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Exploring the use of number line in additive problem solving by applying the Statistical Implicative Analysis

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Résumé. La présente recherche étudie l'utilisation de la droite arithmétique dans la solution des problèmes additifs de changement par 1134 étudiants d'école primaire de 6 à 9 ans dans les classes 1, 2 et 3. Parmi ces étudiants un total de 356 étudiants des trois classes ont reçu une intervention sur l'utilisation de la droite arithmétique coordonné avec d'autres représentations dans la résolution des problèmes additifs, c.-à-d. groupe expérimental, alors que 778 étudiants ont suivi un programme d'études mathématique habituel, c.-à-d. groupe témoin. Les résultats ont prouvé que les étudiants ont employé la droite arithmétique largement, mais pas toujours convenablement en résolvant les problèmes additifs. Le programme expérimental a eu des effets positifs dans le développement de la capacité d'employer la droite arithmétique dans le cadre de la résolution des problèmes additifs. L'application de l'Analyse Statistique Implicative (ASI) a fourni des résultats intéressants au sujet des interdépendances de l'utilisation de la droite arithmétique et de la résolution des problèmes. Spécifiquement, elle a fourni l'évidence pour un compartmentalization entre l'inclination des étudiants de pas utiliser la droite arithmétique et leur succès de résolution des problèmes. En d'autres termes, le choix pour ne pas employer la droite arithmétique dans la résolution des problèmes additive n'a pas soutenu les étudiants en atteignant la réponse correcte. Employant correctement la droite arithmétique s'est avéré avoir un potentiel limité de mener au succès. Quoique ce rôle positif de l'utilisation correcte de la droite arithmétique ait diminué avec l'âge, il a été augmenté par le programme expérimental dans la première et deuxième classe. Le programme expérimental n'a pas différencié les relations limitées et faibles entre la capacité d'employer la droite arithmétique et le succès de résolution des problèmes dans la troisième classe. Les résultats ci-dessus suggèrent que l'introduction de la droite arithmétique comme représentation auxiliaire dans la résolution des problèmes additifs et l'utilisation d'elle systématiquement dans ce contexte mathématique puissent contribuer non seulement à une utilisation plus compétente de la droite arithmétique, mais à une meilleure compréhension de la structure mathématique de ces problèmes et donc à une achèvement plus élevée de résolution des problèmes, particulièrement dans les étudiants de 6-8 ans.

Abstract. The present study investigates the use of number line in the solution of additive change problems by 1134 primary school students from 6 to 9 years of age in grades 1, 2 and 3. Among these students a total of 356 students from the three grades received an intervention on the use of the number line in coordination with other representations in additive problem solving, i.e. experimental group, while 778 students attended the usual mathematical curriculum, i.e. control group. Findings showed that students used the number line to a considerable extent, but not always appropriately in solving the additive problems. The experimental program had positive effects in the development of the ability of using the number line in the context of additive problem solving. The application of the Statistical Implicative Analysis (SIA) provided interesting results about the interrelations of the use of the number line and problem solving performance. Specifically, it provided evidence for a compartmentalization between students' inclination of not using the number line and their problem solving success. In other words, choosing not to use the number line in additive problem solving did not support the students in reaching the correct answer. Using correctly the number line was found to have a limited potential to lead to success. Even though this positive role of the correct use of number line declined with age, it was enhanced by the experimental program in the first and second grade. The experimental program did not differentiate the limited and weak relations between the ability of using the number line and problem solving success in the third grade. The above findings suggest that introducing the number line as an auxiliary representation in additive problem solving and using it systematically in this mathematical context may contribute not only to a more proficient use of the number line, but to a better understanding of the mathematical structure of these problems and therefore to higher problem solving performance, especially in students of 6-8 years of age.

