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Preservice teachers' approaches in function problem solving: A comparative study between Cyprus and Italy [1],[2]

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Résumé. Le but de cette étude était de contribuer à la compréhension de l' approche algébrique et " coordonnée" des futures enseignants chypriotes et italiens qui se développent en résolvant des tâches de fonction et d' examiner quel' approche est davantage corrélé avec les capacités des futures enseignants dans la résolution des problèmes. Les participants étaient 260 futures enseignants chypriotes et 206 italiens. Un test composé de sept tâches – quatre tâches simples sur les fonctions et trois problèmes complexes a été administré. L'analyse statistique implicative a été appliquée pour évaluer la relation entre l'approche des futures enseignants et leur capacité de résoudre des problèmes. La plupart des futures enseignants chypriotes avaient l'habitude de suivre l'approche algébrique afin de résoudre les tâches simples sur les fonctions. En revanche, un plus petit pourcentage des italiens futures enseignants a employé l' approche algébrique tandis qu'un plus grand pourcentage employait une approche coordonnée. Les enseignants qui pouvaient employer l'approche coordonnée ont eu de meilleurs résultats dans la résolution des problèmes.

Abstract. The aim of this study was to contribute to the understanding of the algebraic and "coordinated" approaches Cypriot and Italian pre-service teachers develop in solving function tasks and to examine which approach is more correlated with teachers' ability in problem solving. Participants were 260 Cypriot and 206 Italian pre-service teachers. A test consistin of seven tasks – four simple function tasks and three complex problems- was administrated. Implicative statistical analysis was performed to evaluate the relation between teachers' approach and their ability to solve problems. Most of the Cypriot teachers used an algebraic approach in order to solve the simple function tasks. In contrast, a smaller percentage of Italian pre-service teachers used an algebraic approach while a larger percentage used a coordinated approach. Teachers who were able to use the coordinated approach had better results in problem solving.

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Περίληψη. Στόχος της μελέτης αυτής ήταν να συμβάλει στην κατανόηση της «αλγεβρικής» και της «αλγεβρικής-ολιστικής» προσέγγισης που αναπτύσσουν οι Κύπριοι και οι Ιταλοί υποψήφιοι δάσκαλοι κατά την επίλυση έργων συναρτήσεων και να εξετάσει ποια προσέγγιση παρουσιάζει τη μεγαλύτερη συσχέτιση με την ικανότητα επίλυσης προβλήματος. Το δοκίμιο που χορηγήθηκε αποτελούνταν από 7 ασκήσεις – 4 απλά έργα συναρτήσεων και 3 πιο πολύπλοκα έργα. Χρησιμοποιήθηκε η Συνεπαγωγική μέθοδος ανάλυσης για εντοπισμό των σχέσεων μεταξύ των προσεγγίσεων που χρησιμοποιούν οι υποψήφιοι δάσκαλοι και της ικανότητάς τους στην επίλυση προβλήματος. Προέκυψε ότι οι περισσότεροι Κύπριοι υποψήφιοι δάσκαλοι χρησιμοποιούν την αλγεβρική προσέγγιση για να λύσουν απλά έργα συναρτήσεων. Αντιθέτως, ένα μικρότερο ποσοστών Ιταλών υποψήφιων δασκάλων χρησιμοποίησε την αλγεβρική προσέγγιση, ενώ ένα μεγαλύτερο ποσοστό χρησιμοποίησε τη αλγεβρική-ολιστική προσέγγιση. Οι υποψήφιοι δάσκαλοι που ήταν σε θέση να χρησιμοποιήσουν τη αλγεβρική-ολιστική προσέγγιση παρουσίασαν καλύτερα αποτελέσματα στην επίλυση προβλήματος.

1. Introduction and theoretical framework

The concept of function is central in mathematics and its applications. It emerges from the general inclination of humans to connect two quantities, which is as ancient as mathematics. The understanding of functions does not appear to be easy. Students of secondary or even tertiary education, in any country, have difficulties in conceptualizing the notion of function. The understanding of the concept of function has been a main concern of mathematics educators and a major focus of attention for the mathematics education research community (Dubinsky & Harel, 1992; Sierpinska, 1992). A factor that influences the learning of functions is the diversity of representations related to this concept (Hitt, 1998). An important educational objective in mathematics is for pupils to identify and use efficiently various forms of representation of the same mathematical concept and move flexibly from one system of representation of the concept to another.

