicated. You might or might not actually be aware of components, but just that there are several square roots. The right hand side may attract attention because of its simplicity, leading to an affective response of disbelief or surprise, but it might actually be overlooked due to the absorbing or distancing complexity of the left hand side. When attention is allowed to play over the symbols, a relationship between the numerator and denominator might come to the fore. Some people are tempted into multiplying out the denominator, while others notice that one of the factors in the denominator is the conjugate of the numerator, so multiplying numerator and denominator by the numerator will eliminate the first bracket in the denominator. Some people might notice that the $\sqrt{6}$ is the product of $\sqrt{2}$ and $\sqrt{3}$ and so there is some experience a ray of hope that the answer might indeed be 1. Others might go further and experience an association with the famous paper by Dedekind (1912) challenging people to prove that $\sqrt{6}$ does actually equal $\sqrt{2} \times \sqrt{3}$.

Associations, connections and links which ‘come to mind’ can be accounted for through either or a mixture of two processes: *metonymic* triggering and *metaphoric* resonance. These grammatical terms were re-introduced by Jakobsen (1951) and exploited by Lacan (1985) and Lakoff and Johnson (1980). Metonymies are associations which play on the surface of meaning, usually highly idiosyncratic and often based on syntactic elements such as homonyms. They are grammatical constructions in which one thing is referred to by some association such as a property or a specific part (literally, ‘change of name’). The verb *trigger* is used with metonymies because the links are largely within the affect-emotional domain, very rapid and often not even conscious. By contrast, *metaphor*, which means literally ‘carrying across’ involves describing one thing in terms of another. Underneath every metaphor there is an extended analogy (originally a mathematical term for proportion), which is why metaphor is concerned with structure and meaning. The verb *resonance* is used to indicate the structural, semantic basis of how metaphors bring things to mind.

One form of attending is gazing, not really focused on anything in particular, yet taking in the whole. An assonant description is *holding wholes*. These ‘wholes’ may be parts of other entities which have been discerned. Thus you can gaze at a geometric diagram, waiting for it to ‘speak to you’ or at some particular part of a diagram. The experience is of ‘waiting for things to come to mind’. Similarly you can gaze at a collection of symbols, and you can hold a situation in your mind and allow your subconscious to work away at it. You can gaze at the expression above without focusing on particular components. This may last only a few micro-seconds before you make distinctions, or discern details.

Making distinctions, which can be assonantly referred to as *discerning details*, is a structure of attention, a way of attending. Elements or aspects which might serve as useful sub-wholes are distinguished and identified, often through ‘recognition’ either metonymically or metaphorically. Discerning details is neither algorithmic nor logically sequential. It doesn’t happen all at once. You don’t ‘suddenly discern details’, and then stop discerning details; rather, your attention is caught by some detail, allowing you to distinguish some aspect

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from some other aspect, and these distinctions participate in and contribute to subsequent attending. You may return briefly to gazing, whether at the detail or at some larger part. Discerning details applies to hearing and sensation as well as to sight. Most often attention darts around like a butterfly, settling for a brief moment, and then moving on. One of the things that has to be learned is to direct your own attention (using will) rather than be subject to will-o'-the-wisp movement. As learners of mathematics, people need to become familiar with worthwhile details to look out for; as teachers of mathematics people need to become familiar with ways of attracting and holding attention on worthwhile details; as teacher educators people need to become familiar with ways of drawing teachers' attention to worthwhile details, to ways of directing learner attention, as well as to integrate these with pedagogical constructs and theories.

An excellent example is afforded by the use of a simple partly shaded diagram to direct attention to the ways in which it is possible to change both the unit used to compare two things, and what constitutes the whole. In the figure, below, find something which is two-fifths of something else; something which is three-fifths of something else; something which is five-thirds of something else; something which is five-halves of something else (Thompson, 2002).



The task develops from there. But many people are so inured to seeing the whole of the figure as the only whole that they balk at the third and fourth parts.

It is sometimes hard to imagine that you haven't always discerned all the details of which you are currently aware, but if you put yourself in an unfamiliar situation you may become aware that experts are attending to details un-noticed by novices. This 'awareness' is of course vital for teachers and teacher educators. A significant if not major part of learning is discerning important details or aspects not previously not discerned, including what it is that constitutes 'important'. Thus the algebra expert detects errors without even checking the calculations, and spots possible avenues of enquiry invisible to novices who are not distinguishing the same sorts of things. In the surds-fraction earlier, an expert discerns numerator, denominator, equal signs, the solitary 1, the repetition of the square-roots of 3 and 2 and the single square-root of 6 and immediately recognises some relationships.

Recognising of relationships between discerned elements is often an entirely automatic development from discerning details, but it is very difficult to be aware of a relationship between two or more terms when those terms are embedded in a mass of symbols or geometrical objects and your attention is not discerning at that level. Recognising relationships refers to specific relations between specific elements. In the fraction strip, it is the relations between 2, 3 and 5 which are crucial. In the surds-fraction someone might, for example, detect the presence of more than one square root of 3, or see the numerator and one term in the denominator as differing by a sign. A relationship can come to mind in its entirety, or there can be a coagulation of disparate discerned elements. The feature of relationships here is that they are embedded in the particular.