Περίληψη. Η παρούσα έρευνα εξετάζει τη χρήση της αριθμητικής γραμμής στην επίλυση προσθετικών προβλημάτων αλλαγής σε 1134 μαθητές Δημοτικού, 6 – 9 χρόνων, από την Α΄, Β΄ και Γ΄ τάξη. Οι 356 από τους μαθητές αυτούς δέχθηκαν παρεμβατική διδασκαλία για τη χρήση της αριθμητικής γραμμής σε συνδυασμό με άλλες αναπαραστάσεις στην επίλυση προσθετικών προβλημάτων (πειραματική ομάδα), ενώ οι υπόλοιποι 778 μαθητές παρακολούθησαν την προβλεπόμενη από το Αναλυτικό Πρόγραμμα διδασκαλία (ομάδα ελέγχου). Τα αποτελέσματα έδειξαν ότι οι μαθητές χρησιμοποίησαν την αριθμητική γραμμή σε ένα αξιόλογο βαθμό, αλλά όχι πάντοτε με το σωστό τρόπο για την επίλυση των προσθετικών προβλημάτων. Το παρεμβατικό πρόγραμμα είχε θετικές επιδράσεις στην ανάπτυξη της ικανότητας χρήσης της αριθμητικής γραμμής στο πλαίσιο της επίλυσης προσθετικών προβλημάτων. Μέσα από τη χρήση της Συνεπαγωγικής μεθόδου ανάλυσης προέκυψαν ενδιαφέροντα αποτελέσματα για τις σχέσεις μεταξύ της χρήσης της αριθμητικής γραμμής και την ικανότητας επίλυσης προβλήματος. Συγκεκριμένα, προέκυψαν στοιχεία στεγανοποίησης μεταξύ της τάσης των παιδιών να μη χρησιμοποιούν την αριθμητική γραμμή και την επιτυχία τους στην επίλυση προβλήματος. Με άλλα λόγια, η επιλογή να μη χρησιμοποιήσουν την αριθμητική γραμμή στα προσθετικά προβλήματα δε βοήθησε τους μαθητές να φτάσουν στη σωστή απάντηση των προβλημάτων. Η χρήση της αριθμητικής γραμμής φάνηκε να έχει μια περιορισμένη δυνατότητα στο να οδηγήσει σε επιτυχία στα προβλήματα. Παρόλο που ο θετικός ρόλος της ορθής χρήσης της αριθμητικής γραμμής μειώνεται με την ηλικία, εντούτοις ο ρόλος της αυτός τονίστηκε μέσα από το παρεμβατικό πρόγραμμα στην Α΄ και Β΄ τάξη. Το παρεμβατικό πρόγραμμα δε διαφοροποίησε τις περιορισμένες και ασθενείς σχέσεις μεταξύ της ικανότητας χρήσης της αριθμητικής γραμμής και της επιτυχίας στην επίλυση προβλήματος για τους μαθητές της Γ΄ τάξης. Τα παραπάνω αποτελέσματα δείχνουν ότι η εισαγωγή της αριθμητικής γραμμής ως βοηθητικής αναπαράστασης για την επίλυση προσθετικών προβλημάτων και η συστηματική της χρήση σε μαθηματικά συγκείμενα μπορεί να συμβάλει όχι μόνο σε μια πιο αποτελεσματική χρήση της αριθμητικής γραμμής, αλλά και σε μια καλύτερη κατανόηση της μαθηματικής δομής τέτοιων προβλημάτων και συνεπώς σε καλύτερη επίδοση στην επίλυση προβλήματος, ειδικά για μαθητές 6 – 8 χρόνων.

1. Introduction

It is well documented that students encounter difficulties in solving arithmetic word problems (e.g., Verschaffel & De Corte, 1993; Elia & Gagatsis, 2005; Coquin-Viennot & Moreau, 2007). An important reason for this phenomenon is the complexity of the structure of arithmetic problems and specifically the variation in the placement of the unknown element (Adetula, 1989; Elia, 2009).

In elementary mathematics the number line is a representation that is widely used for the teaching of basic whole number operations and arithmetic in general (Klein, Beishuisen & Treffers, 1998; Gagatsis, Kyriakides & Panaoura, 2004). In this study, the number line is seen as a special kind of visual representation (see Figure 1), because it may stand for an internal arrangement of the relations between numbers. The number line is not a simple representation: Many authors consider the number line as a geometrical model, which involves a continuous interchange between a geometrical and an arithmetic representation (Gagatsis, Shiakalli & Panaoura, 2003; Shiakalli & Gagatsis, 2006; 2005). With respect to the geometric dimension, the numbers shown on the line correspond to vectors and to the set of the discrete points of the line. According to the arithmetic dimension, points on the line are numbered so that the distance between two points depict the difference between the corresponding numbers.

Despite the widespread use of the number line as an aid to operations on numbers, doubts about its usefulness have been raised (Gagatsis et al., 2003; Hart, 1981). Ernest (1985) maintains that there can be a mismatch between students' understanding of whole number addition and their understanding of the number line model of this operation. The simultaneous presence of the geometric and the arithmetic conceptualization of number may limit the effectiveness of the number line and thus hinder the performance of students in arithmetical tasks (Gagatsis et al., 2003).

An example of a very interesting phenomenon concerning the use of number line in arithmetical tasks that has been revealed by using the ISA is the phenomenon of compartmentalization. In a research with primary school students, Gagatsis et al. (2003) have proposed different questionnaires including simple additions and subtractions with whole numbers. The first questionnaire used the symbolic expression for representing the additions and subtractions, the second involved conversions from the symbolic expression to the representation of number line and the third one comprised of the inverse conversions, that is, from the representation of the number line to the symbolic expression. The application of the ISA has revealed a clear separation between the implicative chains concerning the tasks in the different questionnaires. That is, no implication has been observed between tasks of the three different questionnaires. This has been defined by Gagatsis et al. (2003) as a compartmentalization between representations. The same phenomenon has been observed some years later in the research of Shiakalli and Gagatsis (2005) in a different experimental setting.

A number of studies suggest that prior to the use of the number line as an aid in the teaching of mathematics, children should be taught about it so that they can use it efficiently in carrying out numerical operations (Ernest, 1985). What's more, Shiakalli and Gagatsis (2006) have found that systematic teaching of the use of the number line in conducting addition and subtraction helped students of early primary school grades to moderate their tendency to treat numerical tasks with the number line and similar tasks without the number line in isolation. The instruction they received led them to a "de-compartmentalized" performance signifying the development of the understanding of the relationship between the number line and symbolic mathematical expressions and of their coordination.