The use of multiple representations has been strongly connected with the complex process of learning in mathematics, and more particularly, with the seeking of students' better understanding of important mathematical concepts (Dufour-Janvier, Bednarz, & Belanger, 1987; Greeno & Hall, 1997), such as function. Given that a representation cannot describe fully a mathematical construct and that each representation has different advantages, using various representations for the same mathematical situation is at the core of mathematical understanding (Duval, 2002). Ainsworth, Bibby and Wood (1997) suggested that the use of multiple representations can help students develop different ideas and processes, constrain meanings and promote deeper understanding. By combining representations students are no longer limited by the strengths and weaknesses of one particular representation. Kaput (1992) claimed that the use of more than one representation or notation system helps students to obtain a better picture of a mathematical concept.

The ability to identify and represent the same concept through different representations is considered as a prerequisite for the understanding of the particular concept (Duval, 2002; Even, 1998). Besides recognizing the same concept in multiple systems of representation, the ability to manipulate the concept with flexibility within these representations as well as the ability to “translate” the concept from one system of representation to another are necessary for the mastering of the concept (Lesh, Post, & Behr, 1987) and allow students to see rich relationships (Even, 1998).

Duval (2002, 2006) maintained that mathematical activity can be analysed based on two types of transformations of semiotic representations, i.e. treatments and conversions. Treatments are transformations of representations, which take place within the same register that they have been formed in. Conversions are transformations of representations that involve the change of the register in which the totality or a part of the meaning of the initial representation is conserved, without changing the objects being denoted.

Some researchers interpret students’ errors as either a product of a deficient handling of representations or a lack of coordination between representations (Greeno & Hall, 1997; Smith, DiSessa, & Roschelle, 1993). The standard representational forms of some mathematical concepts, such as the concept of function, are not enough for students to construct the whole meaning and grasp the whole range of their applications. Mathematics instructors, at the secondary level, traditionally have focused their teaching on the use of the algebraic representation of functions (Eisenberg & Dreyfus, 1991). Sfard (1992) showed that students were unable to bridge the algebraic and graphical representations of functions, while Markovits, Eylon and Bruckheimer (1986) observed that the translation from graphical to algebraic form was more difficult than the reverse. Sierpinska (1992) maintained that students have difficulties in making the connection between different representations of functions, in interpreting graphs and manipulating symbols related to functions. Furthermore, Aspinwall, Shaw and Presmeg (1997) suggested that in some cases the visual representations create cognitive difficulties that limit students' ability to translate between graphical and algebraic representations.

The theoretical perspective used in this study is mainly based on the studies of Even (1998) and Mousoulides and Gagatsis (2004). Even (1998) focused on the intertwining between the flexibility in moving from one representation to another and other aspects of knowledge and understanding. The participants were 152 college mathematics students who were also prospective secondary mathematics teachers. In the first phase of the study they completed an open-ended questionnaire. In the second phase ten of them were interviewed. This study indicated that subjects had difficulties when they needed to flexibly link different representations of functions. An important finding of this study was that many students deal with functions pointwise (they can plot and read points) but cannot think of a function in a global way. The data also suggested that subjects who can easily and freely use a global analysis of changes in the graphic representation have a better and more powerful understanding of the relationships between graphic and symbolic representations than people who prefer to check some local and specific characteristics. This finding cannot be generalized since in some cases a pointwise approach proved to be more powerful. In the case of problem solving a combination of the two methods is most powerful.

