Abstract: In this paper some aspects related with the success or the failure of the integration of information technology in mathematics education are discussed. The paper focuses on the need of establishing a new special methodology of work in math education that takes into account the intensive use of information technology tools and choosing those who adapts better to a certain social reality without depending on only one software. This work methodology should take into account besides the classic problems in mathematical education the new ones derived due to the use of computer programs, that is, the transition from a paper-and-pencil environment to a computer-supported learning environment. These problems involve the specific problems about the use of particular software and the use of Internet and should embrace a wide spectrum of topics: classroom activities, tasks, assessment, appropriate spaces, selection of the software, laboratories, projects, language problems, questions of equity, design of utilities, elaboration of quick guides, exams’ revision, work with students outside the classroom, research, results of research, training teachers, and so on.

1. Introduction

In the last decade of the last century, in many countries of the world, we begin to live a dramatic change in the didactics of the mathematics as a result of the massive diffusion of information technology. In many mathematics courses of different school levels from kindergarten to the university information technology is used like a daily tool. Although still is not a majority the number of teachers that incorporate this tool in its lessons, the number of who use it grows everyday and everything seems to indicate that this number will continue to grow in the next years. This tendency has put and it continues putting to the didactics of mathematics new problems linked to the use of this powerful tool.

We begin a new millennium with a bigger access to powerful computers and software, with a vast access to Internet and its web sites of mathematics, with books written with the assumption that math software will be used by students and teachers, with multimedia products distributed by means of CD's, with a great quantity of International Meetings addressed to the discussion of specific problems, with several research journals devoted to spread investigations, proposals and reflections, with institutions and governments that offer significant support for experimentation, with specific curricular propositions (for example the curricular discussions of the NCTM), and so on.

This constitutes so we can call a "macro" environment but here emphasis is made in the "micro" environment (closely related to the previous one): the need to design methodologies of integration of the information technology that look for the success of the process, that is to say, methodologies that can help students to learn mathematics.

2. The passage to the paper and pencil environment to the information technology environment

As it is pointed out in López (2001a) the use of modern computer algebra systems and other software pose a new paradigm for teaching, learning, and doing applied mathematics and that it is no longer sufficient to acknowledge the power of such software while still adhering to the
paradigm of paper-and-pencil environment. López points out (2001b) «Modern computer algebra systems implement nearly all the calculations found in these disciplines, and should be used as the tool of first recourse for teaching, learning, and doing applied math». We really believe that this statement made about the CAS (Computer Algebra Systems) it can be extended without problems to the use of information technology in general: it is necessary to change paradigm with all that that implies. It is necessary to point out that not all the mathematics professors have accepted this new paradigm, the passage of one to another is not automatic but rather it requires of a transition period (maybe the one that we are living) for teachers and students. For this reason our strategy consists on accompanying the students in its trip from the paper and pencil environment to the digital environment. One of our main goals is to see the software like a daily use tool (just as the paper use in the paper and pencil environment).

3. The integration methodology

An important part of our strategy is following a strategy of “gray box”: white box of the processes that we want discuss with a combination of black box of the previously discussed processes, processes of precedent courses or processes that will be discussed in the future. Indeed we not only use automatisms, some times students continues to use a step by step technique by hand or with software.

The first part of the integration process it is connected to the concept of didactic transposition in the sense of Chevallard (1985), there are a lot of examples about this, see for example any number of the The International Journal of Computers Algebra in Mathematics Education. But this is not enough, it is required, later on, think about the didactic engineering in the sense of Artigue (1989) with the purpose of studying in a scientific way (or at least in rational way) the didactic phenomena, that is to say, to conduct an investigation activity to verify the theoretical proposals results.

In this sense we describe next several features about our method of integration of software in regular courses of mathematics at university level, this methodology constitutes a platform for future research projects who will try to answer a “simple” question ¿with the use of software students learn “better” mathematics? (deliberately, we use the nebulous term of better thinking in the skills that define so that we call to know mathematics). Allusions to traditional courses refer to the courses based on the paradigm of paper and pencil environment.

3.1 Quick guides

If we consider some mathematics books associated with the use of software we can observe that, independently that along the text and along the courses that make reference to this type of texts are introduced in an specific way some features with regard to the use of the software, Authors feel the need to dedicate exclusive chapters of their texts to introduce and illustrate different basic features of the software in use.

