We Need Learning Tasks That Support Sense Making

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Teachers Provide Learning Tasks

Successful teachers provide learning tasks that will give their students the opportunity to play the sense-making game and, over time, to learn how to play this game at progressively higher levels.

The Sense Making Game

The sense making game is about using knowledge and experience in integrated, creative, authentic, and purposeful ways to solve problems, conduct inquiries, carry out investigations, and perform experiments. The sense making game is about using these processes to come to a better understanding of things. Sense making in the mathematics classroom is, however, about more than coming to understand mathematical concepts and procedures. It is also about such things as making sense with mathematics, making sense when communicating with others, making sense of situations, making sense of people's actions and ideas. It is this game that the student needs to be able to play if they are to successfully address the challenges and opportunities of life in and outside of school.

Sense Making Through Rich Learning Tasks

I define any learning task as *'rich'* if it gives students the opportunity to

• use (and learn to use) their knowledge in an integrated, creative, and purposeful fashion to conduct inquiries, investigations, and experiments and to solve problems and in so doing,

• acquire knowledge with understanding, and in the process,

• develop the attitudes and habits of a life-long sense maker

Put more simply, a learning task is rich if it allows students and teachers to play the sense-making game. Rich learning tasks are of central importance to the teacher and the student. They support a sense-making culture in the classroom. [For a detailed examination of rich learning tasks, see Reference #1.]

Pseudo-Rich Learning Tasks

I find that most commercially-produced learning tasks are not very rich. They, typically, do not support sense making. At best, they let students nibble at the edges of sense making. Most learning tasks advertised as giving the student the opportunity to problem solve, inquire, experiment, and investigate, in reality, are structured in ways that force the student to proceed through these tasks in a lockstep unreflective manner, forcing the student to react rather than act, respond rather than reflect, follow rather than navigate, never really use or experience sense making processes. The only people getting the opportunity to make sense, unfortunately, are the authors of the tasks. In setting out tasks for the student, the authors do most of the interesting bits, the important bits, the challenging bits, the creative bits, the significant bits, the authentic bits, the bits in which the student needs to gain experience, the bits that could empower the student and allow them to make sense, the bits that would give the student insight into the nature of the discipline and what learning is really like. The student is usually tossed some crumbs, some of the easier bits, bits that occupy the student and take them to the end of the task without real engagement, understanding, or payoff. The authors typically marginalize the roles of both the student and the teacher. They substitute do-able anaemic tasks for tasks worth doing. They forget, as the educator Caleb Gattegno advises (see Reference #2), to subordinate teaching to learning. The unwritten message attached to most of these *pseudo-rich* learning tasks seems to be either that students and teachers don't have (or don't think they have) permission or the time to work in a sense-making environment, that students can't be trusted to play this game well or that the teachers can't be trusted to engage/assist students in sense making.

'How Long Would It Take You To Count To One Billion?'

In the following section I examine a sample learning task, entitled '*How Long Would It Take You To Count To One Billion?'*

I selected this example task for the following reasons:

- It is written by a respected and experienced author (see Reference #3),
- it comes from a professionally-produced modern resource,
- it is part of a Ministry of Education resource linked to a specific curriculum (see Reference #4),
- it is part of a package of resources designed to support a large-scale (math reform) teacher in-service initiative, and
- many would classify the task as exemplary.

I restrict my examination of the task to an attempt to answer the question, *How well does the task support /*

encourage / allow student sense making?

How Well Does This Sample Task Support Sense Making?

[*Please read the sample task displayed on the next two pages before continuing with this examination.*]

1. <u>Expectations for Sense Making</u> The author lists 4 curriculum expectations, selected from the Gr 7 Number Sense strand. These should not be the only expectations associated with this learning task. Other expectations (taken from the same policy document / Reference #4) must be included *if* sense making is to be taught / emphasized / encouraged, especially,

• *"students ... should learn to examine their own thinking processes and to try a different strategy if they are having difficulty solving a problem."*

• *"Problem solving is a trial-and-error process that involves starts and stops, successes and failures, and the examination and rejection of some solutions. It is important for the teacher to model this process in the classroom, joining in the search for answers and thinking out loud."*

• *"An emphasis on reasoning must pervade all mathematics instruction."*

• *"The freedom to explore and the process of exploration itself are essential elements in the maturation of the student's capacity for mathematical reasoning."*

I can readily see that the task was designed with the first four expectations in mind. I have difficulty finding evidence that the task was also designed with the latter four expectations in mind.

2. <u>The Lesson Launch</u> The author suggests that the teacher spend 5 minutes to launch the lesson, taking some of that time to review 'some fundamental relationships between time units.'

