Art, Mathematics and Architecture for Humanistic Renaissance: the Platonic Solids

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ABSTRACT
Platonic solids and the polyhedra have been connected with the world of art and architecture in different cultures and through many centuries. For some Renaissance artists, for example Leonardo da Vinci (1452 -1519), Albrecht Dürer (1471-1528), Piero della Francesca (around 1420-1492), this solids provided the models to demonstrate the properties of the symmetry and to apply the laws of the perspective. For others, the polyhedra were the symbols of deep religious or philosophical truths. In the last centuries, they have also influenced the architecture and the design. For example, Bruno Munari (1907- 1998) and the husband and wife design team composed by Charles Eames (1907-78) and Ray Eames (1912-88) were designers fascinated by the polyhedral shapes. The aim of this paper is to describe an overview of the use of Platonic solids and polyhedra in different periods and in different cultures.

1. INTRODUCTION
The regular polyhedra, also known as the Platonic solids, are the three-dimensional bodies whose surfaces consist of identical, regular polygons (e.g., equilateral triangles, squares, and pentagons) which meet in equal angles at the corners [3]. There are five Platonic solids: the Tetrahedron, the Octahedron, the Hexahedron (Cube), the Icosahedron and the Dodecahedron, as illustrated in figure 1.

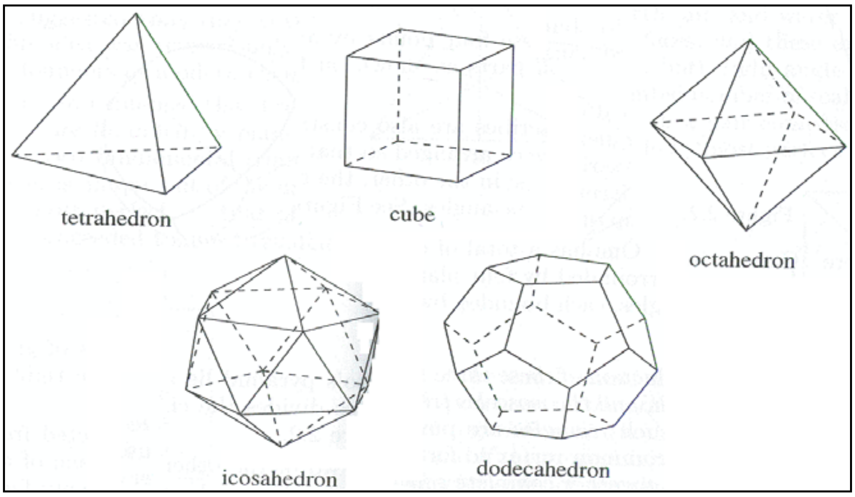

Figure 1 The Platonic solids.

The adjective "Platonic" derived by the Greek philosopher Plato (427-348 B. C.), that has described the five regular solids in his *Timaeus*, and he also illustrated and explained the construction of the five regular solids based on the "*most beautiful of all many triangles*" (Timaeus, 54a). The Greek philosopher found an important association in the *Timaeus* between the Platonic solids and the Empedocles' Four Elements: the fire, the earth, the air, and the water (and the universe) [1, 6, 13]. This association has influenced the philosophers, the artists, and the mathematicians of the Renaissance period. Plato constructed the five solids using simple rules and simple polygons: "*The first will be the simplest*" (Timaeus, 54e–55a) the tetrahedron "*which is the original element and seed of fire*" " (Timaeus, 56b). "*The second species of solid is formed out of the same triangles…*" (Timaeus, 55a). The octahedron "*let assign the element which was next in the order of generation to air*" (Timaeus, 56b) and "*the third to water*" (Timaeus, 56b) the icosahedron. The cube: "*To earth let us assign the cubical form…to earth is the most immovable of the four*" (Timaeus, 55d- 55e), and the dodecahedron "*There was yet a fifth combination which God used in the delineation of the Universe*" [5, 6]. The Platonic Correspondences are the following:

Tetrahedron → Fire,
Icosahedron →Water,
Octahedron→Air,
Cube→ Earth,
Dodecahedron → The Quinta Essentia (the "Universe") [1, 2, 5].

There is also an interesting connection with the Four Humour Theory of Hippocrates (460-377 B. C.). Hippocrates was "the father of the medicine", and he conceived a theory based on blood, phlegm, yellow bile and black bile. His theory is in correspondence with the Empedocles' Four Elements Theory. In this way, existed a connection between the Hippocrates' theory and the Platonic solids, too [1, 14].

The correspondences between the Platonic solids, the four elements, the four humour, and the colours are the following:

Tetrahedron →	Fire →	Blood →	Red
Cube →	Earth →	Black bile →	Black
Octahedron →	Air →	Yellow bile →	Yellow
Icosahedron →	Water →	Phlegm →	White

2. PLATONIC SOLIDS IN THE ANCIENT AGE

We can meet the polyhedral shapes in the arts and in the architecture across the centuries and in different cultures. In the Neolithic period some carved stones (around 2000 B. C.), discovered in Scotland, were shaped like polyhedra [8, 11].