There is a subtle, but potentially important shift from recognising specific relationships between specific elements, to *perceiving properties*. When you are aware of a possible relationship and you are looking for elements to fit it, you are perceiving a property. Particular relations are seen as instances of general properties or abstract concepts.

Mathematical reasoning proceeds only when learners identify properties that can be used as the basis for reasoning. Without this, learners tend to dredge up everything they know, even when it has not yet been proved formally, or when it leads to circular reasoning. Formal reasoning identifies properties as either axioms or as ‘already deduced’, and then proceeds to use only them in further reasoning. This is *reasoning on the basis of specified properties*.

The significance of these subtly different forms of attention lies in the disparity of comprehension which can take place when teacher and learners are attending either to different things, or to the same thing(s) but in different ways. Thus if learners are trying to discern the elements that a teacher is relating together, they are unlikely to appreciate the relationship as well; if learners are recognising relationships between specific elements but not perceiving them as properties, then again they may not appreciate the import of what is being said and done.

A novice teacher observing a lesson may miss all sorts of subtle details evident to a more experienced teacher or educator. In particular, they may be unaware of what learners are attending to, or how, and oblivious to what the teacher may be attending to, or how. A teacher caught up in attending to learners’ responses may not be attending to the negative (or positive) effects of the implicit choices they are making in how they interact with the learners. Similarly, an educator caught up in attending to teachers’ responses may not be aware that their own behaviours may be influencing those responses in unexpected or unintended ways.

Overt and explicit directing of attention (“look here”, “can you see ...?”) are very local and ephemeral phenomena. There is no reason to expect that such direction will alert learners to the possibility of attending similarly in the future. Indeed, the refinements of the structure of attention introduced earlier serve as a reminder that even if learners are attending to the same thing as the teacher, they may not be attending in the same way.

Awareness

The word *aware* is usually used in connection with some degree of consciousness, so the word *awareness* then picks up that association. However, one of Caleb Gattegno’s many insights is that as organisms, we have many ‘awarenesses’ which lie below the surface of consciousness (Gattegno, 1973, 1987, 1988). For example, somatic functioning such as breathing, perspiration, and heart-rate, not to say digestion and lymphatic defenses all take place without our being conscious of them. Functionings like these which are controlled by the soma deserve to be considered awarenesses because they are dynamically responsive rather than mechanically determined. There are also internalised, habituated or automatic functionings to which we no longer need to attend which are also dynamically responsive and deserve to be thought of as ‘awarenesses’. Awareness may be restricted in the degree to which the individual is specifically and explicitly aware and in what they could articulate about it. Thus a child may coordinate pointing and number-word-speaking without articulating anything to do with ‘counting’, nor even ‘I am coordinating pointing and speaking’; a learner may draw axes, plot points and

sketch straight lines through specified points without articulating anything to do with ‘graphing’.

Awarenesses are influenced by and also influence affective responses, such as dislike (or stronger) of surds in the surd fraction. Finally there are functionings which arise through the use of human powers for making sense (literally as well as figuratively), which are developed and extended through social interaction, and which can also be observed as actions. For example, actions such as stressing and consequently ignoring different parts of the fraction strip, or treating the surd equation as an object to manipulate are functionings driven by awareness. Awarenesses form the backbone of disciplined modes of mathematical enquiry whether at the level of algebra, geometry, topology and so forth. or at more refined levels associated with specific topics. Lakoff and Nunez (2000) exploit this in their conjecture that most mathematical concepts are based on bodily sensations and movement.

Gattegno (1987, 1988) proposed that disciplines such as mathematics or physics arise when people become aware of the fact that they have actions that they can carry out, explicit functionings, which he called awarenesses, even though they may be implicit and below the level of consciousness. Thus a discipline arises when people become aware of awarenesses which enabled or guided them to act in certain situations (Mason 1998). The second order awareness means that they can begin to act upon and with the base awarenesses, to study them as phenomena, and, in Vygotskian terms, explicitly choose to make use of them. As people become aware of their below-the-surface functionings which are activated by the tasks they undertake and by the activity in which they engage and participate, they develop competence in and a sense of the associated discipline. Thus, the discipline of counting emerges from becoming aware of and making use of actions such as pointing, speaking, coordinating pointing and speaking, and speaking a particular word-sequence; the discipline of fractionating emerges from relating different wholes via a common unit.

Awarenesses trigger actions without our having to initiate them consciously, and often without any conscious knowledge at all. Since the term *conscious* is itself ambiguous, referring both to being awake rather than asleep, and to being mindful rather than oblivious or functioning in automatic mode (Claxton, 1984, 1997; Langer, 1997), I shall use the terms *explicit awareness* to refer to awareness which could be articulated, and *implicit awareness* to refer to awareness that is either not yet or no longer readily articulable. The importance of Gattegno’s insight is that when we say that we are ‘aware’ of something, we obscure the origins of that ‘conscious perception’, and we overlook a myriad of functionings of which we are not explicitly aware but yet which comprise the awarenesses of our organism as a whole, since they initiate action of which we only later become explicitly explicitly aware.

