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The role of technologies in the argumentative and demonstrative process

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Summary. In this article I am dealing with the possible implications of the usage of computer calculation tools when demonstrating mathematical properties in the school practice. The point is to verify whether the use of educational software encourages the passage from conjecture to demonstration, or it is just a quick way to the solution of the problems. The Theory of Situations (Brousseau, 1997) is the referent theory. This work will present the results of statistics protocols, on the base of a quality and quantity analysis, (R. Gras, 2000) given to students in their last year of secondary school. The methods used are, a descriptive analysis (working on EXCEL sheets) and the analysis of possible variables (working with CHIC).

Key works: technology, proof, strategies.

Résumé. Dans cet article, nous allons considérer les effets possibles en utilisant en classe des outils de calcul informatique pour démontrer les propriétés mathématiques. Le point de vue est celui de vérifier si l'utilisation de logiciels éducatifs favorise le passage de la conjecture à la démonstration, ou s'il est juste un moyen rapide de solution des problèmes. Nous nous référons à la théorie des situations (Brousseau, 1997). Ce travail présente les résultats des protocoles statistiques, sur la base d'une analyse de la qualité et de la quantité, (R. Gras, 2000). Les sujets étaient tous étudiants du dernière année d'école secondaire. Les méthodes utilisées sont l'analyse descriptive (travail sur des feuilles Excel) et l'analyse de variables possibles (avec l'utilisation du logiciel CHIC).

Mots clés : technologie, preuve, stratégies

1 Introduction

This work concerns the school practice use of technology with students in their last three years of secondary school (16 – 18 years old). This research will describe the choice students did about the type of tool and the language to use in solving problems of analytical geometry. To this end, I carried out an experiment on 60 students about their reasons for choosing and managing an educational software, GeoGebra. The aim of my research is then to highlight the implicit ideas of the students during the:

- recognition of a problem that must be solved and demonstrated;
- identification of the most effective strategy to solve a problem;
- the software management process: speculation, argumentation, demonstration.

1.1 Theoretical background

These reflections are made necessary, in my opinion, by the massive usage of calculating tools and the huge availability of computer systems used by students today. Moreover, mathematics is nowadays making a widespread use of computer systems, so that we can talk of "experimental mathematics"; such a perspective does imply a serious reflection on the mathematical demonstration and its application on an epistemological basis (Hanna, 2000). The teaching and the learning practice is obviously influenced by such a change; what is the role of the rigorous formal mathematical demonstration, then? Could this latter be "softened" by "more heuristics" strategies (Mason, 1991)? The institutional educational indications about the curricula have changed (Hanna, 2000) even about maths; the students’ equipments have changed (they all have a computer, at least) and the classroom tools have changed as well. It is therefore right to consider some changes in the mathematical demonstrations at school.

1.2 The experimentation

The experiment was carried out on 60 students (aged 16-18) from different parts of Italy, who were given a guided test; they were asked to follow a precise interactive and multimedia path. There were four steps implying the use of different semiotic registers (natural language, graphics, tabular forms, algebraic language) during which the students related about their argumentative process.

The steps were:

1. giving two problem texts and time to write conjectures;
2. presentation of two solving approaches to the first problem (explanatory text and video mode);
3. giving time to the students to solve the second problem;
4. students writing their considerations about the suggested strategies.

Stage 1: the presentation of two strategies

I first presented two problems frequently analysed in the Italian secondary school curriculum. These problems are not difficult to be solved, but they require a good model using skills and an ability in changing different semiotic registers quickly.

The task was:

1. determine, among all rectangles with the same perimeter, the one with the maximum area;
2. determine, among all the triangles with a given value of the hypotenuse, which one has the maximum ratio between the sum of the sides and the hypotenuse.

I immediately said that the purpose was not to check the students' problem solving skills. The aim was to create a learning context in which the resolution of these simple frequent mathematical problems could imply the problematic choice of a solving process rather than another. The students were not asked to solve problems, they were just given some time (15 mis) to speculate about them. Then I provided two different possible strategies to use. Students were also provided with computer software such as Excel, GeoGebra, Derive. The first strategy is the graphic-analytical one, present in different textbooks, and then used in different ways according to the age. The two problems could be solved with the same strategy, I’m now quoting only the first problem given, because the solution approach is the same. The first strategy is the graph-analytical one, commonly used in any textbook, modified in accordance to the student’s age.