Mousoulides and Gagatsis (2004) investigated students' performance in mathematical activities that involved principally the second type of transformations, that is, the conversion between systems of representation of the same function, and concentrated on students' approaches as regards the use of representations of functions and their connection with students' problem solving processes. The most important finding of this study was that two distinct groups were formatted with consistency, that is, the algebraic and the geometric approach group. The majority of students' work with functions was restricted to the domain of algebraic approach. This method, which is a point to point approach giving a local image of the concept of function, was followed with consistency in all of the tasks by the students. Only a few students used an object perspective and approached a function holistically, as an entity, by observing and using the association of it with the closely related function that was given. Moreover, an important finding of the study was the relation between the graphical approach and geometric problem solving. This finding is consistent with the results of previous studies (Knuth, 2000; Moschkovich, Schoenfeld, & Arcavi, 1993), indicating that a geometric approach enables students to manipulate functions as an entity, and thus students are capable to find the connections and relations between the different representations involved in problems. Specifically, students who had a coherent understanding of the concept of functions (geometric approach) could easily understand the relationships between symbolic and graphic representations in problems and were able to provide successful solutions.

In this study the concept of function is viewed from two different perspectives, the algebraic and the coordinated perspective. The algebraic perspective is similar to the pointwise approach described by Even (1998) and the one described by Mousoulides and Gagatsis (2004). In this perspective, a function is perceived of as linking x and y values: For each value of x, the function has a corresponding y value (Moschkovich et al., 1993). The coordinated perspective combines the algebraic and the graphical approach. In this perspective, the function is thought from a local and a global point of view at the same time. The students' can "coordinate" (flexibly manipulate) two systems of representation, the algebraic and the graphical one.

The purpose of this study is to contribute to the understanding of the algebraic and coordinated approaches Cypriot and Italian pre-service teachers develop and use in solving function tasks and to examine which approach is more correlated with teachers' ability in solving complex problems.

2. Method

The analysis was based on data collected from 260 Cypriot and 206 Italian pre-service teachers. The subjects were for the most part students of high academic performance, although they are not mathematical oriented, admitted to the University of Cyprus and to the Universities of Bologna and Palermo on the basis of competitive examination scores. Concerning the teaching of functions in the two countries a high percentage (almost 21-30%) of the material included in the curriculum of both countries deals with this concept. Furthermore, while the mathematics textbooks in Cyprus have as a main goal the acquisition of knowledge the mathematics textbooks in Italy have as a main goal the development of problem solving abilities and are based on investigations. A test was administrated to all the participants (Monoyiou & Gagatsis, 2008a; Monoyiou & Gagatsis, 2008b). The test consisted of seven tasks. The first four tasks were simple tasks with functions (T1a, T1c, T2a, T2c, T3a, T3c, T4a, T4c). In each task, there were two linear or quadratic functions. Both functions were in algebraic form and one of them was also in graphical representation. There was always a relation between the two functions (e.g. f(x)=2x, g(x)=2x+1). The participants were asked to interpret graphically the second function. The other three tasks were complex problems. The first problem consisted of textual information about a tank containing an initial amount of petrol (600 L) and a tank car filling the tank with petrol. The tank car contains 2000 L of petrol and the rate of filling is 100 L per minute. Teachers were asked to use the information in order to give the two equations (Pr1a), to draw the graphs of the two linear functions (Pr1b) and to find when the amounts of petrol in the tank and in the car would be equal (Pr1c). The second problem consisted of textual and algebraic information about an ant colony. The number of ants (A) increases according to the function: $A=t^2+1000$ (t=the number of days). The amount of seeds, the ants save in the colony, increases according to the function S=3t+3000 (t= the number of days). Teachers were asked to use the information in order to draw the graphs (Pr2a) of the quadratic and linear functions and to find when the number of ants in the colony and the number of seeds would be equal (Pr2b). The third problem consisted of a function in a general form of $f(x) = ax^2+bx+c$. Numbers a, b and c were real numbers and the f(x) was equal to 4 when x=2 and f(x) was equal to -6 when x=7. Teachers were asked to find how many real solutions the equation ax^2+bx+c had and explain their answer (Pr3). The test was administered to Cypriot and Italian pre-service teachers in a 60 minutes session.

