1. Teachers’ mathematics knowledge

Key idea
- To teach mathematics, it is necessary to know (well) mathematics.

Questions
- What do mathematics teachers know...
  - About numbers and operations, proportion, equations, functions, number theory concepts, geometry, proof?

Beliefs and conceptions studies

Key idea
- All mathematics teaching stands on a philosophy of mathematics (Thom, 1973)

Questions
- What are the most important beliefs/conceptions/views/perspectives/images that frame practice?
- How can we change teachers’ beliefs/conceptions?

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Overriding image: the deficient teacher…

Striking issue: this topic is researched since a long time, with very little progress. Why?
Cognitive psychology studies

Key ideas
- The teacher is an expert.
- The teacher can be empowered by knowledge about students thinking

Questions
- What are the teachers’ cognitive structures?
- How does the teacher make decisions in the classroom?

Empirical research
- Squier, frame, script, agenda, routine
- Knowledge, Goals, Beliefs, Decision Making
- Students’ concepts and cognitive processes in specific areas

Authors
- Borko 1990
- Schoenfeld 2007
- Carpenter & Fennema 1989
- Even & Tiorsk 1992 (…)

Theoretical frameworks
- Cognitive psychology
- Mathematics education

Unsolved issues: In teachers’ thinking and decision making, what is the role of non-cognitive elements, such as affective aspects, professional culture, classroom context?

PCK studies (pedagogical content knowledge)

Key idea - “Content” has an essential role in teachers’ professional knowledge, but the teacher has a special way of knowing it

Recent development – Breaking MKT into CCK, SCK, HK and breaking PCK into KCT, KCS, KCC

Questions - What are the main elements of PCK and how are they learned?

Empirical research
- Teaching of specific topics or areas of the curriculum (number, algebra, functions…)

Authors
- Lampert 1990 / Ball 1991
- Eisenhardt et al. 1993
- Linares 1993
- Even & Tiorsk 1995
- Ball et al. (2005, 6,7,8,9…)

Theoretical frameworks
- Mathematics education
- Cognitive psychology

Unsolved issues: What is PCK? What is its nature? How does it develop?
- Lacks emphasis in the level of action
- Does not consider the community of teachers
- Does not consider affective, motivation or passion
- Does not include elements such as students, community, curriculum.

Professional knowledge for teaching mathematics

What is the nature of PK?
- Formal/declarative or intuitive/practical?
- Mostly informed by theory or mostly informed by practice?
- How do theory and practice combine?

What is the content of PK? (hat is “good” PK?)
- Who validates it? The professional community of teachers?
- Researchers in mathematics teacher education?
- Social/government institutions external to mathematics teaching and researchers?
- By what processes is PK validated?

How do teachers develop PK?
- What is the role of theory? What is the role of experience?
- How do they combine?
- What conditions promote its development? Hinder it?

Mathematics teacher craft knowledge

Academic knowledge
- Formal, declarative
- May be assessed through tests
- Theoretical

Common Sense
- Experiential
- Cultural and context-bound
- Practical

Craft knowledge refers to the professional knowledge used by the teachers in their day-to-day classroom teaching; action-oriented knowledge which is not generally made explicit by teachers, which they may indeed find difficult to articulate, or which they be unaware of using. (Ruthven and Goodchild, 2008)

Professional (craft) knowledge
- Is performative (knowing in action)
- In part explicit, in part tacit
- Is recognized by the professional group of practitioners (and by outsiders)
- May integrate theory and practice
**2. Studies of teachers’ practices**

**Key idea**
- The professional role of teachers is carried out through teachers’ practices

**Questions**
- What are teachers’ practices?
- What conditions frame teachers’ practices?
- How may teachers’ practices develop?

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<tr>
<td>Boaler 2003 (...)</td>
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**Issues:** What are “good” practices? Who decides that? What is the role of theory?