Sense making, in part, involves connecting relevant concepts to the situation at hand and identifying needed relationships associated with these concepts. Reviewing units of time and relationships between them puts the cart before the horse. Students need to be exposed to the situation first and, only after they have had this opportunity, identify units of time and relationships between some of these units to help them produce their estimates. Successful sense makers don't wait for someone else to point out prerequisite skills, concepts, and relationships before tackling a task.

3. <u>Exploration and Problem Solving</u> When the author suggests that the *Paired Activity* part of the lesson be allotted 15 minutes, including work on Exercises 1 and 2 on the 2^{nd} student activity page, you can be reasonably certain that the opportunity for students to get involved in sense making is neither serious or significant. I think students will find a 5-10 minute opportunity to explore, discuss, model, manipulate, test, justify, revise, and communicate quite limiting, unsatisfactory, and unsatisfying.

The author's use of a *Hint* component in Exercise 5 places emphasis on getting the answer rather than on making students better sense makers. Successful sense makers know that there is no 'read the hint' strategy when practicing real sense making outside the classroom.

4. <u>Assumption Making / Assumption Checking</u> Part of the problem solving process usually involves making some assumptions about a situation. Being told on the first student activity page, "*Imagine that you were able to do this without stopping to eat , drink, or sleep,* removes the opportunity for the students themselves to make some important / realistic assumptions about the situation.

Being told to '*Estimate how many years it would take you to reach one billion'* asks students to accept the idea that the answer has a magnitude that should be measured in years and to accept this fact before the student has had a chance to come to a realization of the magnitude of time involved in the counting task. (They are also told in a picture of an elderly, inappropriately stereotyped, lady that the answer is likely many years in magnitude.)

Then, to be asked on the second student activity page, *'Do you think you could count one number every second? Explain why or why not. If not, how long do you think it would take?'* would cause confusion by overturning the earlier understanding that assumptions / constraints needn't be realistic. (They were told earlier to count and ignore the need to eat, drink, or sleep.)

5. <u>Teacher Modeling the Problem Solving Process</u> When the author says "*it helps to use analogies such as: A million hours ago was late in the 19^{th} century, but a billion hours ago was over 100 000 years ago - a prehistoric time when the human population was small and sparse*." the author is really telling us that he has a love and a flair for math. This is not modeling problem solving but sharing the end results / product of a problem solving process. This form of 'showing off' does not give the student the opportunity to observe the teacher in the act of 'searching for answers and thinking out loud' nor does it allow the student to come to an understanding of how the teacher or author arrived at such a 'neat' conclusion.

Just after the *Context* section the author says, *"To estimate how long it would take to count to one billion, students need to estimate how long it would take to say each number. The easiest way is to assume one number per second and adjust the answer later by simply multiplying by the appropriate factor."* Then in the *Paired Activity* session, the author directs the teacher to "*Circulate around the class to assist pairs of students who are experiencing difficulty obtaining estimates. Distribute page 33* [the secon d student activity page] *to students who need help and*

Activity 2 – Teacher Edition

How Long Would it Take You to Count to One Billion?

Expectations Addressed

N 7-2 compare and order integers (e.g., on a number line).

N 7-4 explain numerical information in their own words and respond to numerical information in a variety of media.

N 7-6 perform three-step problem solving that involves whole numbers and decimals related to real-life experiences, using calculators.

N 7-16 explain the process used and any conclusions reached in problem solving and investigations.

Context

This activity is dedicated to helping students understand the relative sizes of a million and a billion. As Paulos (see p. 96) has noted, many people regard millions and billions as huge numbers that are beyond normal comprehension and consequently make statements such as, "The Canadian debt is 583 million dollars…or is it 583 billion dollars? I can't remember, but it's huge." To help students understand the relative magnitudes of millions and billions, it helps to use analogies such as:

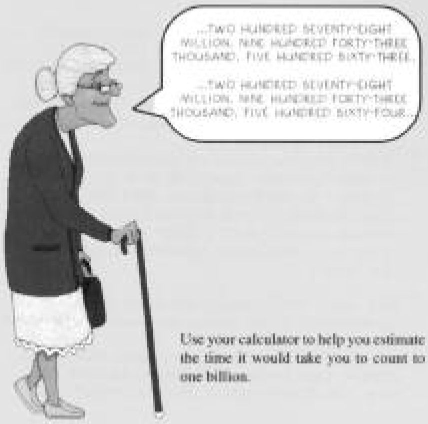
> A million hours ago was late in the 19th century, but a billion hours ago was over 100 000 years ago – a pre-historic time when the human population was small and sparse.
>
> If a marble were magnified so that its diameter were one million times as large, it would be a sphere with a diameter of about 13 km. If the marble were magnified so that its diameter were one billion times as large, it would be a sphere the size of the Earth.

Activity 2 – Student Page

How Long Would it Take You to Count to One Billion?