The results concerning teachers' answers to the four tasks were codified by an uppercase T (task), followed by the number indicating the exercise number. Following is the letter that signifies the way teachers solved the task: (a) "a" was used to represent "algebraic approach – function as a process" to the tasks, (b) "c" stands for teachers who adopted a "coordinated approach – function as an entity". A solution was coded as "algebraic" if teachers did not use the information provided by the graph of the first function and they proceeded constructing the graph of the second function by finding pairs of values for x and y. On the contrary, a solution was coded as coordinated if teachers observed and used the relation between the two functions in constructing the graph of the second function. In this case teachers used and coordinated two systems of representation. They noticed the relationship between the two equations given and they interpreted this relationship graphically by manipulating the function as an entity. The following symbols were used to represent teachers' solutions to the problems: Pr1a, Pr1b, Pr1c, Pr2a, Pr2b and Pr3. Right and wrong answers to the problems were scored as 1 and 0, respectively. Concerning problem 3, the strategy followed by the teachers in order to reach a solution was also investigated. The variable "Pr3sg" was used to represent a graphical solution of the problem while the variable "Pr3sa" was used to represent an algebraic solution.

For the analysis of the collected data the similarity statistical method (Lerman, 1981) was conducted using a computer software called C.H.I.C. (Classification Hiérarchique, Implicative et Cohésitive) (Bodin, Couturier, & Gras, 2000). Two similarity diagrams and two implicative diagrams of teachers' responses were constructed (Gras, Peter, Briand, & Philippe, 1997). The similarity diagram, which is analogous to the results of the more common method of cluster analysis, allows for the arrangement of the tasks into groups according to the homogeneity by which they were handled by the students. This aggregation may be indebted to the conceptual character of every group of variables. The implicative diagram, which is derived by the application of Gras's statistical implicative method, contains implicative relations that indicate whether success to a specific task implies success to another task related to the former one. It is worth noting that CHIC has been widely used for the processing of the data of several studies in the field of mathematics education in the last few years (e.g., Evangelidou, Spyrou, Elia, & Gagatsis, 2004; Gagatsis, Shiakalli, & Panaoura, 2003; Gras & Totohasina, 1995). Descriptive analysis performed by using SPSS. The descriptive analysis gave valuable information concerning the percentages of correct or wrong responses given by the teachers. Furthermore, Hierarchical Cluster analysis (Ward's method) was also performed in order to categorize the teachers into groups according to the use of the coordinated or algebraic approach. The univariate analysis of variance (ANOVA) was also employed in order to determine differences between the groups of teachers (coordinated, algebraic or various approaches group) concerning their problem solving ability.

3. Results

The main purpose of the present study was to examine the mode of approach Cypriot and Italian pre-service teachers used in solving simple tasks in functions and to investigate which approach is more correlated with solving complex mathematical problems.

Table 1, shows Cypriot and Italian pre-service teachers’ responses to the first four tasks. The majority of Cypriot teachers chose an algebraic approach to solve the first three tasks. In Task 4, 41.1% of Cypriot teachers used an algebraic approach and 49.3% used a coordinated approach. In this task a coordinated approach seemed easier and more efficient and as a result the percent of teachers who used this approach was higher in comparison with the other three tasks. In general, the algebraic solution was predominant in the answers of the Cypriot teachers.

Almost a third of the Italian pre-service teachers used an algebraic approach and another third used a coordinated approach. In general, the Italian pre-service teachers gave more incorrect responses than the Cypriot pre-service teachers, used more the coordinated approach and less the algebraic.

Table 1: Cypriot (C) and Italian (I) teachers’ responses to the first four tasks.

Tasks (%)		Algebraic approach	Coordinated approach	Other/Wrong answers
1	C	70	26.6	3.4
	I	38.8	43.2	18
2	C	65.8	30.8	3.4
	I	36.9	39.8	23.3
3	C	72.7	19.7	7.6
	I	38.4	30.1	31.5
4	C	41.1	49.3	9.6
	I	29.6	36.4	34

In the case of Task 1 ($y=2x$, $y=2x+1$), some teachers who used an algebraic approach found the points of intersection with x and y axis and constructed the graph. Others constructed a table of values in order to help them construct the graph. The teachers who used a coordinated approach compared the two equations and mentioned that the slope was the same and the two functions are parallel. Then they referred to the fact that the points of the second function are “one more” than the points of the other. Some of them found a point in order to verify their assertion.