Trying to distinguish between explicit and implicit awareness is fraught with difficulty. For example, it has been suggested that a multitude of experiments point to the fact that consciousness trails behind action and affect (Mandler, 1989). Nørretranders (1989) goes further and claims that it is an illusion that we (consciously) make choices at all. Rather, choices made in the moment, on the fly, as distinct from reasoned or debated choices, are made below the surface of awareness-attention-consciousness. Consciousness then makes up a narrative which gives itself the ‘starring’ role. Choices involving reactions or responses to stimuli are actually made on the basis of past

experience before we become aware of them *as* choices. It is more convenient therefore to follow Gattegno and to use *awareness* to refer to the sensitivities of the organism to detect change and to respond to that change through action. Those actions may be well rehearsed habits or functionings, or may be cobbled together from fragments of such actions to suit the situation.

The significance of awarenesses is that development as conceived by Vygotsky (Valsiner, 1988, p. 334) lies in transformation of a person's ability to act in some way 'in himself', into an ability to act that way 'for himself'. Thus development, in Vygotskian terms, refers to becoming able to make a conscious, explicit choice, rather than simply being triggered or cued into action. Put another way, development has to do with the growth of conscious control and participation, not the extension of facility with a particular technique or procedure. When people refer to the zone of proximal development (ZPD) as imminent changes in behaviour (solving problems or doing tasks with the help of a relative expert) they misunderstand Vygotsky's intentions. What they are referring to really is a zone of proximal behaviour which is in some sense a projection of the ZPD into the enactive component of the psyche (Mason, Drury & Bills, 2007).

Vygotsky's perspective is compatible with the proposal by Gattegno (1987) that 'only awareness is educable, and I added 'only behaviour is trainable' and 'only emotion is harnessable' in line with the Upanishad image of the chariot. Gattegno meant that functionings can be integrated (through subordination of attention), but that participation in a discipline (such as mathematics, or more particularly synthetic geometry or elementary algebra) occurs when you become aware (explicitly) of those awarenesses (functionings). Similarly, Vergnaud observed what he called *theorems-in-action*: children acting as if they knew a theorem or a technique such as the commutativity of addition or multiplication, even when they patently could not articulate it, nor was there any evidence that they were aware of it at more than a functional level.

Development, that is, education of awareness, comes about when implicit theorems-in-action become explicit, open for questioning and possible justification or refutation, and made use of in new ways or in new situations in which they might not have come to mind previously. This is what some people refer to as 'applications' and what behavioural psychologists referred to as transfer (Detterman & Sternberg, 1993, Lobato, 2006). Situated cognition stresses that 'knowing' arises in and because of a particular situation. It tried to account for transfer and its absence by describing learning as situated. However, the same issue then arises in terms of how situativity extends and broadens to encompass previously unfamiliar situations. Ference Marton (see Marton & Booth, 1997, Marton & Trigwell, 2000, Marton & Pang, 2006) sees learning a concept as becoming aware of aspects of examples which can be varied while still remaining examples. These are referred to as dimensions of (possible) variation which provides a more precise formulation of situativity than is usually available (Marton 2006). All of these articulations are, at heart, concerned with educating awareness, implicitly and explicitly. Thinking in terms of attention and 'what comes to mind' in a situation, whether triggered metonymically or resonated structurally through metaphor, highlights the importance of ways in which metonymy and metaphor can be supported and strengthened, and more generally, how noticing an opportunity to act can be developed (Mason, 2004).

Taking the perspective described here means that learners need to experience and integrate into their functioning various actions which lie at the core of mathematical topics and themes. Setting tasks, concomitant activity, accumulated experience and working on that experience is the surface structure of mathematical classrooms. Below the surface lies the essential purpose of the tasks, activity and experience, for at the heart of mathematics lie core awarenesses. Thus coordinating the number names experienced as a form of 'counting poem' with pointing, together with discerning a 'unit' are the actions which underpin the awareness which constitutes counting; the action of discerning a repeated unit lies at the core of measuring as well as counting; the action of combining and breaking up underpins addition and subtraction; the actions of scaling and of repeating underpin multiplication; the projection of motion along a path into motion in two (usually orthogonal) directions and the coordination of action in two directions to form a path underpins graphing; the action of selection from a range of possibilities underpins probability, and so on.

Each mathematical topic is based on core actions which learners can carry out under instruction (otherwise they cannot be expected to make much sense of the topic!). These actions constitute the core awarenesses around which the topic is built. A learner needs to encounter the use of previously familiar actions, possibly in a new form or with extensions or variations; an effective teacher needs to be aware of these awarenesses; an effective teacher educator needs to be aware of how to locate and bring to the surface these core awarenesses, derived from the literature and from experience. Tzur (this volume) provides one articulation of how this can be done.

