1. Mathematics teacher practice

Planning
- Curriculum objectives
- Classroom structure (introduction, exploration – discussion)
- Tasks
- Materials
- Organization of students’ work
- Management of time
- Assessment

Doing
- Introduction and negotiation of the work and classroom norms (contract)
- Communication in the classroom
- Negotiation of mathematical meanings
- Classroom climate

Reflecting
- The curriculum objectives were met? The students learnt what was sought?
- The tasks and materials were appropriate? The classroom structure and organization of students’ work was all right?
- The (unforeseen) classroom events were dealt with properly?

Overview
1. Teacher practice, craft knowledge, and curriculum change
2. Studies on teachers’ knowledge, practice, and identity
3. Issues on teacher’s knowledge, practice, and identity
4. Collaboration
5. Researching our own practice
6. Teacher education: strategies and frameworks
7. Questions
Curriculum change

Students learn from their mathematical experience and their reflection on their experience.

Direct teaching
Tasks
- Standard task: Exercise.
- Situations are artificial.
- For each problem there is a strategy and a correct answer.
Roles
- The teachers show “examples” so that students learn “how to do”.
- Students receive “explanations”.
- Teachers and textbooks are the only authorities in the classroom.
Communication
- Teachers pose questions and provide immediate feedback (sequence I-R-F).
- Students pose “clarification” questions.

Exploratory learning
Tasks
- Variety: Explorations, Investigations, Problems, Projects, Exercises...
- Situations are realistic.
- There are several strategies to tackle a problem.
Roles
- Students receive tasks to discover strategies to solve them.
- The teacher asks the student to explain and justify his/her reasoning.
- The student is an authority.
Communication
- Students are encouraged to work and discuss with their mates (groups or pairs).
- Frequent classroom discussion (addressing significant work).
- Meanings are negotiated in the class.

2. Teachers’ mathematics knowledge

Key idea
- To teach mathematics, it is necessary to know (well) mathematics.
Questions
- What do mathematics teachers know...
  …About numbers and operations, proportion, equations, functions, number theory concepts, geometry, proof?

Empirical research
- Primitive models
- Concept image/concept definition
- Conceptual knowledge/procedural knowledge

Authors
- Tiross, Graeber & Glover, 1986
- Vinner 1986
- Tall & Hershkowitz 1980

Theoretical frameworks
- Mathematics
- Cognitive psychology

Overriding image: the deficient teacher…
Striking issue: this topic is researched since a long time, with very little progress. Why?

Beliefs and conceptions studies

Key idea
- All mathematics teaching stands on a philosophy of mathematics (Thom, 1973)
Questions
- What are the most important beliefs/conceptions/views/perspectives/images that frame practice?
- How can we change teachers’ beliefs/conceptions?

Empirical research
- Conceptions about:
  - Mathematics
  - Education
- Students’ learning
- Mathematics teaching

Authors
- Thompson 1982, 1992
- Cooney 1985
- Lerman 1983 / Ernest 1989
- G. Törner 1995
- H. Guimarães 1988 (…)

Theoretical frameworks
- Philosophy
- Epistemology
- Psychology

Unsolved issues: What is the relationship between beliefs/conceptions and practices?
What are the processes that constitute beliefs/conceptions?

Cognitive psychology studies

Key ideas
- The teacher is an expert.
- The teacher can be empowered by knowledge about students thinking
Questions
- What are the teachers’ cognitive structures?
- How does the teacher make decisions in the classroom?

Empirical research
- Square, frame, script, agenda, routine
- Knowledge, Goals, Beliefs, Decision Making
- Students’ concepts and cognitive processes in specific areas

Authors
- Borko 1990
- Schoenfeld 2007
- Carpenter & Fennema 1989
- Even & Tiros 1992 (…)

Theoretical frameworks
- Mathematics education

Unsolved issues: In teachers’ thinking and decision making, what is the role of non cognitive elements, such as affective aspects, professional culture, classroom context?
PCK studies (pedagogical content knowledge)

Key idea - “Content” has an essential role in teachers’ professional knowledge, but the teacher has a special way of knowing it.

Recent development - “Specialized knowledge of content” (SCK)

Questions - What are the main elements of PCK and how are they learned?

<table>
<thead>
<tr>
<th>Empirical research</th>
<th>Authors</th>
<th>Theoretical frameworks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teaching of specific topics or areas of the curriculum (number, algebra, functions…)</td>
<td>Lampert 1990 / Ball 1991</td>
<td>Mathematics education</td>
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<td>Cognitive psychology</td>
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<td>Llinares 1993</td>
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<td></td>
<td>Even &amp; Tirosh 1995</td>
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Unsolved issues: What is PCK? What is its nature? How does it develop?