In the case of Tasks 2 ($y=x^2$, $y=x^2-1$) and 3 ($y=x^2+3x$, $y=x^2+3x+2$), teachers who used an algebraic approach found the real solutions of the second equation and the minimum point and constructed the graph without using the first graph. In contrast, teachers who used a coordinated approach first compared the two equations and realized that they are parallel. Then they mentioned that the minimum point in the first case is “one down” and in the second case “two above”. Some of them found another point in order to draw the graph precisely. In the case of Task 4 ($y=3x^2+2x+1$, $y=-(3x^2+2x+1)$), the teachers who used an algebraic approach found the point of intersection with y-axis and the maximum point. The participants who used a coordinated approach compared the two equations and mentioned that the two functions are “opposite” and “symmetrical” to the x-axis. In this task, an algebraic approach was more complicated due to the fact that the equation does not have real solutions. Most of the teachers, after an unsuccessful effort to find the points of section with x-axis drew the graph using a coordinated approach.

Table 2 shows Cypriot and Italian pre-service teachers’ responses to complex problems. Teachers’ performance was moderate.

Table 2. Cypriot (C) and Italian (I) teachers’ responses to the complex problems.

Problems (%)		Correct answer	Incorrect answer
1a	C	42.7	57.3
	I	30.6	69.4
1b	C	35.4	64.6
	I	28.2	71.8
1c	C	37.7	62.3
	I	26.2	73.8
2a	C	32.3	67.7
	I	25.2	74.8
2b	C	28.5	71.5
	I	29.1	70.9
3	C	22.7	77.3
	I	12.6	87.4

In Problem 1 only 42.7% of the Cypriot teachers and 30.6% of the Italian teachers managed to use the information given in order to give the two equations. A smaller percentage constructed the two graphs correctly (35.4% and 28.2% respectively) and found their point of intersection (37.7% and 26.2%). In Problem 2 only 32.3% of the Cypriot and 25.2% of the Italian teachers managed to construct the graphs and only 28.5% and 29.1%, respectively found their point of intersection. In this problem in order to find the point of intersection the teachers had to solve a second degree equation and that caused difficulties. Problem 3 was quite difficult for the teachers since only 22.7% and 12.6% respectively managed to solve it correctly.

The Cypriot pre-service teachers performed slightly better than the Italian teachers in problem solving. Concerning the third problem we furthermore investigated the strategy teachers followed in order to reach a solution. A small percentage of the Cypriot and Italian pre-service teachers, 15.4% and 7.3% respectively, used a graphical approach while a bigger percentage (31.9% and 14.6%) used an algebraic approach.

Cypriot and Italian teachers’ correct responses to the tasks and problems are presented in the similarity diagrams in Figure 1 and 2 respectively. More specifically, two clusters (i.e., groups of variables) can be distinctively identified. The first cluster in both diagrams consists of the variables “T1a”, “T2a”, “T3a” and “T4a” which represent the use of algebraic approach. In the Cypriot teachers’ diagram the first cluster also includes variable “Pr3sa” that corresponds to the algebraic solution of problem 3. The second cluster, in both diagrams, consists of the variables “T1c”, T2c”, “T3c”, “T4c”, “Pr1a”, “Pr1b”, “Pr1c”, “Pr2a”, “Pr2b”, “Pr3”, and “Pr3sg” and refers to the use of the coordinated approach, the problem solving and the graphical solution of problem 3.