If a teacher is not aware of relevant core awarenesses, then they are not in a position to choose appropriate tasks, nor to choose appropriate pedagogical strategies and didactic tactics. For example, partitioning a rectangle into equal pieces with vertical divisions, or with horizontal divisions is a core action which underpins the awareness of fractions as operators on objects. An associated pedagogical awareness is the fact that many learners have a predisposition for vertical partitions. Many require continued exposure to horizontal ones as well so that they develop the flexibility to choose for themselves, which is essential for using the rectangular area model for fraction operations (Kyriakides, 2006).

One of the classic interventions used by relative experts to enculturate novices into particular practices, is often referred to as *scaffolding and fading* (Seeley Brown *et al.* 1989). A teacher repeatedly uses a particular prompt or question with learners, and then begins to use less and less direct prompts or meta-questions such as "what question am I going to ask you?" or "what did you do last time in this sort of a situation?", until the teacher need only rarely if at all remind learners of the prompt: the prompt has been internalised and become a spontaneous action (Mason, 1991). At first astonished by the question, learners become aware of the actions they were carrying out 'in themselves' as being available to carry out 'for themselves'. The prompts become internalised and available for use, at least in situations in which the prompt comes to mind, by the learners themselves. Requisite awarenesses are integrated into functioning so that they are likely to come to mind when required, in the form of actions to choose to carry out. This is perhaps the

paradigmatic manner in which learners develop (in the sense of Vygotsky), although sometimes prompts and questions will come to mind even without being subjected to a process of deliberate scaffolding and fading. Here the propensities and other attitudinal components, together with the sensitivities to notice (even if below the surface of explicit awareness) play a major role in whether and how readily awarenesses trigger actions, or in other words, the extent to which someone has 'learned'.

Awareness & Attention

Awareness and attention are closely related. Someone may be attending to something in a particular way but unaware explicitly of the what or the how. This makes it particularly difficult for teachers and educators to work out what the people they are working with are attending to and aware of. For example, when you express a relationship in words, it is often highly ambiguous as to whether you are expressing a generality, that is, perceiving a property that can hold in different situations, or whether you are forced by language to express yourself in generalities, but in fact you are referring to the foci of your attention, namely specific elements and relationships between them (Mason, Drury & Bills, 2007). When people are caught up in action, in the 'doing', they may be completely oblivious to relationships that are present, although some of these relationships may be enabling the action to be performed.

For example, if a learner says

“to subtract one fraction from another [meaning ‘the ones I am looking at’] I make the bottoms the same and then adjust the tops and subtract the second from the first”

the words are necessarily general. They may be thinking generally, but they may be thinking and attending only to the particular. Even if they are thinking generally, they may not be aware of what they are saying *as* a generality. This is what Noss and Hoyles (1996) are pointing to by using the label *situated abstraction*. If the learner had said

“I multiply the 4 by the 5 and the 10 by the 2 and adjust the 3 by multiplying by 5 as well, and the 7 by 2, giving me 15 minus 14 equals 1 so the result is 1/20”

then there is stronger indication that the learner may be dwelling in the particular, but in fact they may also be exemplifying a generality of which they are aware. Thus a learner may have no sense of generality at all, and be dwelling entirely in the particular; there may be a vague but inchoate sense of generality; there may be an ‘almost articulable’ or ‘tip of the tongue’ sense of generality; there may be a strong sense of generality as a procedure, or as a procedure based on a generative appreciation of how the procedure works; there may be varying (and even unstable) degrees of confidence, varying and unstable degrees of facility, and various assumptions about the *range of permissible change* in the variation permitted by the generality (Mason, Drury & Bills, 2007).

Notice that the ambiguity inherent in interpreting someone’s utterances as statements of generality or of particularity applies at every level. A teacher working with learners needs to be aware of the potential gap between what learners appear to be saying and how they are actually attending; the educator working with teachers needs to be aware of the potential gap between what

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teachers appear to be saying about their teaching or about their learners, and how they are actually attending. As is well known, there are often gaps between espoused, intended and enacted beliefs about teaching and learning (Cooney, 1985) even when the researcher's own interpretations of these is taken into account (see Taylor & Dirkx, 2002). Educators need to be cognisant of their awareness when drawing on pedagogical constructs and didactical tactics so as to be alert and sensitive to how teachers might be attending to and interpreting what is being said: as generality or as particularity.

It is important to point out that the process of emerging awareness is not a monotonic process of constant improvement. Articulating your current sense of something of which you are only beginning to become aware is often fragmentary to the point of incoherence. In the context of beginning algebra, Nicolina Malara (2003) referred to this as 'algebabble' by analogy with young children's babble prior to bursting into speech; Janet Ainley (1999) called it 'emergent algebra'. More generally it has been described as a cyclic process of manipulating confidence-inspiring objects, getting-a-sense of something, and struggling to articulate it (Floyd *et al.* 1981; Mason & Johnston-Wilder, 2006).