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**Professional knowledge: Production modes**

- **Pedagogical tradition**
  - Practice -> practice
  - Observation of models
  - Exercises
  - No research

- **Modern tradition**
  - Theory -> practice
  - Learning in “school way” (taught courses)
  - Research done outside the profession

- **Inquiry tradition**
  - Theory <-> practice
  - Professional problem solving
  - Projects
  - Research done inside the profession

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**2. Mathematics teacher professional practice**

**Planning**
- Curriculum objectives
- Classroom structure (introduction, exploration – discussion)
- Tasks (Adapting/Designing)
- Resources
- Organization of students' work
- Management of time
- Assessment

**Doing**
- Introduction and negotiation of the work and classroom norms (contract)
- Handling mathematical growth (challenging/supporting/observing)
- Handling classroom communication (questioning/explaining/negotiating meanings/orchestrating discussions)
- Regulating the classroom climate

**Reflecting**
- The curriculum objectives were met? The students learnt what was sought?
- The tasks and materials were appropriate? The classroom structure and organization of students' work was all right?
- The (unforeseen) classroom events were dealt with properly?
Variety: Explorations, Investigations, Curriculum change

Tasks
- Standard task: Exercise.
- Situations are artificial.
- For each problem there is a strategy and a correct answer.

Roles
- The teachers show "examples" so that students learn "how to do".
- Students receive "explanations".
- Teachers and textbooks are the only authorities in the classroom.

Communication
- Teachers pose questions and provide immediate feedback (sequence I-R-F).
- Students pose "clarification" questions.

Exploratory learning

Tasks
- Variety: Explorations, Investigations, Problems, Projects, Exercises...
- Situations are realistic.
- There are several strategies to tackle a problem.

Roles
- Students receive tasks to discover strategies to solve them.
- The teacher asks the student to explain and justify his/her reasoning.
- The student is an authority.

Communication
- Students are encouraged to work and discuss with their mates (groups or pairs).
- Frequent classroom discussion (addressing significant work).
- Meanings are negotiated in the class.

Studies of teachers' identities

Key idea
- Identity connects cognitive, affective, and social issues and offers a new perspective to consider teachers' knowledge, practices, and development.

Questions
- How does a teacher identity develop from pre-service, to beginning, to experienced teachers?

Empirical research
- Teachers' professional knowledge
- (Political and institutional) constitution of teachers' identity
- Pedagogical practices and beliefs

Authors
- Walshaw 2004
- Goos 2005
- Oliveira 2004

Theoretical frameworks
- Cognitive perspective
- Artistic perspective
- Sociocultural studies
- Narrative and biography
- Mathematics education

Issue: Is there a teachers' identity? Or many identities (primary, secondary mathematics, university...)?

Professional identity

Link among the individual, the social, and the professional

Individual identity
- Images of the profession / Images of self
- Ways of assuming the professional roles
- Values about education and teaching
- Relationship with mathematics
- Relationship with students and colleagues
- Reflexivity
- Professional agency

Group identity (social)
- Professional culture
- Norms and values
- Knowledge specific of the profession
- Processes of induction

Professional practices
- Teaching
- Institutional
- Associative
- Professional development

Social and institutional context

Constitution of a Professional identity

Preservice teacher education

Participation in other professional spaces

Professional practice at the school (with colleagues, students, parents, the community)

Teaching practices (interaction with students)

Ponte and Chapman (2008)
Theoretical perspectives about the mathematics teacher

1. What is the nature of TK?
2. How is TK appraised?
3. What is the relation between TK and TP?
4. How does TK develop?
5. How does TI develop?