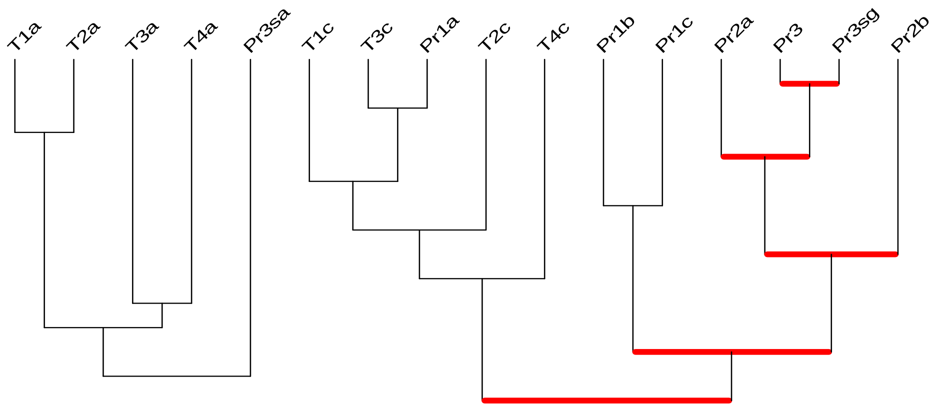

Figure 1 : Similarity diagram of the Cypriot teachers’ responses.

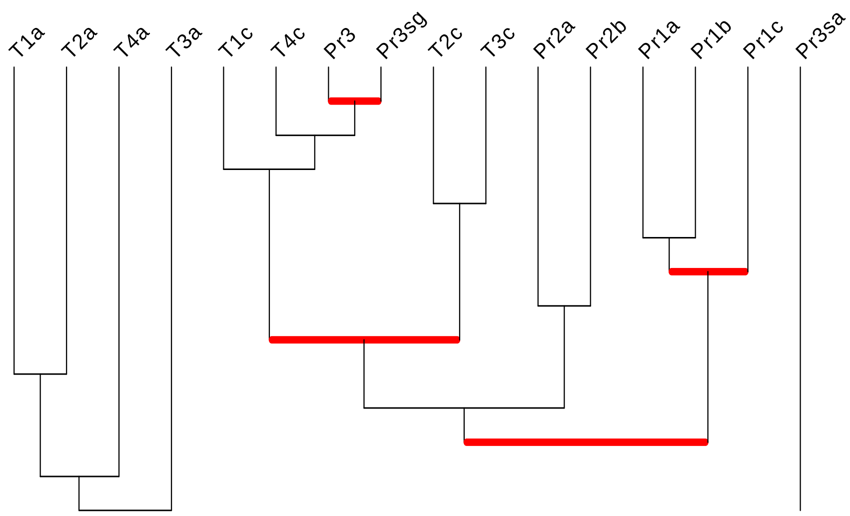

Figure 2 : Similarity diagram of the Italian teachers’ responses.

From both similarity diagrams it can observed that the second cluster includes the variables corresponding to the solution of the complex problems with the variables representing the coordinated approach and the graphical solution of problem 3. More specifically, students’ coordinated approach to simple tasks in functions is closely related with effectiveness in solving problems and with the graphical solution of a problem. This close connection may indicate that students, who can use effectively different types of representation- in this situation both algebraic and graphical representation- are able to observe the connections and relations in problems, and are more capable in problem solving. It is noteworthy the fact that the similarity clusters presented in the two diagrams are almost the same indicating that the connections and relationships between the approaches and problem solving are very strong, despite the difference in mathematics curriculum of the two countries.

Figures 3 and 4 illustrate the implicative diagrams of the variables. The results of the implicative analysis are in line with the similarity relations explained above. In Figure 3, two separate "chains" of implicative relations among the variables are formed. The two groups of implications correspond to the two similarity clusters of the diagrams presented above.

