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# Pre-service teachers' performance on the learning of probability distributions and the role of projects: A multilevel statistical analysis

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**Abstract.** In this paper the problem of pre-service teachers' approaches in solving tasks in probability distributions is studied. Three statistical methods, namely Principal Components Analysis (PCA) that is a symmetrical method, Hierarchical Clustering of Variables and Implicative Statistical Analysis that are dissymmetrical methods, by comparing their outcomes try to clarify the role of projects in relation to pre-service students' performance on the learning of probability distributions. Statistical data were obtained from 98 Greek pre-service teachers from the Western Macedonia University that formed the experimental group and 132 Greek pre-service teachers from Macedonia University that form the control group, enrolled in two basic statistics courses. The control group participates in teacher-centred teaching environment. The experimental group participates in student-centred teaching environment that includes lectures and projects related with tasks concerning the creation of probability distributions. Results indicate the difficulty of control group in defining the difference between discrete and continues distributions. In addition the estimation of the Poisson distribution parameter λ troubled the majority of the control group while the experimental group was dilated to participate in the experiment and showed higher success rate.

**Résumé** Dans cet article nous étudions le problème des approches des professeurs professionnels qui résolvent des tâches impliquant des distributions de probabilité. Nous avons recueilli des données statistiques à partir de 98 professeurs professionnels Grecs de l'université occidentale de Macédoine qui ont constitué le groupe expérimental et de 132 professeurs professionnels Grecs de l'université de Macédoine qui constituent le groupe de commande. Ceux-ci sont inscrits dans deux cours de statistiques générales. Le groupe de commande participe à l'environnement d'enseignement du type centré sur le professeur. Le groupe expérimental participe à l'environnement de enseignement du type centré sur l'étudiant qui inclut des conférences et des projets connexes aux tâches relatives à la création de distributions de probabilité. Les résultats montrent la difficulté du groupe de commande en définissant la différence entre discret et continu pour les distributions. En outre l'évaluation du paramètre λ de distribution de Poisson a préoccupé la majorité du groupe de commande tandis que le groupe expérimental était dilaté pour participer à l'expérience et à l'indice de réussite plus élevé montré.

**Keywords**: Pre-service teachers, learning, probability distributions, projects

## 1. Introduction

There is an increasing recognition that statistical and probabilistic concepts are among the most important unifying ideas in mathematics. Nowadays probability and statistics are part of mathematics curricula for primary and secondary schools in many countries. The reasons for this development are related to the usefulness of statistics and probability for daily life (Chadjipadelis, 2003), its instrumental role in other disciplines, the need for basic stochastic knowledge in many professions, and its key role in developing critical reasoning (Batanero, et al., 2004).Understanding of probabilistic and statistical concepts does not appear to be easy and teachers apply many methods to accomplish their goal, projects included (Anastasiadou, 2010).

Over the last decade, the use of real world projects in introductory statistics courses has increased in popularity. The use of (individual or group) projects for teaching Statistics can, under suitable conditions, help in correcting erroneous perceptions and misapprehensions (Chadjipadelis and Gastaris, 1995). According to Zeleke et al. (2006) real world projects provide students with an opportunity to learn the entire process of a statistical research. The use of projects in an introductory statistics has gained a great deal of attention in the recent statistics reform movement. Many statistics educators consider the use of projects as an authentic assessment tool for assessing student’s understanding of the ‘big picture’ of statistical concepts. According to Ponte (1990), working with projects has the biggest advantage of letting the student free to decide what he or she wants to research, and presenting the subject matter as a tool for developing the project. The use of projects in introductory statistics has been reported in many literatures. For instance, Chadjipadelis (1998a, 1998b) and Chadjipadelis and Primerakis (1998) argued that assigning projects to students sets a frame which encourages interactivity between the instructor and the students, and motivates students to explore the field. Guimaraes et al. (2005) mentioned that projects provide students with a coherent didactical contract, allowing the creation of a secure class climate, to develop autonomy and responsibility as well as their critical sense. In addition Abrantes (1994) supported that important project work allows less motivated students to become involved in the tasks and achieve higher performances, participating actively in the construction of the project.

## 2. Aim and research questions

This study uses three statistical methods, namely Principal Components Analysis (PCA) that is a symmetrical method, Hierarchical Clustering of Variables and Implicative Statistical Analysis that are dissymmetrical methods, by comparing their outcomes in order to shed light on the role of projects in relation to pre-service students’ performance on the learning of probability distributions. Principal Components Analysis (PCA) is a very well statistical method that many researchers used in order to measure students abilities and cognitive capabilities in relation to probability and statistics understanding (Anastasiadou, 2009) while others prefer to used Hierarchical Clustering of Variables or Implicative Statistical Analysis (Anastasiadou, 2004; Anastasiadou & Chadjipantelis, 2008; Anastasiadou, 2008). This study tries by comparing these methods to evaluate their results by examining the differences and the similarities of their outcomes. The following objectives are of highly importance in this study and concern the advantages of using ASI and its relation with the classical statistical PCA method and the benefits of each method separately.