The whole point of tasks and emergent activity is that learners become aware of mathematical concepts, techniques, themes, heuristics, ways of thinking, and the use of their own powers in making sense of mathematics and making mathematical sense of phenomena. The more explicit their awareness, the more likely it is that tasks, activity and intervention will be informed and guided by teacher/educator awareness of core elements of mathematical thinking. If attention is taken up by the interaction, there is little room for meta-awareness and hence the use of pedagogical strategies which will direct learner/teacher attention to useful objects and in useful ways.

Attitude

Attitude is another much used term which covers a multitude of meanings. Its etymological origins lie in the word *aptitude* in the sense of 'having the quality of likeliness or appropriateness'. Here it is taken as a synonym for the affective or emotional component of the psyche, the horses in the chariot metaphor described earlier. Strongly attitudinal or affective components include a veritable alphabet of terms: alignments, assumptions, beliefs, desires, dispositions, likes and dislikes, orientation, perspective, stance, *weltanschauung*, wishes, and so on. Attitude includes not only where the force comes from (as in the horses; see also Piaget, 1954/1981) but also the way in which that force is processed through the individual's selves which are active in the situation, producing reactions (automated or habituated actions) and responses (freshly formed or chosen actions). Of course those 'selves' develop through, and because of, interaction in the social milieu.

Selves

Following a long and ancient tradition extending back at least to Plato, I find it helpful to see people not as individuals, but as assemblages of competing selves (Bennett, 1964; Minsky, 1986). Each 'self' is a cluster of typical emotional states and associated habituated or automatic behaviours, with concomitant awarenesses which come to the surface. Different selves are

dominant at different times. For example, the 'person' who leads a seminar is not the 'person' who buys petrol on the way home; the person who fills their briefcase with work they imagine they will do at home is not the person who rejects doing homework once at home: they may have the same outward body, but the gestures, postures, voice tones, vocabulary, way of relating to people, things that are attended to and thoughts that arise may sometimes be quite different in nature. Some people find 'multiple selves' disturbing and prefer to see these as aspects of one self, referred to as 'I'. But detailed investigation into the source of the word 'I' when it is uttered reveal that most often it is a habituated grammatical construction with no referent. Its use is primarily a form of social reassertion of the currently dominant self (Bennett, 1964; Mason, 2002). James (1892) made the point nicely: "if from the one point of view I am one self, from another I am quite as truly many" (p. 202, quoted in Bullough, 2005 p. 146).

Each self therefore can be thought of as an interconnected and self-amplifying collection of propensities, behaviours, indeed a mini psyche. The term *attitude* then functions as a label for motive properties of which ever self is dominating at any given moment. Teachers are well aware that learners invoke different selves in different lessons, and that to connect with a learner so that an experience has some significance requires sensitivity to the learners' currently dominant selves, including invitations and prompts to access a more appropriate self. Adolescents particularly are engaged in the process of discovering that they can have some control, can exercise some choice about which self they bring into play, and that this can have an enormous impact on what they get from lessons. Similarly, educators are well aware that teachers invoke different selves at different times, with the result that sometimes there is eager identification and take up of proposals, and other times hostility or outright rejection. Where teachers cannot imagine themselves taking up an idea, or where learners cannot imagine themselves enacting a proposal, reluctance or rejection are likely outcomes.

When people are together in social groupings there is a social-amplification-attenuation effect. Sometimes there is a sufficient degree of alignment so that it is reasonable to speak of a 'group attitude'. But it is also possible for some people to react against proposals from the outside, for a range of reasons, and so act as a brake on social alignment. As Jaworski (2006) has pointed out, a functioning community of inquiry will experience a 'critical alignment' which is not necessarily a feature of a community of practice. Teachers sensitive to these subtle movements are better able to nudge and cajole, to massage a group into effective mathematical work. Educators sensitive to these subtle movements are better able to work effectively with groups of teachers.

Similarities and Differences in Different States

The terms *attention*, *awareness* and *attitude* provide a useful vocabulary for discerning similarities and differences between the experiential states of people acting as learners, teachers, and teacher educators. People rarely function solely in a single mode: many learners are remarkably aware of pedagogic choices; many teachers work on mathematics for themselves as well as with learners; many teacher educators work on mathematics and on teaching, both for themselves as well as with others. The fact that human beings are complex organisms embedded in a multitude of environments, coupled with the elaboration of aspects of attention, awareness and attitude suggests to me that teachers who are not learning about their learners, and who are not challenging

Being Mathematical With and In Front of Learners themselves mathematically cannot expect to remain fresh and effective for long. Conversely, teacher educators who are not challenging themselves mathematically as well as pedagogically similarly cannot expect to remain fresh and effective for long. For teacher educators this means working on mathematics alone and with others, working on their own pedagogy by using pedagogical constructs to elaborate their practice as well as using their practice to elaborate constructs; for teachers this means working on mathematics alone and with others as well as working on their pedagogical practices and didactic tactics. In short, this can be re-cast as educating awareness, sensitising attention and enriching attitudes to mathematics, to learning mathematics and to teaching mathematics.