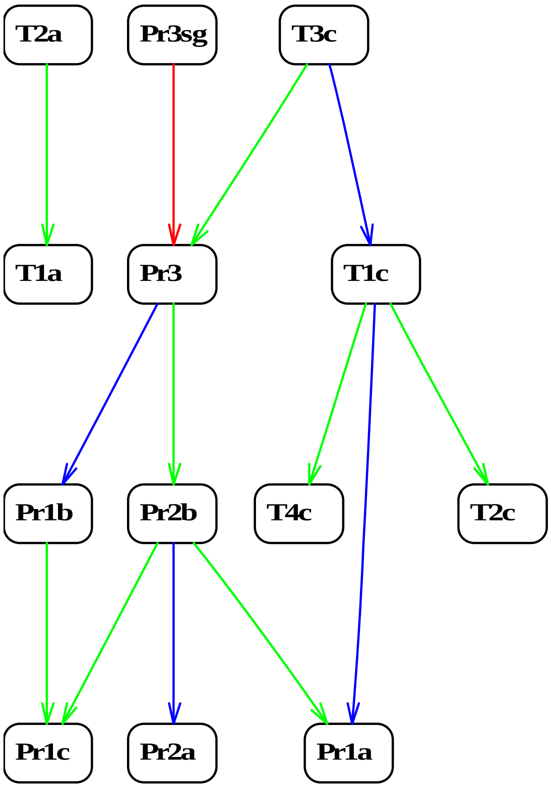

Figure 3 : Implicative diagram of the Cypriot teachers' responses.

Chain A involves the variables concerning the use of algebraic approach (T2a, T1a). Chain B refers to variables concerning the use of the coordinated approach, the variables concerning the solution of the problems and the variable concerning the graphical solution of problem 3. Chain B indicates that teachers who used a coordinated approach to solve Task 3 and a graphical approach to solve problem 3 and succeeded in those tasks also solved correctly the three problems.

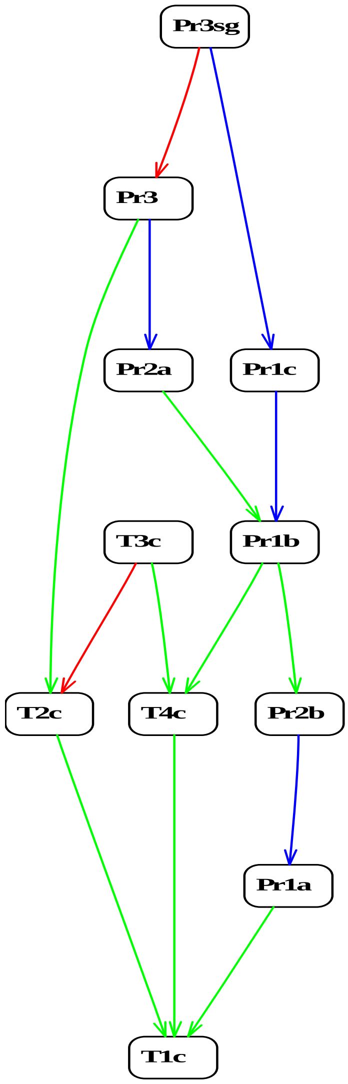

Figure 4 : Implicative diagram of the Italian teachers' responses.

In Figure 4, only one chain is formed. This chain indicates that teachers who used a graphical approach to solve problem 3 also solved correctly the problems and used a coordinated approach in the simple function tasks. According to the above diagrams, students who can coordinate two systems of representation and flexibly move from the one to the other, have a solid and coherent understanding of functions and therefore are able to solve complex problems.

In order to examine whether the Cypriot and Italian teachers who used a coordinated approach to solve the simple functions tasks performed better in problem solving the Ward's method of hierarchical cluster analysis was used. The teachers were clustered into three distinct groups. Concerning the Cypriot teachers, in the first group 42 teachers were clustered who used systematically the coordinated approach ($\overline{X}_{coordinated}$=0.89, $SD_{coordinated}$=0.13; $\overline{X}_{algebraic}$=8.929E-02, $SD_{algebraic}$=0.12). In the second group 95 teachers were clustered who used extensively the algebraic approach ($\overline{X}_{coordinated}$=6.842E-02, $SD_{coordinated}$=0.12; $\overline{X}_{algebraic}$=0.87, $SD_{algebraic}$=0.13). In the third group 123 teachers were clustered who used other approaches or used equally the algebraic and coordinated approach ($\overline{X}_{coordinated}$=0.17, $SD_{coordinated}$=0.18; $\overline{X}_{algebraic}$=0.22, $SD_{algebraic}$=0.21).