2. The research

Despite the extensive research on the role of number line in arithmetic operations, limited attention has been given to the role of number line in arithmetic problem solving. The present study aims to contribute to our understanding of the use of the number line in additive problem solving, and specifically in the solution of change problems. This study is a part of a larger research project concerning the use of multiple representations in additive problem solving (Elia, Gagatsis & Demetriou, 2007). In the context of this study, we developed and organized in the three first grades of primary school an experimental program focusing on the use of the number line in coordination with other representations in additive problem solving and compared its outcomes with the results of the usual mathematical curriculum. The study sought answers to the following research questions:

1. To what extent do the students use the number line in additive problem solving? How does the proportion of students who use the number line vary with age and instructional approach, that is, between the students who received an intervention on the use of the number line in additive problem solving, i.e. experimental group, and students who attended the usual mathematical curriculum, i.e. control group?

2. What are the relations between the use of the number line and answer success to additive change problems? How do these relations vary with age and instructional approach?

3. Method

A total of 1134 primary school students (599 boys and 535 girls) in Cyprus participated in the study. The sample involved students from 6 to 9 years of age from three grades (first grade: 387, second grade: 370, and third grade: 377). The control group consisted of 778 students, 275 students in first grade, 251 in second grade and 252 in third grade. The experimental group included 356 students, 112 students in first grade, 119 students in second grade and 125 students in third grade.

The aim of the experimental program was the introduction of the number line as an auxiliary representation for the solution of different types of additive problems. In particular, the focus was on one class of additive arithmetic problems: one-step change (measure-transformation-measure) problems, which describe a transformation or a change (an increase or a decrease) in a starting situation to result in a final situation. Varying the unknown and the direction of the transformation in a change problem generates different situations. Therefore, change problems include a total of six situations (Pa, Pb, Pc, Na, Nb, Nc), distinguished by whether the problem describes a positive transformation or a negative transformation and by the placement of the unknown. The unknown may be one of the three amounts (starting amount-a, transformation-b or final amount-c).

The content of the instruction involved these six types of change problems, as well as, three different forms of representations for introducing the problems and facilitating their understanding and solution. The three representations were gradually introduced in three phases: First phase, physical objects, second phase, number line and third phase, verbal description and symbolic expression, that is, numerical sentence. In the second and third phase the relationships among the already used representations were discussed. Each representational phase included all of the six types of change problems. The sequence of the problems in each phase was progressive according to the difficulty level of their structure and with children's prior knowledge.

After the intervention, students of the experimental group and students of the control group were asked to solve the six types of change problems (Elia et al., 2007; Elia, 2009) in verbal description with a number *l*ine, that is, the problems LPa, LPb, LPc, LNa, LNb and LNc. Figure 1 shows the LPb problem that was used in this study.

I had had 7 pounds before my birthday. I received some more money on my birthday. Now I have 16 pounds. How much money did I receive on my birthday? (Problem with number line-LPb)

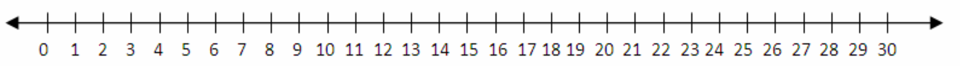

Figure 1: Problem with a positive transformation and with the unknown on the transformation (LPb).

For the analysis of students' responses in the present study, the SIA (Gras et al., 1996; Gras, Peter, Briand & Philippe, 1997) was used. In this paper, the results of the implicative analyses are described. Although similarity analyses were also conducted on the data, due to space limitations the similarity diagrams could not be included or discussed here. It is noteworthy, however, that the results of the similarity analyses are in line with the outcomes of the implicative analyses that are discussed below.

4. Results

4.1. Students' success and number line use

Table 1 shows students' success rates per problem for the control and the experimental group in each grade.

Table 1. Students' success percentages.

Type of problem	Grade	N	LPa %	LNa %	LPb %	LNb %	LPc %	LNc %
Control group	6-7 years	275	29.8	29.8	25.1	38.5	68.7	45.8
	7-8 years	251	66.9	70.1	62.2	70.1	88.4	80.9
	8-9 years	252	85.3	79.8	70.6	80.2	94.4	90.5
Experim.group	6-7 years	112	35.7	34.8	35.7	47.3	67.9	59.8
	7-8 years	119	80.7	69.7	73.1	79.0	89.9	80.7
	8-9 years	125	78.4	72.0	74.4	77.6	94.4	86.4

The placement of the unknown appears to be an important factor for the difficulty level of the problems in each group of students. Problems with the unknown on the final amount (c) are easier than the rest of the problems. The problems LPa, LNa and LPb appear to be more difficult than LNb, probably because the direction of the transformation they involve, i.e. positive or negative, is not congruent with the operation between the two numbers of the problem that is needed to reach an answer. This is not the case for LNb, in which, the transformation and the arithmetic operation are congruent to each other. Specifically, the transformation that is described is negative, while the operation needed is subtraction.

The change problems turned out to be rather difficult for a great proportion of first graders and rather easy for the majority of second and third graders. Students of the first and second grade that received the experimental program were more successful than students who attended the usual mathematics curriculum. The positive effects of the experimental program were stronger in the solution of the problems of higher complexity, that is, with the unknown on the initial amount or the transformation. The experimental program did not differentiate the experimental group students' success rates from the control group students' success rates in the third grade.