For example Malek-Madani (1998) dedicates their first chapter to Mathematica and the second one to Matlab; Coombes et al. (1995) dedicate chapters 2, 3, 4 and 8 to introduce Mathematica; Gray, Mezzino & Pinsky (1997) use chapters 2, 4 and 10 for the same goal.

It is no easy to solve the problem of training students in the use of software due to the limitations of time. Coombes et al. (1995) point out the need of minimising the time required in the learning of the computation platform in use, collaboration teacher-student and student-student are looked with this goal, different proposals are set up for try to learn in the quickest way the use of a software, for example as those described in Bennett (1995) with regard to Derive, or in Shuchat & Shultz (2001) with regard to Mathematica.

This is a didactical challenge that comprehend several problems of not easy solution, because it involves the information technology culture of teachers and students, the educational
training, the design of strategies for the integration of software in each topic, the didactics of teaching the software itself. And all this in the usual time for an ordinary course. The experience obtained with the experimentation lead us to suggest that the best strategy is that students learn mathematics while they use software and vice versa that students learn how to use software while they do math. This principle reflects the absolute priority to mathematics in the training: the use of software adapts to the professor and course needs.

Our proposal includes quick guides of the software for a quick training of students, the manual use is reserved for students that want to deepen in some aspects or for teachers consultation. The quick guides are designed specifically by the professor for the content and the methodology of their courses and thinking specifically about the type of students with those that he will work (this influences, for example, the type of language used). The quick guide, as easy to use and not very extensive material, substitutes the manual use to the most of students and it is structured in such a way of introducing some basic skills in the use of the software step by step, sometimes following approaches of easiness, sometimes following the order of the syllabus.

In our case, and based upon the experience, we work with the hypothesis that students don’t know the software, if this is not the case, it is possible to advance quickly and the proposed exercises are useful for reinforce skills already achieved by students.

3.2 Utility files

These files are designed to follow the explanation of the topics discussed in a course. Not all of them become accessible to the students (for example, those automatisms that are forbidden to students) of those accessible ones, some of them contain prerequisites, other files go with labs or lessons or are review material and they can used as distance lessons putting them in a web site.

Thinking about the change of paradigm pointed out at the beginning of this paper a great advantage of working in this way it refers to build digital data with all the ancillaries materials used in a course, data that can be used $n$ times, for $n$ people, in $n$ different places.

3.3 Labs

They are the most important element in our methodology of integration. The central idea that is behind their design is the following one:

\begin{quote}
we use the students previous mathematical knowledge to teach them how to use software, then we use the acquired knowledge about the software like a new support to acquire new mathematical knowledge.
\end{quote}

With an intention, evidently, recursive that allows to begin an spiral process. Some features of the design are the following ones:

- the design is not centred in the software but in the mathematical content. For this reason skills in the use of software are introduced when the mathematical discussion requires them.
- they are not designed for autonomous use from students, instead we part from the hypothesis that teacher will work with students in an appropriate place.
- they are designed for students but we have used too for training teachers. Then we use hidden text: students receive a copy without printing the hidden text while teachers receive a copy with the hidden text (that includes comments, suggestions and results). In some cases, for teachers, sections called observations or problems and some advantages of the use of software in this topic are inserted, here more punctual observations are made.
- they are designed to improve the students communication skills when they use software.
3.4 The process of redesign

We create a diary of the labs sessions. This is a central tool in the planning process, here we document mistakes in the implementation process, we observe the advance level, and so on. Indeed, besides the current variables in a traditional course (mathematical and didactical) we should add those that depend on the use of information technology, primarily at two levels, those related to:

- the general information technology culture and
- the specific software used during the course.

Indeed, still now it happens that a certain percentage of students don’t have some basic skills, for example, some of them can have problems with a keyboard (different configurations, names, position and symbol of important keys, use of the ASCII code and so on).

3.5 Home assignments

Naturally, all the classic tasks must be reviewed searching to adapt them to the new powerful tools. For example in that exercises that can be solved manually, changes the amount of algorithms made by hand, the number of different cases that can be approached with software increases sensibly, increases too the possibilities of visualization and exploration and so on.

3.6 Web sites

There is also a companion web site for the course (for example the site www.uaq.mx/matematicas/ed for our curse of differential equations or the site www.uaq.mx/matematicas/c2 for our curse of Vector Calculus) where students can have access to lesson notes, home assignments, sample exams, exams results. The web sites constitute a central part in the new communication ways by means of mathematics software files.