Concerning the Italian teachers, in the first group 55 teachers were clustered who used systematically the coordinated approach ($\overline{X}_{coordinated}$=0.90, $SD_{coordinated}$=0.12; $\overline{X}_{algebraic}$=5.455E-02, $SD_{algebraic}$=0.10). In the second group 29 teachers were clustered who used extensively the algebraic approach ($\overline{X}_{coordinated}$=5.172E-02, $SD_{coordinated}$=0.10; $\overline{X}_{algebraic}$=0.82, $SD_{algebraic}$=0.11). In the third group 122 teachers were clustered who used other approaches or used equally the algebraic and coordinated approach ($\overline{X}_{coordinated}$=0.12, $SD_{coordinated}$=0.19; $\overline{X}_{algebraic}$=0.16, $SD_{algebraic}$=0.19).

In order to examine whether there are statistically significant differences between the three groups (coordinated, algebraic and various approaches) concerning their problem solving ability, two univariate analyses of variance (ANOVA), one for the Cypriot and one for the Italian pre-service teachers, were performed. Overall, the effects of teachers' group were significant for the Cypriot (Pillai's $F_{(2, 257)} = 77.79$, $p<0.01$) and the Italian (Pillai's $F_{(2, 203)} = 78.14$, $p<0.01$) pre-service teachers. Table 3 presents the mean and standard deviation of problem solving of the three groups for Cypriot and Italian pre-service teachers. Both Cypriot and Italian pre-service teachers who used the coordinated approach had better performance than the other pre-service teachers in problem solving.

Table 3: The mean and standard deviation of the problem solving for the three groups of Cypriot and Italian teachers.

Groups	Cypriot pre-service teachers		Italian pre-service teachers	
	Problem solving		Problem solving	
	$\overline{X}$	SD	$\overline{X}$	SD
1: Coordinated approach	0.82	0.31	0.65	0.33
2: Algebraic approach	0.35	0.34	0.15	0.28
3: Various approaches	0.15	0.26	9.699E-02	0.25

4. . Discussion

A main question of this study referred to the approach Cypriot and Italian pre-service teachers use in order to solve simple function tasks. Most of the Cypriot teachers used an algebraic approach in order to solve the simple function tasks. In contrast, a smaller percentage of Italian pre-service teachers used an algebraic approach while a larger percentage used a coordinated approach. A quite large percentage of Italian pre-service teachers gave incorrect responses to the four simple function tasks. From a closer look in the tests of the Italian pre-service teachers' some of them used the coordinated approach rather occasionally and superficially. Furthermore, they probably were affected by the didactical contract indicating that all the data given in a problem or exercise must be used in order to reach an answer.

A coordinated approach is fundamental in solving problems even though many students have not mastered even the fundamentals of this approach. This finding is in line with the results of other studies that suggest that many students deal with functions pointwise (Even 1998; Bell & Janvier, 1981). Students can plot and read points, but cannot think of a function as it behaves over intervals or in a global way. These studies also indicate that a global approach to functions is more powerful than a pointwise approach. Students who can easily and freely use a global approach have a better and more powerful understanding of the relationships between graphic and algebraic representations and are more successful in problem solving. Cypriot pre-service teachers' preference in the algebraic solution is probably the Cypriot curricular and instructional emphasis dominated by a focus on algebraic representations and their manipulation. (Dugdale, 1993). In their textbooks, students are usually asked to construct graphs from given equations using pairs of values. As a result, students fail to connect algebraic and graphical representations and therefore fail to develop a "global-coordinated" approach.

Teachers' performance in problem solving was moderate. Cypriot teachers performed better than Italian teachers. This finding suggests that in order to give a correct solution to a complex function problem the students must be able to handle different representations of function flexibly. Furthermore, an important finding of this study is the relation between the coordinated approach, the problem solving and the graphical approach. The data from both countries suggest that students who have a coherent understanding of the concept of function can easily understand the relationships between symbolic and graphical representations and therefore are able to provide successful solutions to complex problems. It is noteworthy that this close relationship between the coordinated approach and problem solving ability is strong, despite the differences exist in the curriculum of the two countries.

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