The percentages of students who used the number line (correctly or incorrectly) in the various problems ranged between 34% and 45% of the total sample, while the corresponding percentages of the correct number line use ranged between 18% and 32%. Table 2 illustrates analytically the percentages of the students in each group who used the number line correctly or incorrectly in the various types of change problems.

The following remarks apply not only for the general use of the number line, but also for the correct use of it. The number line was more commonly used by the experimental group students relative to the control group students. This was clearer for the second and third graders. In the control group, the use of the number line decreased with age development. In the experimental group, however, the students used the number line to a similar degree irrespectively of age.

As regards students' inappropriate use of the number line, in the experimental group the incorrect use of the number line was less frequent than the incorrect use of the number line in the control group for most of the problems. In the solution of some problems the incorrect number line use was more common within the experimental group relative to the control group. This indicates that more students of the experimental group tried to use the number line, probably as a result of their participation to the experimental program, but unsuccessfully. In the control group, as students got older, the incorrect use of the number line decreased. In the experimental group a similar pattern appears in most of the problems, with first graders presenting clearly higher percentages of incorrect number line use relative to the older students.

Table 2. Students' use of the number line per problem.

Type of problem	**Grade**	**N**	**LPa**		**LNa**		**LPb**		**LNb**		**LPc**		**LNc**	
			Cor. %	***Inc. %***	**Cor. %**	***Inc. %***	**Cor. %**	***Inc. %***	**Cor. %**	***Inc. %***	**Cor. %**	***Inc. %***	**Cor. %**	***Inc. %***
Control group	6-7 years	275	33.1	*28*	21.1	*24.7*	32	*20.7*	35.3	*24.4*	37.5	*25.5*	36.4	*22.5*
	7-8 years	251	12.7	*12*	8.4	*12*	13.5	*10.8*	12.7	*9.2*	10.4	*14.7*	13.5	*10.8*
	8-9 years	252	11.9	*1.6*	6	*3.2*	9.5	*2.8*	10.7	*1.2*	10.7	*4*	10.3	*2.4*
Experim. group	6-7 years	112	50.9	*20.5*	33.9	*24.1*	54.5	*11.6*	59.8	*13.4*	50.9	*15.2*	58	*13.4*
	7-8 years	119	52.1	*8.4*	30.3	*20.2*	51.3	*5.9*	51.3	*7.6*	43.7	*15.1*	52.1	*6.7*
	8-9 years	125	52.8	*8.8*	31.2	*16.8*	60.5	*4*	61.6	*5.6*	57.6	*14.4*	58.4	*5.6*

4.2. Successfulness of number line use: Results of the SIA

Figures 2 and 3 show the implicative relations among the variables of using correctly the number line, not using the number line and success in the six change problems by the first grade students of the control group and the experimental group, respectively. The two implicative diagrams have many commonalities, even though they concern two different groups of students. These commonalities are described below.

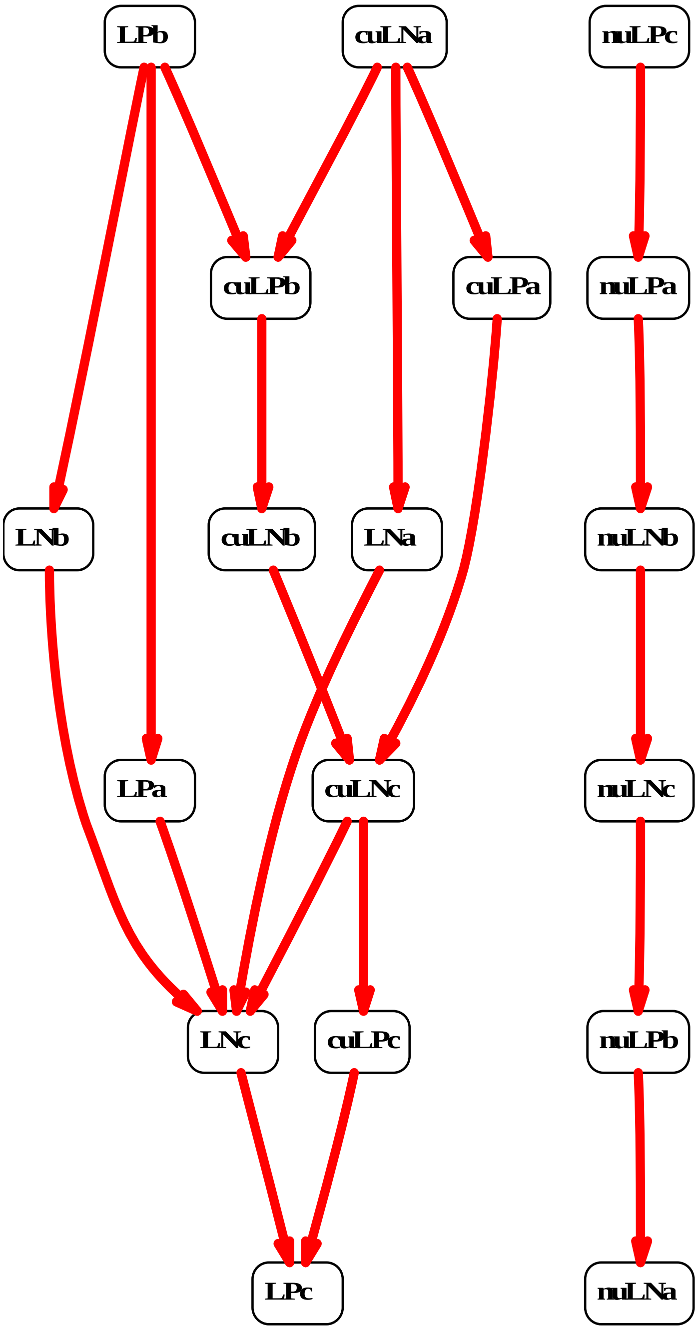

Figure 2. Implicative diagram of the control group of grade 1 (6-7 years old).