- Lacks emphasis in the level of action
- Does not consider the community of teachers
- Does not consider affective, motivation or passion
- Does not include elements such as students, community, curriculum.

Shulman (2003)

Studies of teachers’ practices

Key idea

- The professional role of teachers is carried out through teachers’ practices

Questions

- What are teachers’ practices?
- What conditions frame teachers’ practices?
- How may teachers’ practices develop?

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<td>Classroom interaction</td>
<td>Wood 1996</td>
<td>Cognitive psychology</td>
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<tr>
<td>Teacher intervention / “scaffolding”</td>
<td>Adler 1995</td>
<td>Sociocultural studies</td>
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<tr>
<td>Teacher classroom management</td>
<td>Tzur 2002</td>
<td>Communities of practice</td>
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<td></td>
<td>Jaworski 1991</td>
<td>Narrative and biography</td>
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<td></td>
<td>Boaler 2003 (…)</td>
<td>Mathematics curriculum and mathematics education</td>
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Issues: What are “good” practices? Who decided that? What is the role of theory?

Studies of teachers’ identities

Key idea

- Identity connects cognitive, affective, and social issues and offers a new perspective to consider teachers’ knowledge, practices, and development.

Questions

- How does a teacher identity develop from pre-service, to beginning, to experienced teachers?

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<td>Teachers’ professional knowledge (Political and institutional) constitution of teachers’ identity Pedagogical practices and beliefs</td>
<td>Walshaw 2004</td>
<td>Cognitive perspective</td>
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<td>Goos 2005</td>
<td>Artistic perspective</td>
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<tr>
<td></td>
<td>Oliveira 2004 (…)</td>
<td>Sociocultural studies</td>
</tr>
</tbody>
</table>

Issue: Is there a teachers’ identity? Or many identities (primary, secondary mathematics, university…)?

3. Professional identity

Link among the individual, the social, and the professional

Individual identity
- Images of the profession / Images of self
- Ways of assuming the professional roles
- Values about education and teaching
- Relationship with statistics
- Relationship with students and colleagues
- Reflexivity
- Professional agency

Group identity
- Professional culture
- Norms and values
- Knowledge specific of the profession
- Processes of induction

Professional practices
- Teaching
- Institutional
- Associative
- Professional development

Attitudes

Social and institutional context
Constitution of a Professional identity

(Pontes and Chapman, 2008)

(Pontes and Chapman, 2006)

Professional knowledge for teaching mathematics

Professionl knowledge
- Know / know how / know how to be specific of the teaching activity.
- In part tacit, in part explicit.
- Oriented towards action.
- Informed by tradition and educational theory.
- Shaped by institutions and contexts.

The teacher is a professional that...
- has a specific knowledge that involves mathematics, curriculum, students, didactics (and education, context, self...).
- carries out specific practices (with a know how).
- has a specific identity (a specific know how to be), as an individual and identifying with a group.

Professional knowledge for teaching mathematics

What is the nature of PKTM?
- Formal/declarative or intuitive/practical?
- Mostly informed by theory or mostly informed by practice?
- How do theory and practice combine?

What is “good” PKTM?
- Who validates it? The professional community of teachers? Researchers in MTE? Social/government institutions external to MT and researchers?
- By what processes is PKTM validated?

How do teachers develop PKMT?
- What is the role of theory? What is the role of experience? How do they combine?
- What conditions promote its development? Hinder it?
Professional knowledge: Production modes

- **Pedagogical tradition**
  - Practice -> practice
  - Observation of models
  - Exercises
  - No research

- **Modern tradition**
  - Theory -> practice
  - Learning in "school way" (taught courses)
  - Research done outside the profession

- **Inquiry tradition**
  - Theory <-> practice
  - Professional problem solving
  - Projects
  - Research done inside the profession

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Collaboration (Andy Hargreaves, 1994)

- Combining the efforts of several people to solve a problem – in particular, from professional practice.
- Based on the idea that several people, working together
  - Have more ideas, more energy and more strength to overcome obstacles that one person alone,
  - May draw on the individual competencies,
  - Have to adjust to each other, creating an efficient system of joint work...
- Emerging process, with a common general goal, marked by mutuality, dialog, and negotiation.

- Collaboration is a fundamental strategy to deal with complex problems.

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Projects (Jean-Pierre Boutinet, 1995)

- Set of activities to reach a goal.
- Are based on the agency of the actors.
- Require unity between conception and execution.
- Take into account the singularity of the situation.
- Move permanently between theory and practice.
- Manage complexity and uncertainty.
- Connect individual and collective processes.
- Move between success and failure.