## 3. 3 Method

### *3.1. Principal Components Analysis*

Principal Components Analysis (PCA) is a symmetric method that means that it is based on metric distances and thus the relations between the variables are essentially symmetric. The results of a PCA are usually discussed in terms of component scores and loadings. PCA is mathematically defined as an orthogonal linear transformation that transforms the data to a new coordinate system such that the greatest variance by any projection of the data comes to lie on the first coordinate (called the first principal component), the second greatest variance on the second coordinate, and so on. PCA is theoretically the optimum transform for given data in least square terms.

The calculations of Principal Components Analysis were based on variance- co-variance matrix. The method that was chosen as a choice of missing variables treatment was the method listwise, which excludes from the analysis any case which will reveal a cell, on one or more variables, of the same observed unit, without value, i.e. without information. To define if the sub-scales were suitable for factor analysis, two statistical tests were used. The first is the Bartlet Test of Sphericity, in which it is examined if the subscales of the scale are inter-independent, and the latter is the criterion KMO (Kaiser-Meyer Olkin Measure of Sampling Adequacy, KMO) (Kaiser, 1974), which examines sample sufficiency. The main method of extracting factors is the analysis on main components with right-angled rotation of varimax type (Right-angled Rotation of Maximum Fluctuation), so that the variance between variable loads be maximized, on a specific factor, having as a final result little loads become less and big loads become bigger, and finally, those with in between values are minimized (Dafermos, 2009).

This means that the factors (components) that were extracted are linearly irrelevant (Anastasiadou, 2006). The criterion of eigenvalue or characteristic root (Eigenvalue) ≥1 was used for defining the number of the factors that were kept (Kaiser, 1960, Sharma, 1996, Hair et al., 1995). Essentially, the eigenvalue is the sum of the squares of variable loads on each factor. Actually, it is a measurement of fluctuation quantity that is relevant to the factor.

Model acceptance was based on two criteria: a) each variable, in order to be included in the variable cluster of a factor, must load to it more than 0,50 and b) less than 0.40 to the rest of the factors) (Schene, et al., 1998). Moreover, each factor must have more than two variables. In addition, it was considered, on the basis of common variable Communalities, that the variables with big Communality imply great contribution to the factorial model (Dafermos, 2009). For the statistical data elaboration and check of the questionnaire factorial structure the software S.P.S.S., edition 16 was used (Coakes,et al., 1999).

The evaluation of questionnaire reliability- internal consistency is possible by Cronbach’s index alpha (a) (Cronbach, 1984), which is considered the most important reliability index and is based on the number of the variables/questions of the questionnaire, as well as on the correlations between the variables (Nunnally, 1978). The index alpha (a) is the most important index of internal consistency and is attributed as the mean of correlations of all the variables, and it does not depend on their arrangement (Anastasiadou, 2006).

*3.2. Hierarchical Clustering of Variables and Implicative Statistical Analysis*

Implicative Statistical Analysis is a data analysis devoted to the extraction and the structuration of quasi-implications and originally developed by Gras (Gras & Kountz, 2008). According to Coutourier (2008) Implicative Statistical Analysis establishes the following properties between the variables it handles : 1. relationship between variables that are dissymmetrical, 2. The association measures are not linear and are opades on probabilities, and 3. The user can use graphical representations that follow the semantic of the relationship.

For the analysis of the collected data of this research the Hierarchical Clustering of Variables and Gras’s Implicative Statistical Analysis method was conducted using a computer software called C.H.I.C. (Classification Hiérarchique, Implicative et Cohésitive) (Bodin, Coutourier & Gras, 2000). For the needs of the present study, Similarity, Hierarchical amd Implicative diagram have been released by the application of C.H.I.C. software on the research data. C.H.I.C. , given a set of data, enables to extract association rules. Based on the implication intensity and the similarity intensity C.H.I.C. allows to build two trees and one graph (Coutourier, 2008). The most classical tree is a similarity tree that it is based on the similarity index defined by Lerman (1981) and it is not provide a non-oriented classification. In a similar way, the implication intensity can be used to build an oriented hierarchy tree. More over the implication intensity can be used to define an implication graph. These methods of analysis determine the similarity connections and the implicative relations of the variables. In particular, the hierarchical clustering of variables is a classification method which aims to identify in a set V of variables, thicker and thicker partitions of V, established in an ascending manner. These partitions, when fit together, are represented in a hierarchically constructed diagram (tree) using a similarity statistical criterion among the variables. The similarity is defined by the cross-comparison between a group V of the variables and a group E of the individuals (or objects). This kind of analysis allows for the researcher to study and interpret in terms of typology and decreasing similarity. The clusters of variables which are established at particular levels of the diagram and can be compared to others (Elia & Gagatsis, 2008).