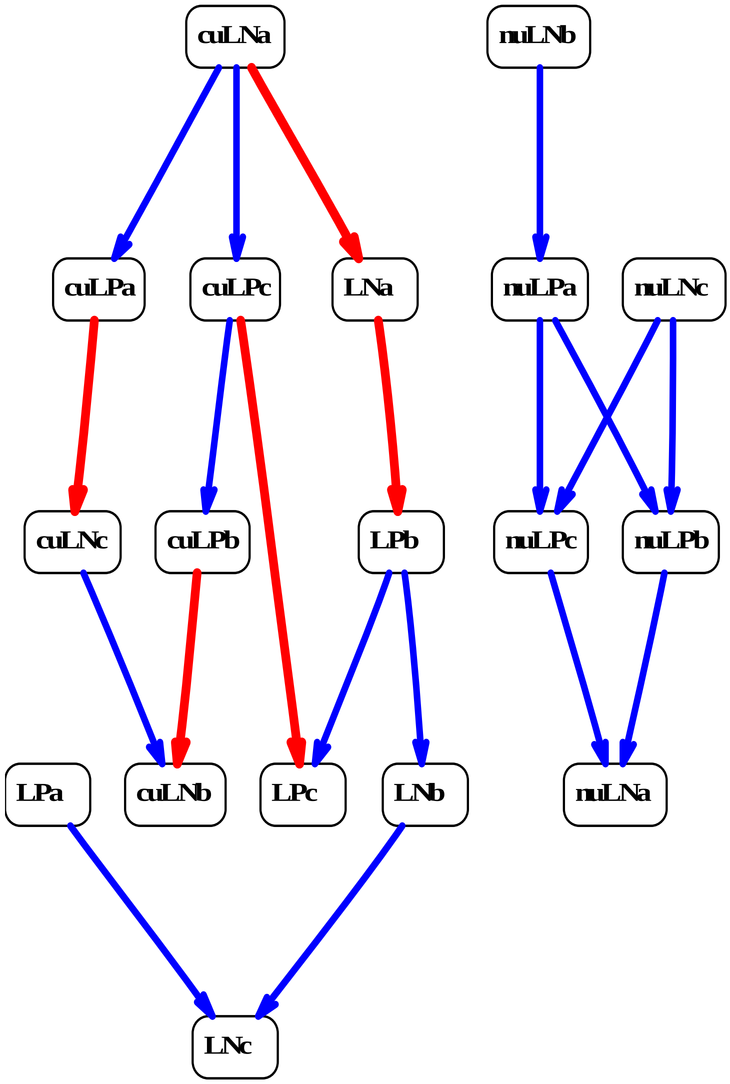

Figure 3. Implicative diagram of the experim. group of grade 1 (6-7 years old).

Notes: a) LPa, LNa, LPb, LNb, LPc and LNc refer to students' success to the corresponding problems, b) cuLPa, cuLNa, cuLPb, cuLNb, cuLPc and cuLNc refer to the correct use of the number line in the corresponding problems, c) nuLPa, nuLNa, nuLPb, nuLNb, nuLPc and nuLNc refer to not using the number line in the corresponding problems, d) The colour of the arrows varies with respect to the strength of the implications, whose estimated probabilities are as follows: the red arrows and the blue arrows correspond to 99% and 95% respectively.

Two distinct groups of implications are established separately in both figures. One of these implication groups links the variables of the correct use of the number line and problem solving success. This implication group indicates that correct use of the number line is connected with the successful solution of the problems. Specifically, in both implicative diagrams the correct use of the number line in LNa and LPc problems appears to be crucial for solving the corresponding problems.

Furthermore, using correctly the number line in one problem implied correct use of the number line in the other problems.

In the other implication group all the variables corresponding to not using the number line in the various problems are linked. The establishment of the second group of implications shows that students who did not use the number line in one problem were inclined not to use the number line in the other problems, as well, and that this stance towards the number line did not lead to success in the various problems. This indicates that there is a compartmentalization between the students' solutions to the problems and their inclination of not using the number line.

A noteworthy difference between the implicative diagrams of the control group and the experimental group is the implication between LPb and cuLPb that is included in the implicative diagram of the control group but not in the diagram of the experimental group. This relation suggests that although an important process feature of students who solved the LPb problem is the correct use of the number line, there is a number of students who used correctly the number line, but did not provide a correct solution to the problem. Another difference is that in the control group diagram the solutions to two problems (LPa, LNb) are not connected directly or indirectly with any variable referring to the correct use of number line, while in the experimental group only one problem (LPa) is not related to such a variable. The above differences indicate that the linkage between a problem's solution and the correct use of the number line is clearer in the experimental group relative to the control group.