> *"…Many educated people have little grasp for [large] numbers and are even unaware that a million is 1 000 000; a billion is 1 000 000 000; and a trillion, 1 000 000 000 000."*
>
> *Innumeracy*
> John Allen Paulos

Suppose you counted to one billion, starting at one and announcing each number in order until you reached one billion. Imagine that you were able to do this without stopping to eat, drink, or sleep. Estimate how many years it would take you to reach one billion. Describe what strategies you used to make your estimate.



Use your calculator to help you estimate the time it would take you to count to one billion.

To estimate how long it would take to count to one billion, students need to estimate how long it would take to say each number. The easiest way is to assume one number per second and adjust the answer later by simply multiplying by the appropriate factor. Page 32 asks the students to use their calculators to convert one billion seconds to minutes, hours, days, and ultimately to 31.7 years. (A review of the relationships among the units of time is recommended in the *Launch* of this lesson, described on p. 31.) Then they can adjust their estimates according to the number of seconds they assume it would take to count each number. Without further guidance, page 33 is given to the students who will work on Exercises ❶ and ❷ to take them step-by-step through the conversion process.

Exercises ❸ and ❹ have students attempt to represent a million and a billion on the same number line so that they realize how miniscule a million is compared to a billion. Finally, in Exercise ❺, students estimate the number of times their hearts will beat in their lifetimes and compare the longevities of their hearts with the longevity of an artificial heart.

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The Lesson Launch 5 minutes

To launch the lesson, ask students questions such as, "What is larger, a million or a billion?" and "How many millions does it take to make a billion?" Then review some fundamental relationships between time units by asking:

- How many seconds in a minute?
- How many minutes in an hour?
- About how many hours in a day?
- About how many days in a year?

Group students in pairs so that no pair has two weak students. Then ensure that each pair has at least one calculator and distribute page 32. Ask students to use their calculators to estimate how long it would take them to count to one billion.

Paired Activity 15 minutes

Circulate around the class to assist pairs of students who are experiencing difficulty obtaining estimates. Distribute page 33 to the students who need help and suggest that they work through Exercises ❶ and ❷. Distribute page 33 to the other pairs of students after they have completed their estimates. Have them work through Exercise ❷ to check the estimate they made when working on page 32.

Individual Activity 10 minutes

When students have obtained reasonable estimates of the time it would take to count to one billion (anything from 31.7 years to about 10 times that much), ask them to work individually on Exercises ❸, ❹, and ❺ and to record their work in their notebooks. The intent here is to ensure individual involvement by having each student attempt to locate one million on a number line up to one billion and, in so doing, develop a sense of the relative magnitudes of millions and billions. Also, each student will have an individual assumption regarding their own longevity and will obtain a unique estimate of the number of times their heart will beat in their lifetime. As students draw their number lines, circulate around the class and discuss with them why it is difficult to locate one million on this number line.

ACTIVITY 2 – STUDENT PAGE

HOW LONG WOULD IT TAKE YOU TO COUNT TO ONE BILLION?

You may use your calculator to help you answer these exercises.

❶ Draw a diagram or display calculations to show how you would calculate the number of seconds in an hour; in a year. Display your answers.

❷ Assuming you could count one number every second, write your answers to the following questions in the spaces provided. Show your work on the side.

The time it would take to count from 1 to 1 billion would be approximately:

a) ____________ seconds.
b) ____________ minutes.
c) ____________ hours.
d) ____________ days.
e) ____________ years.

Do you think you could count one number every second? Explain why or why not. If not, how long do you think it would take? Use this new estimate to estimate the approximate number of years it would take to count to one billion if you did not need to stop for food or sleep.

❸ a) What is one million as a fraction of one billion?
b) What is one million as a percent of one billion?

❹ Numbers from 1 to 1 billion can be represented on a number line like the one shown here. Draw a number line like this and place the numbers 250 000 000 and 500 000 000 on it.

Show the locations of these numbers on your number line.

a) 100 000 000 b) 10 000 000 c) 1 000 000

Describe any difficulty you have locating any of the numbers on your number line.

❺ Eventually, an artificial heart may be developed that is capable of beating up to one billion times. Estimate how many times your own heart will beat in your lifetime.

HINT Make reasonable assumptions about:
- the average number of times your heart beats in a minute.
- the number of years you expect to live.

Do you think your heart has a longer lifetime than such an artificial heart? Explain why or why not.

Closure

For many or all of your students, this may be their first exposure to Fermi problems. If so, some will be uncomfortable with the rough approximations involved in assuming an average number of heartbeats per second, in assuming a particular lifespan, or in assuming that every year has 365 days. In their calculator computations, they will tend to carry all the decimal digits shown on the display and may regard rounded answers as inaccurate. It is important at this point to discuss the problem on page 32 and explain that most calculations in real-world problems involve such estimates and that such problems require "order-of-magnitude" answers. That is, we want to know roughly how an artificial heart with a lifetime of one billion beats compares with a human heart. Is its longevity ten times as great as the human heart or only a fraction? Such questions can only be answered by the Fermi-type assumptions and approximations used in this activity. Invite students to suggest other Fermi problems. For each problem, discuss the information that would be needed and the kinds of assumptions that may be required.