From the foregoing it is evident that the aims and intentions of learners, teachers and teacher educators are different (see also Tzur this volume). Learners' primary concern is functioning in relevant mathematical ways as they work on tasks and become aware of heuristics, themes, strategies and forms of mathematical thinking, as well as encountering and re-constructing for themselves important concepts and techniques. Learners may gradually become explicitly aware of some of these aspects of learning mathematics, but their main concern is that these come to mind when needed so that satisfactory responses to tasks can be achieved. Indeed, most learners act as if they have a theorem-in-action regarding learning, or, put another way, their side of the implicit didactic contract (Brousseau, 1997) is to complete the tasks they are set as best they can. Somehow students assume (rarely explicitly or consciously) that task-completion will produce the learning expected. In this sense, they naturally tend to dwell in the particular, at least until someone, or something in the practices in which they are embedded, draws them out of the particular to reflect upon and access encompassing generalities. Sometimes a task-completion orientation goes so far as obtaining answers by whatever method is available, including copying.

The implicit underlying theory of learning seems to be that learning happens as a result of doing and handing in work. Teachers who focus on assigning tasks, marking learner attempts, and giving answers, with various forms of worked examples and explanations, connive to reinforce this simplistic perspective on learning. By contrast, provoking learners in the use of their own natural powers and familiar actions so as to meet new challenges, and then drawing attention to what learners have achieved, and how, directs their attention to what really matters, and affords opportunities for learners awareness to be educated (Mason and Johnston-Wilder, 2006). From a similar perspective, Tzur (this volume) focuses on the activity-effect relationship experienced in carrying out tasks as the core feature which requires reflection and attention for effective learning to take place. Thus teacher intentions and aims are that learners' attention be drawn to experiences in which powers and heuristics have been used effectively, and mathematical themes, concepts and techniques encountered. Teacher educator aims and intentions are that teachers become aware of ways in which learner attention can be directed appropriately to pertinent experiences. This of course requires that teachers themselves become aware of the powers, themes, heuristics which comprise mathematical thinking, as well as both the concepts and techniques themselves, and the psycho-social components of educating awareness and training behaviour associated with them. In short, then, learners are exposed to teachers who are being mathematical with and in front of their learners, supported and

amplified by being in the presence of teacher educators who themselves are being mathematical and who are able to draw attention to what comprises that mathematical being. To be effective, teachers need to develop ‘awareness-in-discipline’, and teacher educators need to develop ‘awareness-in-counsel’ (Mason, 1998). This unusual label is used to emphasise the states of self-awareness which are necessary in order to be able to function at all the requisite levels, similar to the states needed by effective counsellors.

A narrow perspective on the role of tasks and the nature of learning is likely to lead to a constricted disposition towards mathematical thinking, seeing it as the obtaining of answers through the use of memorised procedures. Positive dispositions, self-image, and confidence in self as a mathematical thinker develop from meeting challenge (Dweck, 2000). Attempts to support learner confidence by giving them simple tasks merely reinforces their self-image and does nothing to open them up to other possibilities (Watson *et al.* 2004; Prestage *et al.* 2007; Houssart, 2004).

Where generality is achieved by varying several parameters, including changing contexts, the learner experiences what Treffers (1987) referred to as *horizontal mathematisation* and has many of the characteristics of metonymy and syntactic or surface learning (Marton & Saljö, 1984). *Vertical mathematisation* arises when attention is directed from relationships to properties, and from properties to characteristic, defining or generative properties which can function as axioms from which all other relevant consequences flow. It has many of the characteristics of metaphor and semantic or deep learning (Marton & Saljö, 1984). Both flows are essential to learning and to developing positive dispositions towards mathematical enquiry and use.

Faced with a mathematical task, learners seek answers. To do this they call upon, usually without reflection or awareness, practices into which they have been initiated if not enculturated through engaging in the social practices of the mathematics classroom in particular and of schooling in general, and through interacting with their teachers and their peers. To teach effectively and efficiently requires working differently. Attention is not on answers but on the approach, the forms of thinking employed. Teachers attend to where and how learners are attending, and choose their interactions with learners so as to direct attention in what the teacher trusts will be useful ways. This accounts for the extensive literature on misconceptions (how to direct learner attention so that classic misconceptions or construals will be circumvented or else corrected through exposure to cognitive dissonance or surprise). Teachers also attend to learners’ motivational-affective state, as is evident in their habitual reactions and their considered responses in lessons at different times of the day and in response to learners’ all too explicitly expressed states of boredom, confusion and frustration. They seek ways to attract learner attention so as to engage them, and to connect to learners’ natural curiosity and desire to explain phenomena.