Figures 4 and 5 illustrate the implicative relations among the variables of using correctly the number line, not using the number line and success on the six problems by the second grade students of the control group and the experimental group respectively.

The implicative diagram of the second grade students of the control group involves three chains of implications. The first implication chain refers to the success to the six problems, with the correct solutions to the more complex problems, that is, the problems with the unknown on the initial amount or the transformation, implying success to the problems with the unknown on the final amount. The second implication chain involves relations within the correct use of the number line in the six problems. This suggests that correct use of the number line in one problem implied appropriate use of the number line in other problems. Specifically, using appropriately the number line in the problem LNa implied appropriate use of the number line in all the other problems. The third implication chain involves relations between the variables of not using the number line in the six problems. The two latter implication chains are not completely distinct from the implication chain referring to problem solving success. In particular, students who used correctly the number line in the problem LPc were successful in solving the problem LNc. Students who gave a correct solution to the most difficult problem LPb were inclined not to use the number line in their solutions.

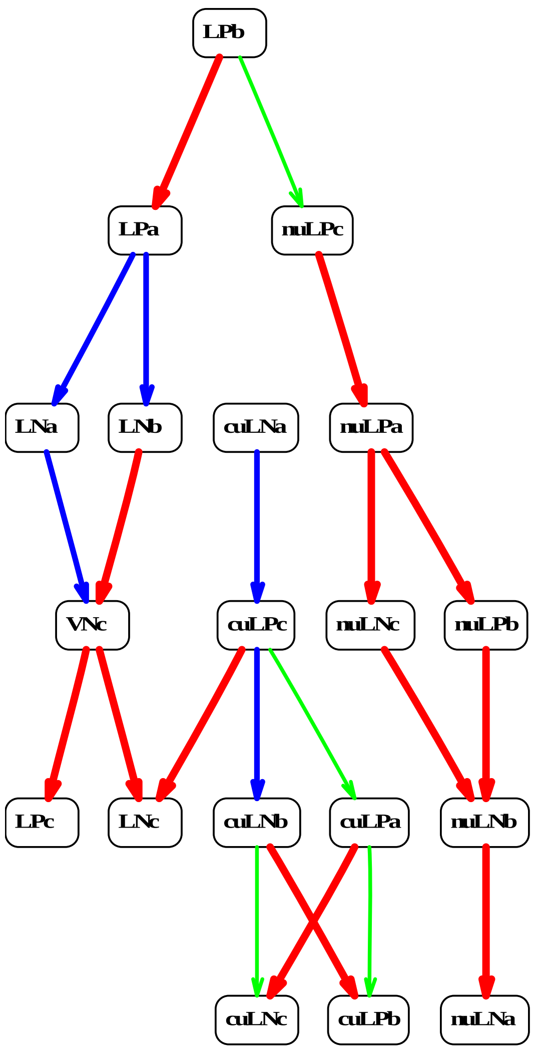

Figure 4. Implicative diagram of the control group of grade 2 (7-8 years old).

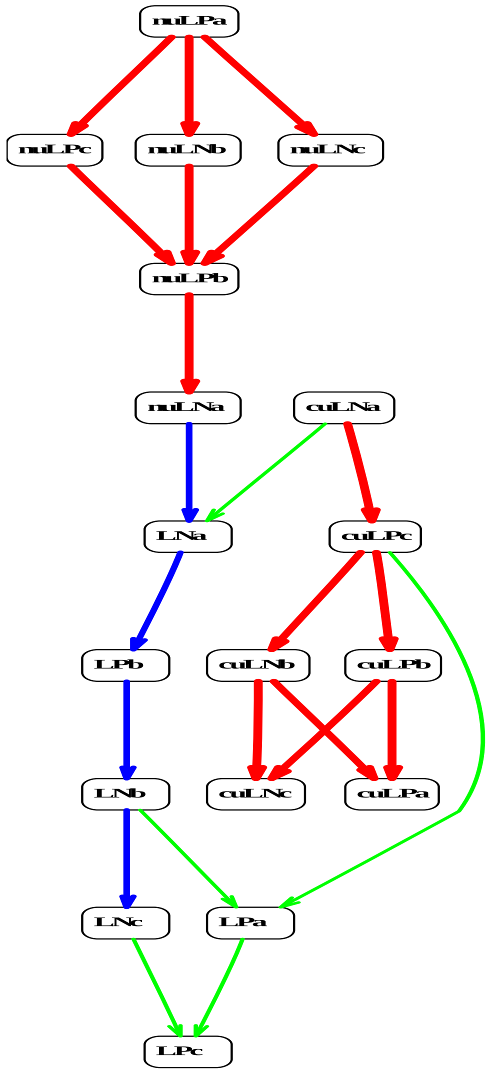

Figure 5. Implicative diagram of the experimental group of grade 2 (7-8 years old).

Note: The estimated probabilities of the implications represented by the red arrows, the blue arrows and the green arrows are 99%, 95% and 90% respectively.

Based on the above observations, the implicative diagram of second graders of the control group suggests that students who used correctly the number line, on the one hand, and students who did not use the number line, on the other hand, did not necessarily provide correct solutions to the problems.