3.7 Exams

In this case, our approach it departs from the idea that no have sense to teach students to work with this new kind of methodologies if after information technology is forbidden in exams. For this reason computers and calculators are allowed in exams, this lead to new problems in all the phases: the design, to make exam and correction, see, for example, the Proceedings of the 6th ACDCA Summer Academy and Balderas (2001) for a more in-deep reflection.

It is true that students use a more powerful tool (reducing the algebraic process by hand), but it is also true that in comparison with traditional courses they are much more demanded in the exams: more depth in the topics, more conceptual and methodological work (for example, how to integrate the use of software in solving mathematical problems), more clearness in writing and communicating their results (because they are demanded to explain all the central parts of the procedures that they are using), more handling of information technology tools (students should give a digital copy of their own exam as well as a printed one, many times accompanied by explained graphics generated with the same or with a different software).

After the exam students have access via Internet to a document where the results of the exams are commented, the main mistakes are pointed out, the intermediate processes are evaluate. They can access too software files with partial solutions.

3.8 Singling out problems and classification

After experiencing each proposal in the classroom, we have identified several problems about the process of integration. We point out that still we are in a very young field for this reason the identification and specification of these problems in specific cases constitute in itself a new and outstanding contribution to the knowledge in the terms exposed by the ICMI (the cursive writing is ours): «It would be good to collect examples of the use of information
technology and software which enrich students experience of mathematics and result in better understanding and learning» (ICMI, 1998, p.93).

Once identified the problems we have classified them according to their specificity in the following four categories:

1) Mathematical: specific of the topic in study, relative to other topics of the same course, relative to the topics of other courses;

2) Didactical: of all type, from the didactic transposition till the cognitive styles, from the didactical contract till the problems with languages.

3) Information technology ones: specific to the used software, relative to general information technology (software and hardware) including the use of Internet (for example the “digital divide”).

4) Practical: specific of the school administration (schedules, regulations, acquisition of software), infrastructure (appropriate rooms, access to computers, technical support), relative to the used rooms (distribution of the computers, quantity of students), relative to the students software access (only in certain rooms, in the whole university, access outside of the school).

3.9 Solution of detected problems

To guarantee the success of the integration, the problems described in the previous point should be studied and solved. In general, the solution of the problems can be looked for by means of

1) Mathematical: personal study, discussion with colleagues.

2) Didactical: personal study, discussion with colleagues, revision of specific bibliography, design of strategies and appropriate materials to attack each specific problem.

3) Information technology: deepening the study of the specific software, deepening the study of other computer tools (that includes software and hardware). In general, searching for an adequate dose of information technology culture.

4) Practical: discussion with authorities, proposing concrete measures of intervention, suggesting an appropriate distribution inside the computer rooms, designing methodologies to adapt the number of students to the number of available computers, proposing the software acquisition.

The aim of solution of these problems leads us to redesign the whole process: we return to identify the new problems, to observe if the proposed solution was satisfactory, and so on.

4. Some results and some observations

As it has been pointed out frequently, with the use of information technology not only changes the curriculum but also the methodology and the progression of the syllabus, for example the NCTM states «The existence, versatility, and power of technology make it possible and necessary to reexamine what mathematics students should learn as well as how they can best learn it» (NCTM, 2000, p.25). With our strategy in the same time of a traditional course we can cover more than the official curriculum. We work in the normal classroom (with and without the use of beamers) and in special rooms where students have hands-on-computer, students are surrounded of digital lessons (before, during and after having developed a topic).

It is evident that with this type of strategies decreases the number of the long, boring and repetitive calculations and then we have a bigger space for a more conceptual discussions.

As it can be observed, not everything changes since different automation levels can be used, then increases the importance of reach an appropriate balance between the manual skills and the software skills. This requires the emergence of new methodologies of mathematical work,
the emergence of new mathematical cultures (see Noss 1998).
On the other hand, we point out that with this kind of methodologies also emerge negative
features that it is necessary to avoid: for example, to change a manual mechanisation by a
software one, the loss of some skills, the excessive dependence of the software, the blind faith
in their results (see Zhao 1998), and so on (see, for example, Lokar, 2000).

5. Final remarks
We learn a lot trying to solve some of the problems that arises in the integration at different
levels. New challenges arise: mathematical, didactical, from information technology, from the
specific software, from communication between people and the knowledge (students and
teachers) as much as between people itself (teacher-student, teacher-teacher, student-student).
As it has been pointed out previously, this only constitutes the starting point from which we
should converge with the classic problems from the didactics of Math that look to guarantee
deep and not only epidermal learning.

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