The problem can be solved by studying the parabola $y(x) = px - x^2$ (where p is the semi perimeter, the dependent variable y is the value of area and the dependent variable x is one the two sides). Students in third year (15-16 years) can now study the associated parabola (this way, they are not strictly linked to object of the problem), that is a concave upside down parabola passing through the origin of the axes. On the x axis we have one of the side of the rectangle, on the y axis we have the area of the rectangle. The vertex indicates the highest point, its abscissa corresponds to the value of the base with the maximum area. The students in their last year (17-18) can study the derivative, and deal with the problem of the analysis of maximum and minimum values. Both processes led to the conclusion that among all isoperimetric rectangles, the one with the maximum area is the one with the same dimensions; that is, a square.

The second strategy implies the use of the software GeoGebra. If we use the slider tool (Figure 1), linking its value to one of the dimension of the rectangle (in this case, the base) we can change the dimension value maintaining a constant value of the semi-perimeter. This also can be associated to the value of an additional slider to show the possible rectangles with different perimeters.

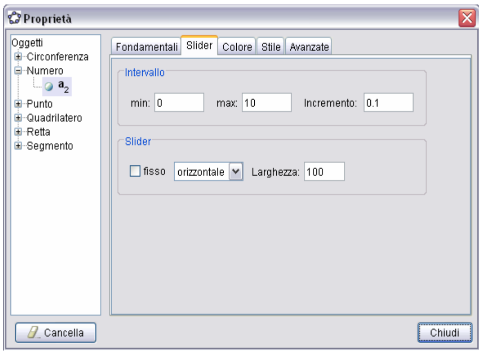


Figure 1

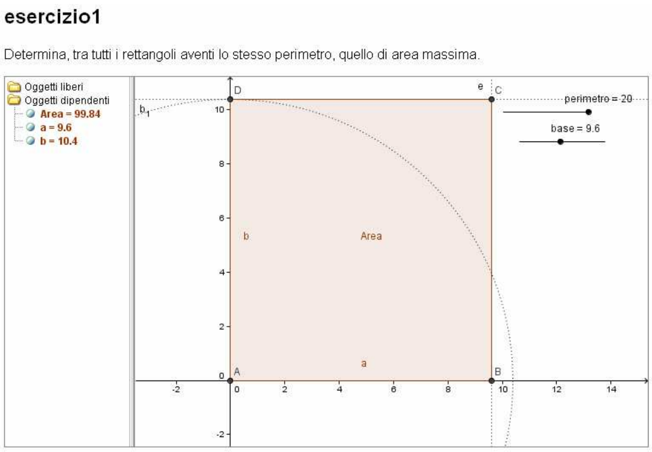


Figure 2

It is necessary to let the students notice that the *min* and *max*, together with the *increase* of the slider, must be determined as a first thing. In the case described in the figure, *min* = 0 *max* = 10, *step* = 0.1, we have exactly 100 squares for each value of the perimeter. By varying the base we immediately have the values of the area (Figure 2). The reason for considering the second problem is its type of solution; while in the first exercise the solution may be represented in rational numbers, in the second case it is an irrational numbers; of course, this would have prevented an approach to the software. Regarding the second problem no strategies were suggested, the students had to try to solve the problem or at least to propose a scheme of argumentation; the use GeoGebra was obviously allowed (Figure 3).

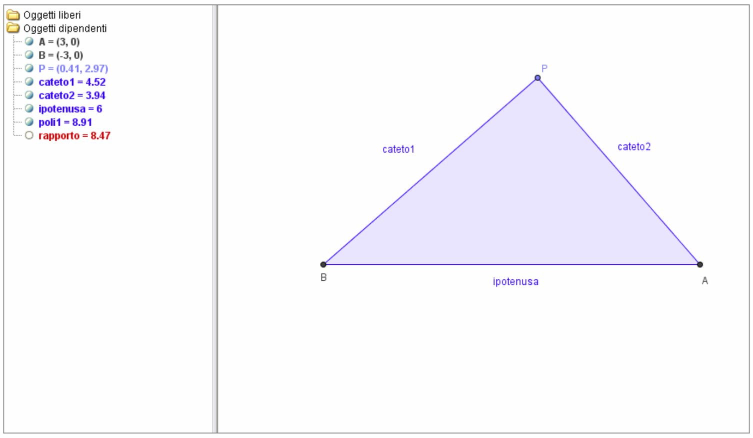


Figure 3

Only at this moment the students were asked to fill in a set of questions.