The implicative diagram of the second grade students of the experimental group involves two chains of implications which are not completely distinct. The first implicative chain involves relations between variables of not using the number line in the problems and of the successful solutions to the problems. The students who did not use the number line in the problems provided correct solutions to the problems.

The second implicative chain consists of relations between the variables of using correctly the number line in the various problems. Two of the variables of this implicative chain (cuLNa and cuLPc) are linked with variables referring to problem solving success, indicating that using appropriately the number line in specific problems (LNa and LPc) leads to successful problem solving. Based on these results, in the experimental group of second grade there are two subgroups of students: First, students who solve the problems without using the number line and second, students who solve the problems by using appropriately the number line.

To sum up, the implicative diagram of second graders in the control group indicated that there is a compartmentalization between using correctly or not using the number line and the solution of the problems. This was not the case in the implicative diagram of second graders of the experimental group, which involved implicative relations between using correctly or not using the number line and problem solving success. This means that the experimental program on the one hand, probably helped a group of students to use the number line efficiently in the solution of the problems. On the other hand, it probably helped some other students to better understand the mathematical structure of the problems and therefore to use solution procedures that are independent of the number line.

Figures 6 and 7 show the implicative relations among the variables of using correctly the number line, not using the number line and success on the six problems by the third grade students of the control group and the experimental group respectively. The two implicative diagrams of the third grade are very similar, even though they concern two different groups of students. The commonalities of the two diagrams are described below.

Three chains of implications are established in both figures. The first implication chain includes the variables of problem solving success to the problems. The second implication chain links the variables of the correct use of the number line in the various problems. In both diagrams there is one implicative relation between the two chains, indicating that the correct use of the number line in specific problems (cuLPb for the control group and cuLNa for the experimental group) is important for solving the corresponding problems.

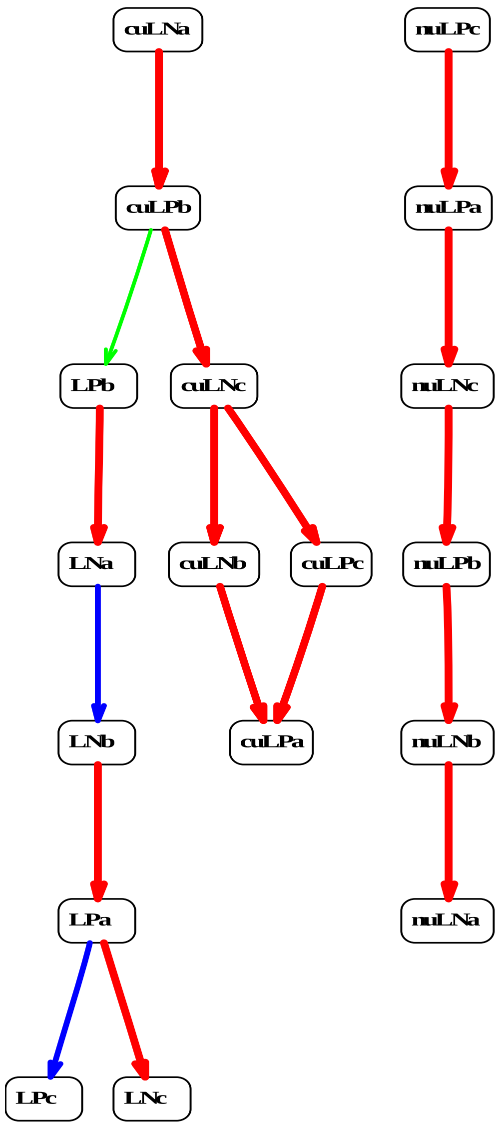

Figure 6. Implicative diagram of the control group of grade 3 (8-9 years old).

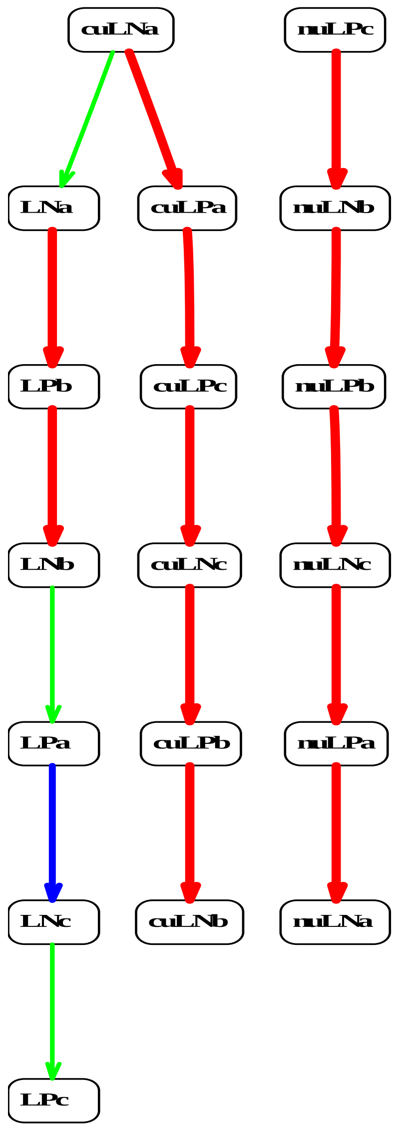

Figure 7. Implicative diagram of the experimental group of grade 3 (8-9 years old).