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suggest they work through Exercises 1. and 2." Neither set of statements ask / encourage the teacher to model the problem solving process. They tend to cast the teacher in the role of the authority with the answers, someone who tells students what to do and how to do things.

"Distribute page 33 to the other pairs after they have completed their estimates. Have them work through Exercise 2 to check the estimate they made on page 32." says to successful students, now that you have your unofficial result and your unofficial solution process I want you to compare them to the real / right answer and to the correct / official / approved way of doing it. Real sense making doesn't work this way. Sense makers reflect on, test, and revise their work in an effort to increase their confidence in their results, to become more certain of their conclusions. Successful sense makers know that there isn't a sense-making strategy called "check your work against the official version." [Rather than comparing their solutions with an official version, students would be better served if they were given a chance to compare / justify / assess estimates and strategies with one another.]

6. <u>Tool Use</u> Part of sense making involves selecting appropriate tools to think with and using these tools in ways appropriate to a given situation. Being told *'Use your calculator to help you estimate the time it would take you to count to one billion."* short circuits this process and may encourage students to interrupt their thoughts about such things as assumptions and solution strategies and prematurely switch to a search for numbers and operations to key into calculators.

7. <u>Communication / Reasoning</u> Students are given very little time or encouragement to communicate clearly, effectively, or comprehensively. Students are given little opportunity to describe and justify the procedures and strategies they used to arrive at their estimate or to indicate how their estimates could be improved.

8. <u>Sharing Criteria for Success / Assessment</u> The author provides rubric 'starting points' for the teacher. There is no evidence to indicate that this important source of meta-cognitive guidance will be shared with students at the beginning of the activity, to guide student action, to help the student reflect on their work, to improve their estimates, and to help students communicate effectively.

9. <u>Sense Making / Story Making</u> A sense making episode should be like a story, with students and teacher acting as authors of / readers of / characters in the story. The *How Long Would It Take You To Count To One Billion?* story should involve such things as setting, events, the involvement of characters, connectedness, coherence, mystery, suspense, questions, actions, interactions, excitement, anticipation, struggle, disappointment, interest, reward, denouement, a coming together, resolution, and satisfaction. The second student activity page tends to disrupt and dissipate the flow and flavor of the story. Exercises 1 and 2 could be left to come out naturally as the story unfolds. Exercises 3 and 4 could become a part of the Lesson Launch. Exercise 5 could be considered the next story in the series, giving students the opportunity to demonstrate how well they can transfer and apply the lessons learned in the 'counting to one billion' task. It is left as an exercise to the reader to consider ways this sample pseudo- rich learning task can be redesigned and enriched so that it will better support sense making.

Conclusion The task, *How Long Would It Take You To Count To One Billion?* is an interesting problem that can motivate worthwhile student activity. It has the potential to be a rich learning task. Like many learning tasks, it all depends on how the task is re-structured, implemented and assessed. As always, much depends on the skills and professional judgement of the teacher. If a teacher were to use the student activity pages related to this task, in their current form and in the ways suggested in the accompanying teacher notes, I doubt the task would encourage or support significant sense making.

Teachers / authors often shy away from selecting / adapting / creating rich learning tasks out of concern that students will be overwhelmed by the complexity of such tasks. Perhaps that is what (at least partially) motivated the author to structure the sample task in the way that he did.

I have found that rich learning tasks generally overwhelm only those who have had no exposure / training / experience with such tasks, overwhelm only those who are lead by teachers who chose not to teach for understanding, and overwhelm only those who find themselves in classrooms lacking in a sense of community. Involving students routinely in pseudo-rich tasks will only ensure that students will be overwhelmed when the object of the game turns to sense making.

References

1. *<u>A Handbook on Rich Learning Tasks</u>* by Gary Flewelling with William Higginson, Centre for Mathematics, Science, and Technology, Queen's University, Kingston, 2002. Contact Bonnie Knox, secretary, 613-533-6211; 613-533-6584 (fax); knoxb@educ.queensu.ca

2. "Reflections on Forty Years of Work on Mathematics Teaching" by Caleb Gattegno <u>For the Learning of Mathematics</u> Kingston: FLM Publishing Association, November 1988

3. *<u>Impact Math: Number Sense & Numeration for Teachers of Grade 7 and 8</u>* , by Brendan Kelly, Ontario Ministry of Education, Toronto, Queen's Printer, 1999

4. <u>The Ontario Curriculum, Grades 1-8: Mathematics</u> Ministry of Education and Training, Ontario, 1997, http://www.edu.gov.on.ca