Teacher identity
- Perspectives, values, norms, ways of being

Teacher (professional) knowledge
- Mathematics
- Mathematics teaching
- Curriculum
- Students’ learning

Teacher practice
- Planning
- Classroom teaching
- Reflecting
- Other professional roles

Combined perspectives
- Cognitive
- Social
- Educational

Cognitive perspectives versus Social perspectives

4. Strategies for teacher education

Problematizing
- Reflection
- Observing/inquiring practice
- Action-research
- Researching our own practice (Noticing)

Changing
- Projects

Interacting
- Networking
- Collaboration
- Communities of practice
- Communities of discourse
- Learning communities

Resourcing
- Using digital resources (MILE etc)
- E-learning and b-learning

The fundamental dilemma in teacher education

Knowledge that the teacher “needs”
- Mathematics
- Children’s learning
- The curriculum frameworks, aims, connections
- Instructional materials
- ICT
- Classroom communication
- Assessment
- Multicultural settings
- Ethno mathematics
- Critical mathematics education
- Inclusive education
- Students with special needs
- History and epistemology of mathematics
- ...

Teachers’ personal processes
- Autonomy
- Professional development
- Teacher reflection (critical)
- Teacher agency
- Ownership
- Transformative learning
- ...

What is the nature of this transaction?

Design of teacher education (TE) programs

Role of the context
- Provide conditions for teacher education / Value teacher education

Needs and interests
- Negotiation of the TE proposal
- Activities and their relation to practice
- End of the course / new challenges

Direct role of the teacher educator / TE setting
- Challenge / New ideas / Resources / Support in doing in practice
- Presence / Empathy / Emotional support

Core of the design of the teacher education process
- Perspective about how the teacher learns and develops professionally - the role of practice, reflection, collaboration, professional projects, etc.
Four critical elements in the design of teacher education settings

**Theory-practice**
Focus on theory... on practice... on the analysis of practice... on changing practice... (projects)

**Teacher agency**
Teacher... teacher agency decides...

**Organization (Space and Time)**
School-Professional settings/Intensive Extended

**Support**
"Isolated teacher"... with colleagues... in the school

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**Example**

The calculator module
(Célia Mercè)

**Underlying principles**
- Teachers' classroom practices as a starting point,
- Attention to the practical needs of teachers in relation to their curriculum practices,
- Strong emphasis on collaborative work among teachers.

**Collective session format**
- 1. Reflection on the tasks previously implemented in the classroom,
- 2. Working on tasks and discussing (deepening teachers' mathematics and mathematics teaching knowledge),
- 3. Discussion of the tasks to propose in the classroom.

**Classroom supervision**
- The teachers could put into practice the tasks collectively planned, keeping the responsibility of organizing and coordinating the classroom work,
- Analysis and reflection by the teacher and the supervisor after each class.

**Session contents**
- Exploring student errors in solving problems; Exploring different representations of rational numbers; Operations with rational numbers....

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**Example**

The calculator module
(Célia Mercè)

**Task**
In a clothing store there are t-shirts that cost 44.50€ and are now with a deduction of 20%. Ana decided to buy two t-shirts and found, after doing some calculations that it would pay 53.40€. When he reached the box the wizard asked for 71.20€. Ana was very confused. So, who is right?

- **Maria** - I think that Ana is right because a t-shirt has a 20% discount, so:
  - 44.50 x 2 = 89
  - 20+20=40%
  - 89 x 0.40=35.60
  - 89-35.60=53.40

- **Francisco** - None is right because: 89-40%= 49€

- **Miguel** - The wizard is correct because:
  - 44.50 x 0,20=8.90€
  - 8.90+8.90=17.80€
  - 89€ - 17.80€=71.20€

- **Marta** - The wizard is correct because: 89-20% = 71.20€

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Sónia: No, she [Marta] has nothing to do with Francisco (...) She did not indicate correctly the problem, so, if she got 71.20, it is because she had to do 20% of 89. Anabela: But is not what is here.

Sónia: Okay, this is what I am saying. She put not the correct indication.

[Lot of talking]
Teacher Educator (TE): But how does she do 89-20% and get 71.20?
Sónia: She did with the machine.