Note: The estimated probabilities of the implications represented by the red arrows, the blue arrows and the green arrows are 99%, 95% and 90% respectively.

In the third implication chain, which is completely separate from the other two chains, all the variables corresponding to not using the number line in the various problems are linked. The third chain of implications shows that students who did not use the number line in one problem were inclined not to use the number line in the rest of the problems, and that this stance towards the number line did not lead to success in the various problems. In other words, there is a compartmentalization between the solution to the problems and not using the number line. The above results suggest that the experimental program in the third grade did not differentiate the relations between the use of the number line and successful problem solving.

The following observations refer to the similarities and variations of the relations between the use of the number line and answer success to the problems with age. Considering the control group, an important difference between the first grade students and the older students is that in the first grade there are more and stronger connections between the correct use of the number line and answer success relative to the second and third grade. This implies that for the first graders the correct use of the number line facilitates problem solving to a greater extent relative to the older students. Not using the number line appears not to contribute to successful problem solving in all the grades, indicating a compartmentalization between the inclination of not using the number line and providing a correct solution to the problems irrespectively of age.

Concerning the experimental group, a commonality is that in all the grades the correct use of the number line leads to success in problem solving. This relationship declines as children get older. However, whereas in the first and the third grade there is a compartmentalization between not using the number line and answer success to the problems, in the second grade the tendency of a number of students not to use the number line leads to success in problem solving.

Finally, in both the experimental and the control group, students who used correctly the number line or did not use the number line in a problem were inclined to follow the same procedure in other problems as well, irrespectively of age.

5. Discussion

This study investigated the use of the number line in additive problem solving by students of 6-9 years of age. A significant proportion (more than 1/3) of the students attempted to use the number line in additive change problems and not all of them could do that successfully. Among the students who attended the usual mathematics curriculum, the inclination to use the number line, as well as the skill to use appropriately the number line in additive problem solving became weaker with age. However, among the students who received an intervention on the use of the number line as an auxiliary representation in coordination with other representations (verbal, symbolic, manipulatives) in additive problem solving, this was not the case. These students’ attempt and competence in using the number line were not only significantly greater relatively to the students of the control group, but remained invariant with age. These findings indicate the positive effects of the experimental program on the ability to use the number line in all three grades. The contribution of the experimental program is further supported by the fact that the students of the experimental group in the first and second grade were more successful in additive problem solving than the students of the control group. However, this was not the case for the third graders. Therefore, although the students of the experimental group developed their ability to use the number line in the context of additive problem solving irrespectively of age, the growth of the ability of solving additive change problems was evident only for the younger students, that is, 6-7 and 7-8 year-old students and not for the older ones. This finding suggests that third grade students at this stage of development do not need auxiliary representations, such as the number line, that are used in teaching and may facilitate the solution process, compared to younger students, as most of the older students are already competent and flexible in solving additive change problems (Elia et al., 2007). This was apparent, on the one hand, from the high success rates of the third grade students of the control group in all the problems, and on the other hand, by the SIA results across the three grades. Specifically, the SIA showed that as students get older the contribution of the correct use of the number line to problem solving success becomes more and more subtle.

The experimental programs in the first and second grade, however, were found to strengthen the relations between the correct use of the number line and successful problem solving or in other words, to stimulate the decompartmentalization between the use of the number line and additive problem solving. This means that the experimental program helped students who previously could not use the number line in the context of additive problem solving to overcome their difficulties in the conversion from the mathematical situation of the problem to the number line (Elia et al., 2007; Gagatsis et al., 2003) and therefore use the number line efficiently as an aid in the solution of the problems. The experimental program in the third grade did not enhance the relations between the use of the number line and successful problem solving, providing further support to the third graders’ use of processes that are less dependent on the mode of representation of the problems.

The SIA showed that a common feature of the various groups of students was the compartmentalization between students’ tendency not to use the number line and their problem solving success. This suggests that students who did not use the number line were more likely to fail in problem solving compared to students who used the number line. An exception was the group of the second grade students who received the experimental program. Although this group of students did not use the number line, they provided correct solutions to the problems. This means that the experimental program helped students to better understand the mathematical structure of the problems and therefore use processes that were less dependent on using the number line as an aid to solve the problems. In other words, the experimental program contributed to a better understanding of the change problems and to the development of problem solving strategies that are independent of representational facilitators. In this group of students, the experimental program probably accelerated students’ problem solving development that would have normally come in a later time.

This study focused on using (correctly or incorrectly) or not the number line in additive problem solving. Valuable information could be given by future research of a more qualitative character that would systematically explore the ways students use the number line in additive problem solving and their role to problem solving success. This study showed that the younger the students, the stronger the relations between the correct use of the number line and problem solving success are. It would be theoretically interesting and practically useful if the exploration of the development of the use of the number line in additive problem solving was extended to younger students, that is, kindergartners. Finally, the absence of a positive impact of the experimental program on the development of problem solving performance and on the links between the correct number line use and success among the third graders is to be noted. Further investigation is needed to explore whether this phenomenon applies for all the older students or varies as a function of mathematical ability or cognitive style (e.g. visualizers or non-visualizers).

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