Projects are the strategy of change par excellence. From the vision to the objectives, to the diagnostic of the problems, conditions and resources, to the plan, action and evaluation.

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A study of a collaborative group

**Cláudia Nunes**

- **The collaborative group**
  - Is an informal group that includes 10 of the 12 teachers of a school,
  - The group exists for about 12 years and usually undertakes a project every year concerning on problem of their professional practice,
  - This project usually has a teacher education component, that yields "credits" through the School TE Center.

- **The research**
  - There was an initial "negotiation" between Cláudia and the group to decide the topic and the activities,
  - Cláudia had to resist to some of the proposals of the group,
  - She had also to resist to some of her impulses to "become a full member" of the group...
  - The focus of research become curriculum management at the classroom level done by the teachers – with attention to the group processes and leadership.
5. Researching our own practice – sources

Reflection (Dewey)
- Careful and active consideration of what one believes or practices, having present the reasons that justify them and the consequences that arise.

Action-research (Lewin, Elliot)
- Activity aiming some change, based in the cycle diagnostic-plan-intervention-evaluation.

Teacher researcher (Stenhouse)
- A teacher that carries out research about his/her own practice or about other issues.

("common") Academic research
- ... And research about practice have different purposes and must be thought of in different ways.

The practice of researching our own practice

Concerning the knowledge generated
- Epistemological critique: why the knowledge produced by teachers can be regarded as valid knowledge?

Concerning the means
- Methodological critique: often lack of clarity and rigor.

Concerning the ends
- Critique of purposes and goals: they are frequently “too” “instrumental”…

The practice of researching our own practice

- Formulation of the problem and study questions,
- Collection of data to respond to that problem,
- Interpretation of the information collected to draw conclusions,
- Dissemination of the results and conclusions.
Proportion – Grade 6
Ana Isabel Silvestre

Teaching unit
- Diversified tasks with emphasis on explorations,
- Realistic contexts and interdisciplinary (youngster’s’ literature),
- Group work, production of group reports.

Duration, materials, evaluation
- 9 blocks of 90 minutes, using worksheets made by the teacher (the textbook was not used),
- Work in the classroom, complemented by work in the computer room (Excel).

Research Design (Teaching Experiment + Case Studies)
- Classroom observations and writing a teacher’s journal,
- Interviews with three pupils at the end of the teaching unit.

Exploring direct proportion relationships – Ana Isabel Silvestre

Is this table representing a direct proportion? Do you want to use Excel to figure it out?

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>2.3</th>
<th>4.3</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>7.5</td>
<td>12.9</td>
<td>15</td>
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</table>

Guilherme observes the table and says that he does not need to use Excel, since he already knows that there is proportionality.

Teacher: How do you know that there is proportionality in this situation?
Guilherme: (Does not use any material) So, first I saw [line] A and [line] B (points to the two lines). I saw the last pair of numbers.

Teacher: What?
Guilherme: (Points to the last column) Multiplying 5 by 3 yields 15... I am almost sure that yes [there is proportionality]. As I did this computation I believe that the same happens with the other numbers and it does, in 7.5 there is 3 times 2.5. It suffices to know the number facts [tabuada] to know that 3 [decimal part of 4.3] times 3 yields 9 [decimal part of 12.9] and 3 times 4 is 12. This is right.

Teacher: Can you identify the proportion constant?
Guilherme: It is 3. If we divide 15 for 5 it yields 3. It also happens with the other [pairs of] numbers.

Proportion, Grade 6
Ana Isabel Silvestre

Results
- Surprise with the contextual situation,
- Involvement with ICT
  - Reports in Excel...
  - Sent by e-mail
- Group work + general discussion, provided a setting for justification.

Findings
- Students show to understand the meaning of their work in the contexts proposed and develop scalar and functional strategies based in the identification of patterns in proportional relationships...
- ... And consider tables as a representation that facilitates reading and understanding data.

Research plan
Neusa Branco

- Preparation of:
  - The teaching unit (strategy, tasks, timing, assessment...)
  - The research tools (to identify students’ thinking and the contribution of the teaching unit in students’ learning).
- Data collection:
  - in different contexts: (i) the classes and (ii) in greater depth, from two students, - with different instruments.
- Analysis and interpretation (linking the different data sets).
Teacher researcher’s conflicts in data collection (Neusa)

**How to collect data in the classroom context?**
- It was important **to know the work** of the students in each task, so
  - I wrote a research journal,
  - Collected students’ written work and
  - Audio recorded moments of classes.
- The **analysis** showed me the concepts students discussed, the strategies they used, and where they had more difficulty, in the classroom.