Other example of an ancient object shaped like Platonic solid is illustrated in figure 2. It is a bronze dodecahedron, 8 x 7 x 7 cm, (2nd century B. C.) which is decorated with circular holes of various diameters in each face and with spheroids at each vertex. It were produced in Roman period. It is preserved in the Rheinisches Landesmuseum in Bonn [5, 8].

Figure 2 Dodecahedron (2nd century B.C.)

3. PLATONIC SOLIDS IN THE RENAISSANCE

The Renaissance is the period of European history at the close of the Middle Ages and the rise of the modern world. The word "Renaissance" derived by the Italian word "Rinascimento" and it represents a cultural rebirth from the 14^{th} through the middle of the 17^{th} centuries.
In the age of the Renaissance, the artists used the Platonic solids and the polyhedra in their productions, and to study the properties of the perspective. For example, the Italian artist Paolo Uccello (1397-1475) has applied the Platonic solids and the polyhedra in his works. It is famous the mosaic, untitled, that he has realized in the Basilica of San Marco (Venice), illustrated in figure 3 [5, 8].

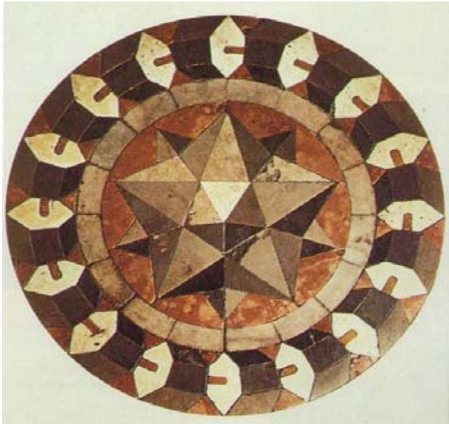

Figure 3 Paolo Uccello, Untitled, mosaic, 80 x 80 cm., Basilica of San Marco, Venice (1425 – 1430)

Leonardo da Vinci (1452 -1519) is an other artist that was fascinated by the mathematics and by the Polyhedra. Leonardo drawn the illustrations for the book entitled *De Divina Proportione* (*On Divine Proportion*) (1509[1]), written by the Franciscan friar Luca Pacioli (1445-1514). *De Divina Proportione* comprises three books: *Compendio de Divina Proportione* (which contains the properties of the golden section, that Luca Pacioli named "divine proportion"), the second book describes the architecture of Marcus Vitruvius Pollio (Marco Vitruvio Pollione) (70-23 B. C.), known as Vitruvio; and the third book, entitled *Libellus in Tres Partiales Tractatus Divisus Quinque Corporum Regularium*, is a translation, in Italian language, of the Piero della Francesca's treatise *Quinque Corporibus Regularibus*. Leonardo drawn about sixty illustrations for the Pacioli's book. In the figures 4a and 4b, there are two examples of Leonardo's polyhedra illustrated for the *De Divina Proportione* .

a) b)

Figure 4 Leonardo's illustrations for the manuscript copies of Pacioli's treatise.

[1] Two manuscripts of the *De Divina Proportione* (dated 1498) have been found in Italy in the Biblioteca Civica of Genova and in the Biblioteca Ambrosiana of Milan.

The German artist Albrecht Dürer (1471-1528), has written an important study on the polyhedral literature in his book entitled: *Underweyssung der Messung mit dem Zyrkel und Rychtschey* (1525). This book, available in English version as *Painter's Manual* [4], was one of the first written works that explains and shows the methods and the properties of the perspective, and it represents an important "milestone" from the 16th century artists.
The most important engraving realized by Dürer was *Melancholia I* (1519). In this work we can find some interesting symbolism [8, 10, 12,15]. For example, the mysterious polyhedron truncated and the magic square associated to the Planet of Jupiter (and connected to the blood humour).
The German astronomer Johannes Kepler (1571-1630) in his book entitled: *Mysterium Cosmograficum* (1595) has conceived a representation of our Solar system where the Platonic solids play a central role. He has associated the Platonic solids to the six planets known in that period (Saturn, Jupiter, Mars, Earth, Venus and Mercury).

4. PLATONIC SOLIDS IN THE LAST CENTURY

The Dutch artist Maurits Cornelis Escher (1898-1972) has studied and applied the mathematics, for example the hyperbolic geometry and the fractal geometry, to create his dream-like worlds. Escher has also applied the polyhedral shapes in his works [8, 14]. For example, in *Reptiles*, lithograph (1943), there is the presence of a dodecahedron (the symbol of the "Quinta Essentia"). The wood engraving *Stars* (1948) includes the compound of different vacuum polyhedra (octahedra and cubes) that contain some chameleons [8].
The Spanish surrealist painter Salvador Dalì (1904-1989) was fascinated by the Platonic solids. In *Crucifixion (Corpus Hypercubus)* (1954) he represented the cross using eight cubes (figure 5), in and his *The Sacrament of the Last Supper* (1955) a vacuum dodecahedron (as symbol of "God") is located over Jesus and his disciples, like a "virtual" embrace (figure 6).