[Lot of talking. Sónia takes a calculator and does computations]
Sónia: Oh! The machine was roguish. If she put this here on the machine she gets 71.20 right away.
Anabela: Oh, yeah. [Confirms in the calculator]
Sónia: I, by chance, was not sure if this would work.
Monica: What?
Sónia: If we put 89-20%...
Marta: But, that... She stopped using the percent technique.
[Confusion among teachers. Some seeming outraged by what the student did]
Key idea 1 - Teacher education based in professional practice

**Teacher education based on practice**  
(Smith, 2001)
- Teacher education seeks to recognize the existing problems in the practical situation that the teacher experiences and frame their solution in the light of theory.

**Teacher education situated “in practice”**
- The materials that represent the teaching activity (students’ work, mathematical/statistical tasks, classroom episodes) are used as opportunities for critique and investigation.
- Teachers develop knowledge analyzing real situations.

**Teacher education based on teachers’ own practice**
- Teachers collect data from their practice and reflect about them with support of the teacher education setting (teacher educator, other teachers).

Key idea 2 - Teacher education based at the school and the professional group

**Diagnostic of students real difficulties**
- The starting point of teacher education process is not “what is new” that one must know, but the real struggles of students.

**Intervention/Professional development projects**
- Establishing verifiable objectives,
- Working in a project and collaborative mode.

**Auto-learning in the group**
- When it is not possible to solve a problem within the group, one seeks external help.

**Organization**
- Joint planning and exchange of experiences,
- The classroom and the subject group (or its formal subgroups) are essential acting spaces.

Key idea 3 – Challenge / support

**Provide examples and reflect good teaching practice**
- Using worthwhile tasks, improving classroom discourse...
- Encouraging students’ reasoning, expecting teachers to assume intellectual risks.

**Create disequilibrium in teachers**
- Challenging conceptions about mathematics, mathematics learning and who can do it,
- Involving necessary moments of discomfort...

**Encourage teachers’ collaboration**
- Defining common goals and combining with individual objectives...
- Negotiating ways of working together...

**Take into account the teachers’ contexts**
- Students, teachers and their current practices and resources available,
- Regulations, educational system guidelines, school calendar, school administration.

**Use teachers’ knowledge and competency**
- About students, curriculum, schools, and communities,
- Also use external contributions, e.g., university staff.

**Sustantability and cohesiveness**
- Set experiences that amplify each other and contribute towards a coherent plan (project),
- Involving a significant time...

Influences on teacher education practices

- Conceptions, knowledge, resources of teacher educators and institutions
- Official regulations and educational policy
- Social context
- Perspectives and results from research
- School context
- Conceptions, needs and interests of teachers
Large scale teacher education

National Teacher Education Plan for primary teachers (1-4 and 5-6) in mathematics
- Coord.: Lurdes Serrazina
- 1 year of work
- weekly meetings and supervisions at school

Teacher education for the new basic education curriculum (1-4, 5-6, 7-9)
- Coord. Curriculum team
- "Workshops" with 20 teachers
- 6 sessions 4 hours each (3-4 months), with the planning, doing, reporting and discussing of a classroom experience.

School based teacher education for the new basic education curriculum
- Involving all teachers of a school-grouping of schools
- Focused on school improvement plans, working on yearly cycles.

Questions for discussion

- Is mathematics teaching moving towards becoming a professional activity? Are mathematics education researchers supporting or opposing it?
- Is the mathematics teachers' identity changing? If so, what are the driving forces of such change?
- Is it possible to investigate teaching without the (implicit/explicit) collaboration of teachers?
- Is it possible to improve teaching without improving mathematics teachers' craft knowledge?
- Is it possible for mathematics educators to work (collaboratively) with teachers and get (full) academic recognition?
- How to design large scale in-service teacher education with real impact in teaching practices?
- What are the critical choices that a pre-service teacher education program (for math / for elementary teachers) faces?

References