The construction of the hierarchical similarity tree is based on the following process: Two of the variables that are the most similar to each other with respect to the similarity indices of the method are joined together in a group at the highest (first) similarity level. Next, this group may be linked with one variable in a lower similarity level or two other variables that are combined together and establish another group at a lower level, etc. This grouping process goes on until the similarity or the cohesion between the variables or the groups of variables gets very weak. Based on this process, it is evident that the shorter the vertical lines in the diagram the stronger they are. The implicative diagram, which is derived by the application of Gras's statistical implicative method, contains implicative relations that indicate whether success to a specific task implies success to another task related to the former one (Elia & Gagatsis, 2008). In addition the implicative graph encloses implicative relations, which indicate whether success to a specific task implies success to another task related to the former one.

### *3.3. Participants-tasks- design of the project*

The sample consisted by 98 Greek pre-service teachers from the Western Macedonia University and 132 Greek pre-service teachers from Macedonia University assumed as experimental group and control group respectively, enrolled in two basic statistics courses. Both groups follow the second year of their studies during the academic year 2008-09. The control group participates in teacher-centered teaching environment. The experimental group participates in student-centered teaching environment that includes lectures and projects related with tasks concerning the creation of probability distributions. For the analysis of the collected data we use a multilevel statistical analysis, a combination of principal components analysis and implicative statistical analysis.

More specifically, both control and experimental group were taught precisely the same disciplines that connected with basic concepts and probability rules, conditional probability, Bayes law, and finally with discrete and continuous distributions by the same professor. The lectures, the presentation and the formation of the two basic statistics courses, and the demonstrated examples during the courses were exactly the same. The project was consisted of concrete parts given by the instructor, by asking students to create real world probability distributions examples/tasks that are possibly related to students' discipline, future working environment or are of interest to students themselves. More concretely after each lecture and presentation of certain probability distribution the instructor asked the experimental group to create its own examples and present them afterwards to following week. Thus students can be corrected and by that any chance errors and misconceptions can be eliminated. In the beginning of the semester a questionnaire has been given to experimental group with regard to their attitudes on the different teaching strategies of probability distributions. In the final examinations the students of two groups had to answer rightly in the following 6 same tasks. All the tasks were equivalent and the students had in their disposal 3 hours in order to complete their examination.

The test consists of following six exercises given below related to the Binomial, Poisson and Normal distribution.

V1nn: The students rent apartments during their studies. The rents follow normal distribution $N(350, 400^2)$. i)Which is the percentage of the apartments that they rent at least 400euro. ii) Which is the percentage of the apartments that they rent 390euro tops .

V2bb: From the records of a medicines company medicine AB cures t the 30% of the cases for which it was subscripted. If a doctor subscribe this medicine to 4 sick persons find the probability that i) It will be effectual to at least 3 of them, ii) It will not be effectual to anyone.

V3pp: The mean value of calls in the secretary desk of a university department during an hour is 12. Estimate the following probabilities: i) Accept no calls during 6 minutes ii) Accept more than 3 calls during 30 minutes.

Task 4: V4pp. A professor contacts with his students through emails. The emails arrive in the professor’ email box with a tempo of one message every 6 hours and he correspond with one message every 8 hours. According to these information, i) Estimate the probability that the professor takes at least 3 messages during a day, ii) If the professor’ box is empty estimate the probability to receive5 messages during a day and not find the time to correspond in every message the same day.

V5bb: A quite large percentage of young children are overweight. A research that took place in a primary school showed that 60% of the children are overweight. We choose 6 children. Estimate the following probabilities: i) No more than one child to be overweight ii) At least one child to be overweight.

V6nb: Students’ weight (kg) follows normal distribution N(68,100). i) Find the percentage of students that weight above 75 kg. ii) We choose 5 students randomly. Find the probability, where only one weights above 75 kg.

## 4. Examples of the project

Below certain characteristic examples that created by the experimental group of the study are presenting and they are distinguished for their originality electing the imagination and the creativity of students.

More specifically certain Characteristic Examples of Binomial Distribution are mentioned below:

Example B1. 40% of infants in the kindergarten present difficult adaptation during the first days. We choose 4 infants randomly. Found the probability:

i) Precisely 3 have difficult adaptation, ii) No one has difficult adaptation

iii) At least 1 infant has difficult adaptation, iv) Estimate EX and VarX of number of infants that has difficult adaptation.

In this example we can clearly see that pre- service students from the department Educational and Social Policy, future schoolteachers of children with special needs, offend a big problem, as that of adaptation of students in the school. The problem of adaptation that appears, to a great degree, in regular students and even more in relation to students with special needs. Thus, a correct example of binomial distribution was created a very closely connected with the future professional occupation.