However, teachers too are practitioners. They carry out actions with greater or lesser explicit awareness. They develop functionings or practices, ways of coping, many of which become habitual and below the surface of explicit awareness, as indeed they must if they are to survive the complex interactions with the energies of children and adolescents. As such they are similar to learners but in a different domain. As well as functioning mathematically, teachers function pedagogically and didactically. To become mathematical, that is, to enter and engage in the discipline of mathematics, it is necessary to

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become aware of the awarenesses which enable and direct functionings which can be construed as mathematical (Gattegno, 1987,; Mason, 1998). For example, explicitly retreating from an overly complex task or situation by trying particular or special cases, building up particular cases so as to see through them to an encompassing generality, or making use of a theme such as invariance-in-the-midst-of-change or doing-and-undoing, begin to become 'second-nature' or automatic as they are integrated into functioning. Brown and Coles (2000) give a vivid description of how learners took over the phrase 'same and different' and turned it into a verb which they explicitly initiated as part of the practice of their classroom when working on new ideas. The same process of integration can be accomplished with any sensitivity to notice, any structure of attention, any awareness (Mason, 1999, 2002).

Distinctions such as that drawn by Skemp (1976) between instrumental and relational understanding, by Marton and colleagues (e.g., Marton & Saljö, 1984) between a surface and a deep approach to learning, between procedural and conceptual knowledge, and between syntactic and semantic appreciation all divert attention away from the richly complex experience of 'having pertinent possible actions come to mind' and towards a single spectrum of technique and comprehension. Valsiner (1988) notes that Vygotsky was trying to direct attention away from what the learner can already do (which is what most tests attempt to discover) to what the learner can soon choose to do unaided but can currently accomplish with the guidance of a more-experienced other. This is where learning, growth, and transformation are imminent, what Vygotsky meant by 'proximal development', and hence where the teacher can be of most value. Tzur (this volume) sees the participatory stage of forming a new (to the learners) concept (through becoming aware of an activity-effect relationship) as a significant step towards learners being able to initiate similar activity themselves, which is the development of importance to Vygotsky.

For example, a focus on tasks and their results avoids the complexity of learning as a transformation of awareness, of what is spontaneously attended to and discerned, of the ways that attention is structured, and of mathematical themes, heuristics and powers which come to mind through metonymic triggering and metaphoric resonance. Such a focus concentrates on task-completion and the accumulation of procedures for every situation, eventually putting an excessive burden on memory and leaving the learner vulnerable to forgetting and to inflexibility in the face of unfamiliar challenges. By contrast, maintaining complexity when working with learners is more likely to provide them with sufficiently rich experiences that their awareness is educated, their attention honed and sharpened, and their attitudes enriched.

Teachers develop functional awareness that enables them to cope with the exigencies and dynamics of a classroom full of will-possessing learners who are themselves full of desires and concerns, interests and dislikes, with varying degrees of coordination between their affect, their behaviour and their intellect. Analogously with learners, as teachers become more and more explicitly aware of their implicit functionings or awarenesses which promote learning, they are in a better position to make choices, to respond sensitively rather than automatically to whatever situation emerges. This response covers the Shulman range from content knowledge to pedagogic subject knowledge and beyond (Shulman 1987 p6).

Becoming aware not simply of the fact of different ways of intervening, but of the fact of subtle sensitivities which guide or determine choices between types and timings of interventions, is what is involved in becoming an effective teacher, a 'real teacher' (Mason, 1998). Such explicit awarenesses give rise to a discipline of teaching, which I refer to as *awareness-in-discipline* to distinguish it from *awareness-in-action (op cit)*. The 'in-discipline' is stressing that the actions being brought from implicit to explicit awareness are in and of the discipline of mathematics rather than actions which constitute the discipline. This 'second-order' discipline that is emerging is really the discipline of mathematics education as experienced by a teacher of mathematics.

Someone acting as mentor to a novice teacher who is teaching a lesson while being observed by the mentor and-or by a teacher educator provides an ideal setting for exploring the complexity of interacting awarenesses. There are mirrored parallels between the novice teacher working with the learners, aware of possibilities not yet accessible to the learners, the mentor working with the novice teacher in the same way, and the teacher educator working with the mentor. In order to make sensible and suitable choices, each must also be aware of the world occupied by the other. Thus the mentor is aware of the mathematics being taught as well as the pedagogical strategies and didactic tactics being used; the teacher educator is aware of the mathematics, the strategies and also the constructs and theories which inform and underpin the strategies. Each must choose how much to try to bring to the surface for those with whom they are working. It is no wonder that teaching and learning are so complex, and perhaps it is even surprising that so much learning actually does take place in connection with attempts at teaching! (see also Jaworski, 2007).

The structures of awareness become even more complex when teachers move into mentoring and then into teacher education. When teachers first make this move, they tend to try to describe what they think they used to do when teaching continuing to see themselves as teachers rather than mentors. Over a period of time they discover that this is insufficient for pre-service or in-service work with prospective or practicing teachers. Something more is required. They become aware that there are useful ways of working with people who are themselves learning to teach, many of which parallel ways of working with learners on mathematics, but which alter subtly with the change of focus from the discipline of mathematics to the second-order discipline of teaching mathematics. In the perspective of teacher educator being developed here, the effective teacher educator aims to direct attention so that participants' attention is drawn out of the actions of doing mathematics and also out of the actions of teaching mathematics, so that awarenesses become explicit. In this way, individuals and their social milieu may serve to educate that awareness, and thus inform actions in the future. This forms a third-order discipline.

