Creating a Constructivist Mathematics Department from a Traditional Setting
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The Marsden Classroom in Context
Until the mid 1990’s the delivery of the Mathematics Curriculum at Marsden State High School was a traditional one. Students sat in rows, worked individually from text sources and copied notes from the blackboard. The lessons were teacher focused and students were skeptical as to when the Mathematics would be useful in their lives, if at all.
The School caters for 1500 students in five year levels, from grade 8 (13 year olds) to grade 12 (17 year olds) and is situated in a suburb of a satellite city adjoining the state capital of Brisbane. The suburb could satisfactorily be called lower socio-economic and often challenging behaviours are brought to classes by a large number of students.

Via the use of Australian Council of Educational Research standardized tests on entry to the school, a profile of student development was found. Approximately 50% of the 300-350 students entering year eight had critical levels of literacy and numeracy, equating to a year 3 (8 year old) level of conceptual development or less.

It was clear that the traditionalist setting was not meeting the educational needs of these students and did little to promote teachers as professionals in the community. Even students operating at their age indicative level were often under performing in the later years of schooling.
The outlook was even more problematic when one considered the necessity to include all students in regular classes (inclusivity), the impact of new technologies and the trend for the academically able to not need as much high school mathematics for tertiary study requirements.
The time had arrived for the classroom to become more than a social experience mixed with small doses of arithmetic. From a cynical teaching approach revolving around the student’s intellectual capacity, background and/or behaviour to an environment where the prime focus is the learning process. The journey had begun!

Where to for Help?
Making a connection with the University of Queensland via Professor Peter Galbraith and Dr Merrilyn Goos was the key change. The research literature in general, and the work of Paul Ernest in particular led to a change in the philosophy of the author. The work of Thom et al. clearly showed that teachers take their own philosophies of mathematics into the classroom with them. These sources were highly significant in the decision to adopt a constructivist philosophy toward education and thereby enable a framework to be built, upon which a paradigm of delivering Mathematics could be formed.

The philosophical change embodied a more learner centered, and socially productive mathematics classroom and acknowledge the acceptance of the following:
- Human beings construct knowledge from their own life experiences.
- Human beings only know something when they have made connections in their own heads.
- Concrete and abstract understandings are at different levels of conceptual analysis.
- New understandings come from perturbing meanings already constructed.
- Increasing the opportunities for students to discuss their mathematical understandings enhances learning.
- The expository method assumes there exists truths to be passed on to learners.

An example
As an example of how a constructivist paradigm can be implemented, the following example is from Group Theory. The activity is from the NCTM Mathematics Teacher April 1996.

“IT’S A SNAP” Rules of the game:
- Two rows of three pegs
- Three rubber bands
- Each rubber band must start at an individual peg in the top row and finish with an individual peg in the bottom row.
- One possible arrangement
• Now find all the possible combinations using the rules.

• Did you get six?

• We now have a set of six elements
• We shall now define the operation “It’s a Snap”
• What does B “It’s a Snap” D look like?
• Now remove the middle set of pegs to find the solution.

This is B “It’s a Snap” D before we operate

• B “It’s a Snap” D = C

After the pegs have been removed

• The task now is to find all the thirtysix possible combinations when we apply
the operation “It’s a Snap” to our six elements
• How can we best organise this information?
• An operation table as per multiplication
• Spending some time on completing the table gives the result below
• The table:

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• The table is a Cayley table
• The table indicates that our six elements along with our “It’s a Snap” operation
forms a non-Abelian group
• A is the identity element
• The table indicates closure
Associativity holds and Inverses exist

**What happened during the activity?**
- Discussion had to take place amongst participants in the activity which enabled a language of discourse to take place
- This discourse is social and allows participants to verify their understandings
- Concrete manipulatives increase enjoyment which increase the likelihood of engagement in the activity
- A scaffolding of the underlying principles is more likely to occur
- This allows a reference point for follow up discussion
- The participants in the activity have been given an opportunity to construct their own meaning
- Students can move to the abstract or move between the concrete and the abstract as suits them
- Further examples can use a similar procedure to allow all learners access to the mathematical principles involved

**Applying the procedure to a whole school program**
- Begin topics with an activity
- Ensure the activity uses concrete manipulatives
- Try to make the activity accessible to all learners
- The best activities have several entry points
- A well chosen activity or set of activities can “cover” a large subset of required syllabus objectives
- Students can gain the big picture from which the subset of skills make more sense
  - In other words students now have a need to know
  - The best activities can be used in all year levels with all learners
  - The students own the learning
  - The students are more likely to ask questions which are meaningful to them
  - The teacher is more likely to glean information about the students
  - In year 8 each topic starts with a kit of 20 – 30 activities
  - Each activity has a set of skills attached which students can tick off on their own checklist
- Teachers have the opportunity to assess students during the process
- A hands on task allows students to take present understandings and perturb them to form new and deeper understandings
  - A task of this nature is now an assessment item in each topic in the junior school
  - A learning culture is more likely to be developed in the classroom
  - Assessment can be taken when the teacher and students are ready
  - Year 8’s can update skills when they feel that they are ready
  - Year 8’s can re-sit tests after negotiation with their teacher
  - Similar principles are applied to all levels
  - After 2-3 weeks working on the activities students are more likely to have a lexicon of words directly related to the topic
  - Teachers are more likely to know where students are at in their conceptual understanding
  - Students are more likely to be engaged

**The teaching Staff**
- They are the captain of the ship of learning, students are the crew and they sail on the sea of mathematics
- Timing of assessment is up to the captain and the crew
- Availability of resources is essential
- Regular professional meetings modelling the procedure
- Have lots of resources on hand
Resources include text, OHP calculators, Graphic calculators, View screens, Computers, scientific calculators, blocks, cubes, dice, counters, worksheets, rotograms, mira mirrors, globes, games, fraction bars, hundred boards, MAB blocks etc.

The Outcomes
- Much less likely to have non-learning classes
- Average LOA’s across the school have increased 10 –15% in the first two years and have continued to improve (see graphs)
- Example: Marsden success rate in Australian Maths Competition compared with State averages
- Staff room discussion is more frequently learner focussed

- More staff use more resources
- Best results in SMA- (A Sound Achievement (SA) is the middle of a 5 point band)
- Questioning techniques demand a re-think

The future
- When outcomes return in the new syllabus they can be check listed to match activities
- The constructivist principles outlined match the thrust of all the Senior Syllabi
- The embedding of the activities in the program give an impetus to staff to change pedagogy

Summary
- Philosophy of Mathematics
- A theory of how people learn
- A constructivist pedagogy in the classroom
- A more student centered approach
- A more professional approach from staff
- Alignment with system requirements
- An ability to adapt to new expectations