Stage two: the choice of the argumentations

Having made conjectures and reflections on the two strategies suggested and having tried to solve the second exercise, the students were asked to answer the following questions, with no specific linguistic expression required:

1. Do you think the two argumentations used to solve the exercise are mathematically correct?
2. Can you find any similarities between the type of argumentation used for physics experiments and the second strategy suggested?
3. Which of the two strategies do you consider more effective? Which would you use in a school test, on your free choice?

2 Data analysis

The protocols were collected and analyzed on the basis of an *a priori* analysis. The *a priori* analisiys is merely an analysis of possible epistemological representations and conceivable behaviors (correct and incorrect) for students. In other words, *a priori* analysis suggests a model based on which it is possible to apply the theoretical framework used to the teaching/learning experience . The *a priori* analysis can identify the variables of the problem situation and the research hypotheses. These hypotheses can be falsified by the statistical analysis and / or qualitative analysis of data. The following are behaviors hypothesized.

1a. The student considers irrelevant the choice between the first or the second strategy because they both lead to the solution.
1b. The student believes the second strategy absolutely wrong because they were provided by their teacher with the first only.
1c. The student believes the second strategy absolutely wrong because the computer processing does not guarantee the result.

1d. The student believes the second strategy correct in the first exercise and incorrect in the second, because in the former the result is precise, while in the second it is approximate.
1e. The student considers correct both strategies, but considers the first as better because the computer is rather rough in the calculations.
1f. The student prefers the second strategy because it allows him to work on mathematical objects present in the problems (squares and rectangles) instead of having to work on other (parabola).
2a. The student believes that there cannot be any relations between a physics experiment and a mathematics one, since the type of logic used is inductive or deductive.
2b. The student considers the two solutions similar, but, while in the physics exercise the value of the curve is an approximation, which best represents all the experimental points, in the maths exercise the curve is created exactly on the points found.
2c. The similarity between the two cases is complete.
2d. No answer.
3a. The First is considered the only acceptable strategy.
3b. The first one is correct, but the second (which should not be used!) provides a faster solution.
3c. They are equivalent.
3d. Since the student would not have been able to make a demonstration using the first strategy, the second is considered as the most effective.

As I said before, any answer became a variable and it was possible to analyse a sample in order to see if our survey query is compatible with the analysis described below. Any variable corresponds to a different behaviour in a specific situation.
Knowing this, the survey hypothesis can be expressed:

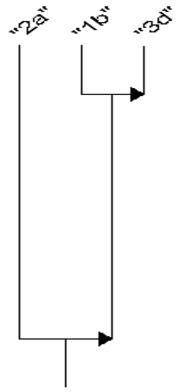
The students able in using both strategies...	**… can distinguish their application**
1c, 1d, 1e	2a, 3a, 3b
Those who cannot make an algebraic demonstration...	**… use the software as a mere arithmetical means**
1a, 1b	3c, 3d

The data obtained interviewing the students can be re-elaborated by C.H.I.C. drawing a chart where each students can be combined with the foreseen behaviour. The number 1 and 0 correspond to the matching or not between the student and a specific behaviour.

Table 1

Studente	**Comportamento 1**	**Comportamento 2**	**Comportamento …**	**Comportamento *n***
1	1	0	0	0
2	0	0	0	1
...				
n	1	0	1	0

A chart similar to this above will be drawn by C.H.I.C., it will show a similarity tree and a hierarchy tree (in the picture only the most significant part is visible) this may lead to the following considerations:



hierarchy tree
Figure 4

Analyzing the hierarchy tree the implication 1b – 3d is obvious with a 80% percentage. Many students seem to be more used to the analytic-algebraic method, but they may have never thought about it. So, they cannot find any analogies between strategies and their languages. They may also be not able to draw the curve using Geogebra autonomously.

About the similarity tree, the data organized using C.H.I.C. leads to different typologies, here corresponding to three different colours:

- Red: the students have problems in managing two linguistic registers at the same time and in recognizing the algebraic steps, they prefer to use Geogebra only because it provides a "clear solution"
- Blue: the students able in using an algebraic language would use the first strategy, avoiding the second one implying the software, because it complies with the "didactic pact".
- Green: the students who can recognize the epistemological differences between the two methods consider the software useful in making up ideas, they think it simplifies the recognition of some properties.