Worlds of Experience and Modes of Action

One way of speaking about the range of structured attention, awarenesses, and attitudes, and structured attention of people in different situations is in terms of occupying or dwelling in different worlds. The image of living in different worlds is of course ancient, probably as old as human beings, as evidenced by shamanic traditions and more modern religions. Nelson Goodman (1978) used the image to consider how people individually and collectively create the worlds they occupy, in a manner consistent with a Buddhist perspective. Francisco Varela and colleagues (1991) used a similar image in similar ways,

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but focusing on the role of embodiment in in-forming and pro-forming such construction. Jerome Bruner (1966) proposed three modes of representation (enactive, iconic and symbolic) and developed the importance of the imaginative as different from the world of material experience (Bruner 1986). If re-presentation is interpreted as indicating the nature of a world of experience, then his three modes can be thought of as three worlds: physical or material, mental, and symbolic, and by metaphoric extension, manipulative, imagistic and intuitive, and as-yet-only-partially-coherent. Thus familiar and expressive symbols can be manipulated, so for such a person, these occupy a manipulative, metaphorically enactive world, whereas ideas which are as-yet abstract and not well connected occupy a symbolic world as yet out of reach in terms of articulation. This is the basis for frameworks such as *manipulating–getting –a-sense-of-articulating* developed from experience with Bruner’s ideas in the Open University Centre for Mathematics Education (Floyd *et al.* , 1981, see also Mason & Johnston-Wilder, 2006). David Tall (2004) has latterly been developing a cognate but slightly different version in terms of conceptual (embodied), proceptual (symbolic) and axiomatic (formal) worlds.

In my own case, I became explicitly aware of mathematical thinking processes such as specialising and generalising, conjecturing and convincing through seeing the film *Let Us Teach Guessing* (Pólya 1962). It released a style of teaching I had experienced at school which promoted interaction with learners, prompting them to undertake the thinking. My awareness expanded to the second-order discipline which was manifested in the writing of *Thinking Mathematically* (Mason *et al.* 1982). However it took me several years of promoting ‘mathematical investigation sessions’ at Open University Summer Schools before I realised explicitly that many very mathematical tutors were not themselves spontaneously prompting learners to specialise and generalise, to conjecture and convince. Thus arose elements of the third-order discipline described above. I then embarked on developing ways to engage people in mathematical thinking, to draw them out of the actions forming their activity so that they became aware of those actions as potential actions to invoke for themselves, and ways of working with colleagues on working in this way with teachers. It is probably the case that finding myself in a distance teaching institution forced me to articulation where in a face-to-face context I might have been content to engage with participants. These different awarenesses may combined with my propensity to see phenomena as metaphorical, to see, for example, the triad of enactive–iconic–symbolic not simply as doing, imagining and symbolising, but more metaphorically (Mason 1980).

Because attention is ‘where we are’, the form and structure of our attention not only determines but *is* the mental world we occupy. The ‘we’ in this case is led by a cluster of competing selves under the leadership of the currently dominant self, so that there is a particular self in its own characteristic world of attention and awareness. What we are aware of, implicitly as well as explicitly determines the fine-grain details of that world of experience. Our attitude in all of its complexity determines what actions will come to mind through a combination of metonymic triggers and metaphorical resonance, whether automatically or as conscious choices.

Alerted by a sensitivity to notice opportunities to initiate actions, learners, teachers and teacher educators sometimes have the chance to participate in making a choice. Among other things, such choices include a mode of interaction: whether to initiate action (say something, do something), to

respond (to wait expectantly, to listen carefully and try to enter the world of the speakers) or to mediate (to influence activity merely through being present and having contributed to a rich milieu of ways of working, access and use of resources, and so on). This is what is meant by ‘being mathematical with, and in front of’ learners.

The world of the learner is essentially to do with locally focused attention to the tasks and consequent activities, but it needs to reach out to the underlying awarenesses. Teaching is about directing learner attention, and not only being consciousness for two or more, but scaffolding and fading support for learners so that eventually they take over the initiative for themselves. It is about being aware of what learners are not yet aware of, and finding ways to prompt them to become aware. Educating teachers is about directing attention to practices and choices, constructs and theories which can inform choices when teaching. Again it is about being aware of what teachers are not yet explicitly aware of, and prompting relevant shifts of attention. It must be made clear that there is *no* assumption that the teacher educator ‘knows’ what the teacher needs to become aware of, for most often what practitioners need at any level, learner, teacher or teacher educator, are prompts which provoke them to become explicitly aware of what they are currently at best implicitly aware of, but which may be evidenced in their practices, their desires, or their aims.

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