**What is the relationship with students in the interview?**
- The researcher prevails but the teacher
  - takes equal care of all the students in class,
  - is always there, behind the researcher, interpreting the students’ signs.

**Distancing**

**Theoretical strategies**
- Using **analytical lenses**: Teachers’ classroom practices, difficulties and understanding of the students.

**Empirical strategies**
- **Relocating the object** (e.g., students’ discourse, students’ activity, group work),
- Generating **records of practice** (video, audio, written...).

**Interational strategies**
- **Collaborating** (teacher/students, teacher/other teachers)
- **Reflecting** with others
  - Teachers who have similar preoccupations/problems,
  - Teachers with different professional experience,
  - Academic researchers,
  - Presentations at research and professional meetings.

**Strategies for teacher education**

**Problematising**
- Reflection
- Observing/inquiring practice
- Action-research
- Researching our own practice (Noticing)

**Interacting**
- Networking
- Collaboration
- Communities of practice
- Communities of discourse
- Learning communities

**Changing Projects**

**Resourcing**
- Using digital resources (MILE etc)
- e-learning and b-learning

**Teacher agency**
- Teacher consumes… negotiates… decides…

**Time scale**
- 1 day… 1 trimester… 1 year… several years…

**Theory-practice**
- Focus theory… on practice… on analyzing practice… on changing practice…

**Sustainability**
- Teacher “lone ranger”… with colleagues… with school support… with university support…
Influences on teacher education practices

Key idea 1 - Teacher education based in professional practice

Teacher education based on practice (Smith, 2001)
- Teacher education seeks to recognize the existing problems in the practical situation that the teacher experiences and frame their solution in the light of theory.

Teacher education situated “in practice”
- The materials that represent the teaching activity (students’ work, mathematical/statistical tasks, classroom episodes) are used as opportunities for critique and investigation.
- Teachers develop knowledge analyzing real situations.

Teacher education based on teachers’ own practice
- Teachers collect data from their practice and reflect about them with support of the teacher education setting (teacher educator, other teachers).

Key idea 2 - Teacher education based at the school and the professional group

Diagnostic of students real difficulties
- The starting point of teacher education process is not “what is new” that one must know, but the real struggles of students.

Intervention/Professional development projects
- Establishing verifiable objectives,
- Working in a project and collaborative mode.

Auto-learning in the group
- When it is not possible to solve a problem within the group, one seeks external help.

Organization
- Joint planning and exchange of experiences,
- The classroom and the subject group (or its formal subgroups) are essential acting spaces.

Key idea 3 – Challenge, support, recognize, empower

Provide examples and reflect good teaching practice
- Using worthwhile tasks, improving classroom discourse...
- Encouraging students’ reasoning, expecting teachers to assume intellectual risks.

Create disequilibrium in teachers
- Challenging conceptions about mathematics, mathematics learning and who can do it,
- Involving necessary moments of discomfort...

Encourage teachers’ collaboration
- Defining common goals and combining with individual objectives...
- Negotiating ways of working together...

Take into account the teachers’ contexts
- Students, teachers and their current practices and resources available,
- Regulations, educational system guidelines, school calendar, school administration.

Use teachers’ knowledge and competency
- About students, curriculum, schools, and communities,
- Also use external contributions, e.g., university staff.

Sustentability and cohesiveness
- Set experiences that amplify each other and contribute towards a coherent plan (project),
- Involving a significant time...
Large scale teacher education

National Teacher Education Plan for primary teachers (1-4 and 5-6) in mathematics
- Coord.: Lurdes Serrazina
- 1 year of work
- weekly meetings and supervisions at school

Teacher education for the new basic education curriculum (1-4, 5-6, 7-9)
- Coord. Curriculum team
- “Oficinas” with 20 teachers
- 6 sessions 4 hours each (3—4 months), with the planning, doing, reporting and discussing of a classroom experience.

School based teacher education for the new basic education curriculum
- Involving all teachers of a school-grouping of schools
- Focused on school improvement plans, working on yearly cycles.

7. Questions for further discussion

- Is mathematics teaching moving towards becoming a professional activity? Are mathematics education researchers supporting or opposing it?
- Is the mathematics teachers’ identity changing? If so, what are the driving forces of such change?
- Is it possible to investigate teaching without the collaboration of teachers? Is it possible to improve teaching without improving mathematics teachers’ craft knowledge?
- Is it possible for mathematics educators to work (collaboratively) with teachers and get (full) academic recognition?
- What is acceptable research on our own practice for a professional group (mathematics teachers)?
- How to design large scale in-service teacher education with real impact in teaching practices?
- What are the critical choices that a pre-service teacher education program (for math / for elementary teachers) faces?

Complex settings

(Krainer, 1998) (Jaworski, 2003)