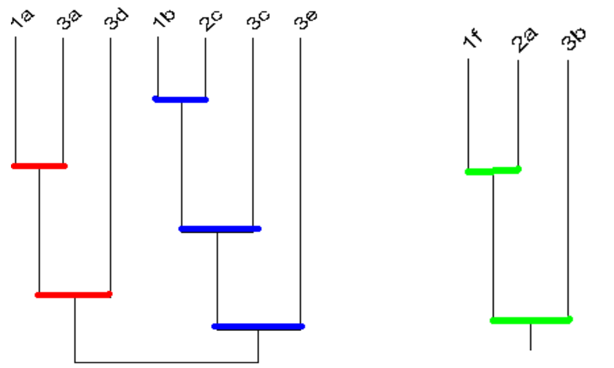


Figure 5

It is blatant that some of the students interviewed (1b ↔ 2c) immediately stated that only the first strategy could be accepted. However, no explanations were provided. This suggests that they preferred to use a strategy because it was suggested and used by their teacher.

A small group (1f ↔ 2a) wrote that the first method was impossible to be applied, while the second strategy was quickly understood. The big difference between the two appears evident, though.

A larger group (1a ↔3a) wrote that the right method was the first, but the second one was not wrong, anyway.

It is really important to focus on this latter group and analyze what they wrote in the protocol; these students would prefer a “traditional” method, because it appears as the most elegant and it is considered as the best to provide a response in any particular case.

Some other students, who showed particular interest and curiosity about the subject matter, consented to be video-interviewed. These students were asked if they considered the use of the software as not exhaustive in describing the functions to be developed in two exercises. In other words, they were asked to comment if the algebraic formalization provided more information than the data calculated using Geogebra. All of them said no. In fact, we can state that the only reason to prefer the first strategy was the idea of an "absolute certainty" (many of them kept on saying, “this way, it is always true"). In addition, they were happier with the possibility of using the method chosen by their teacher. Some of them also appreciated that Geogebra was able to make visible the correspondence between the rectangles, their areas, and the parabola (Figure 4). Some students (the youngest, in particular), having noted how the use of the slider could show the correspondence between the area of the rectangle, the base and the points of the parable and used meaningful words like "power", "charm" and even " elegance "of pure mathematics.

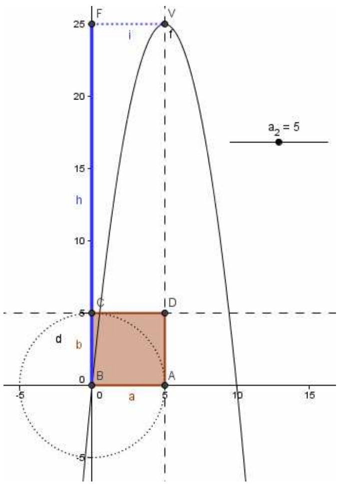


Figure 6

Regarding the second question, many students did not answer it. In my opinion this could mean that they did not perceive it as significant: in fact the question investigated the epistemological value of the concept of demonstration, which, in fact, probably, has never been a subject of these students' school curriculum. The few who answered it emphasized the experimental nature of the second strategy using GeoGebra. Another constant element is the need, due to the Aristotelian logic, for general solutions in continuous and not partial intervals such as those we are forced to consider when using the *slider*. These students insisted on the need to exorcise the possibility of a point of discontinuity. While realizing that the point of discontinuity does not exist, the students believe that it could put the entire argument carried out using the computer and the second strategy on trial.

3 Conclusions

The elaboration of the data we have now introduced let us be positive in considering our work with students. The correspondence between the survey hypothesis and the data encourages further experimentation in this field. In my opinion, all the activities that make the students reflect on the demonstration process cannot be neglected in the practice of teaching and learning mathematics. The use of a software, in my opinion, responds to the need for such a reflection; a free software like GeoGebra is a very useful tool for students when they have to analyze the text of a problem. Such a tool may also force them to quick changes into the semiotic registers used, which is a pivotal point in the observation of "the cognitive functioning of the various mathematical activities " (Duval, 1996). Besides, together with the conversions from algebraic language, graphics, tables to natural language, a further conversion is then required codifying and interpreting the software language. In further studies, I do think it would be important to compare these results with the opinion obtained interviewing some teachers, so that I could be able to make more considerations on this topic.

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