Figure 5 *Crucifixion (Corpus Hypercubus)* (1954)

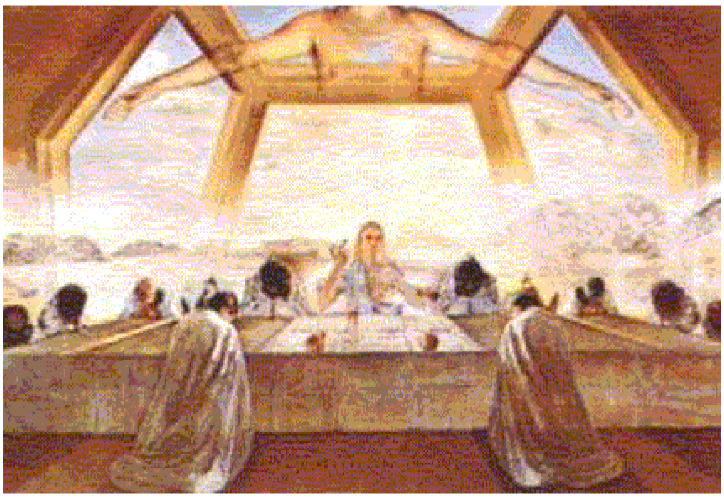

Figure 6 *The Sacrament of the Last Supper* (1955), Dali's oil painting

The Italian mathematician Lucio Saffaro (1929-1998) has realized many scientific works to connect the science and the arts, and he has used the polyhedra as starting point for his theory.

The Italian designer Bruno Munari (1907- 1998) has studied the shapes of the Platonic solids and the polyhedra to apply them in his product of design (ashtrays, lamps). His *Cubic ashtray* (1957) represents a famous example of Italian design (figure 7). Munari did not smoke but he has created a functional ashtray.

Figure 7 *Cubic ashtray (1957)* realized by Bruno Munari

The husband and wife design team composed by Charles Eames (1907-78) and Ray Eames (1912-88) has modified the shape of the America's twentieth century. Their projects and works represented the global expansion of the American culture around the world. In *The Toy* (1951) they have realized a kit to create three-dimensional structures based on polyhedral shapes.

5. PLATONIC SOLIDS IN ARCHITECTURE

Egyptian and Maya pyramids are good examples of polyhedra applied in the ancient architecture. These shapes, their symmetry and their properties, had guaranteed the structural equilibrium.

In the 20th century, polyhedral geometry has been found to be the basis for a wide range of designs, such as Fuller's geodesic domes, deployable buildings and many other types of "nonstandard architecture" [7]. Geodesic Domes are designed by the American architect-engineer Richard Buckminster Fuller (1895-1983). Fuller invented the Geodesic Dome, shows in figure 8, to demonstrate his ideas about housing and "energetic-synergetic geometry" which he had developed during WWII. A Geodesic Dome is shaped like a molecule of carbon [1].

Other modern architects have used Platonic solids and polyhedral shapes in their buildings.

The Swiss Charles-Edouard Jeanneret known as Le Corbusier (1887-1965) is one of the architects that has influenced the modern architecture of the 20th century. He is the "father" of the "modulor" (word derived by the French phrase: "module et section d'or", "module and golden section") a system based on proportions of the human body and connected to the golden section. In his project realized for the *Convent of Saint Marie de la Tourette* (1957 to 1960) at Eveux-sur-Arbresle, near Lyon (France), Le Corbusier has applied the polyhedral shapes to emphasize the "rigour" of the Religious order of the Dominican.

Figure 9 shows a building (1980 to 1981) where is present a polyhedral shape. It has been designed by the Swiss architect Mario Botta. It is located in Pregassona, Canton Ticino, Switzerland. Botta has realized other buildings using polyhedral shapes, for example, *The One-family house* (1971-1973) at Riva San Vitale, Canton Ticino, Switzerland.

Figure 8 Fuller's geodesic dome.

Figure 9 Botta's building in Pregassona (CH).

6. CONCLUSIONS

Through centuries, Platonic solids and the polyhedra have fascinated the artists, the architects, the engineers, and the designers. Mathematics is a science that contains amazing objects, and their interrelations and symbolism are sources of endless beauty, and they help to create an aesthetic sense. As Bertrand Russel affirmed: *"Mathematics possesses not only truth but supreme beauty, a beauty cold and austere, like that a sculpture, sublimely pure and capable of a stern perfection, such as only the greatest art can show*".

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