Example B2. 15% of infants have outside school activities. We choose 4 infants randomly. Estimate the probability: i) Precisely 3 infants have outside school activities, ii) No infant has outside school activities, iii) Maximum 1 infant have outside school activities, iv) Estimate EX and VarX of number of infants that has outside school activities.

Also this example shows that pre-service students do not create classic examples as those that had been taught, but they discover examples through the educational environment. In this example they are reported infants outside school activities and they place very correct questions concerning Binomial Distribution.

Example B3. 30% of infants watch Mike Mouse on TV. We choose 5 infants randomly. Estimate the probability: i) No one of those infants watches Mike Mouse on TV, ii) At least 2 seeing the Mike Mouse on TV, iii) Maximum 3 infants watch Mike Mouse on TV, iv) Estimate EX and VarX of the distribution.

In this example the students are reported in the most popular children's comic, Mike Mouse developed one very separately pleasant example of binominal distribution.

Example B4. Many children spend their free time seeing moved drawings. 60% of children select least one watches the dogs of Dalmatia. We choose 4 children randomly. Estimate the probability: i) At least 3 children to watch the dogs of Dalmatia, ii) Maximum 1 child watches the dogs of Dalmatia, iii) Precisely two children watch the dogs of Dalmatia, iv) Estimate EX and VarX of the distribution.

Also in this example the students show how many effervescent is their imagination and they invent an example taken from the children's movies that so much Lilliputian creatures love. The all examples in regard to the Binomial Distribution that the experimental group manufactured the are extremely connected with the children world.

It is very encouraging the fact that the students did not manufacture the project examples repeating the formal examples of teaching but electing their creativity manufactured examples through the educational environment and rendering obvious that the of Statistics and Probabilities concepts are not abstract concepts that are founded only in the Mathematics science but they are connected with many sectors of people daily and professional life.

The created examples show that the students comprehended deeply the discrete Binomial Distribution. These examples were not corrected by the professor.

Below certain Characteristic Examples Poisson of Distribution are followed:

Example P1. A student of 12 grade in the Greek nations of examination for university entrance in the course Modern Greek makes 4 errors per page. Estimate the probability in a random selected page: i) At least 3 errors are presented in the page, ii) Maximum 3 errors are presented in the page, iii) Precisely 3 errors are presented in the page, iv) Estimate EX and VarX of the distribution.

In this example the students use data undertaken potentially from their experience in the university entrance examinations that they gave potentially associating them with the school life.

Example P2. A student studies at average 2 hours each day. Finds the probability in one randomly selected day to study: i) Precisely 2 hours each day, ii) No hour, iii) Maximum one day, iv) Estimate EX and VarX of the distribution.

In the same length of wave with the previous example and second one appears, reported in daily reading of students.

Example P3. An infant drinks (4-5years old) on average 3 glasses of milk per day. Estimate the probability in one randomly selected day he drinks: i) Only 1 glass of milk, ii) No glass of milk, a iii) very glass milk, iv) At least 1 glass milk.

From the era of secondary education we pass in elementary education. The students with pleasure created an example of Poisson of Distribution that is reported in the nutrition habits of infants.

Example P4. A infant makes on average three phonological errors at the narration of fairy tale per page. Estimate the probability in a randomly selected narration he makes: i) Precisely 2 phonological errors, ii) No one phonological error, iii) No more from 2 phonological errors, iv) Estimate EX and VarX of the distribution.

From the nutrition habits of infants we pass in the phonological errors which make the infants. A very successful example of Poisson distribution with regard to the likely phonological errors that we meet in the infants.

Finally, are mentioned Certain Characteristics of Normal Distribution:

Example N1. The weight of infants follows normal distribution N (30,102). If we take randomly an infant: i) Estimate the probability that the infant has weight bigger than 30 kg, ii) Estimate the probability that the infant has weight between 28 kg and 40 kg, iii) Estimate the probability that the infant has weight precisely 39 kg.

Example N2. The height of infants follows regular distribution N (130,102). If we take randomly an infant estimate the probability: i) The infant has height bigger than 140 cm, ii) The infant has height between 125 cm and 145 cm, iii) The infant has height precisely 137 cm, iv) Estimate EX and VarX of the distribution

The two more achieved examples that manufactured by the students that are related with the weight and the height of infants are reported in the calculation of probabilities of Regular Distribution

Example N3. The dinner of infants (3 months old) includes custard of fruits, the weight of which follows normal distribution N (30,100). If we take an infant randomly, estimate the probability: i) The infant eats in the dinner more than 32 g, ii) he eats in his dinner between 22 g and 24 g, iii) he eats in his dinner precisely 39 g.

That was another achieved example of calculation of probabilities of Normal Distribution is also the example N3, which concerns in the weight of dinners of infants.

Example N4. The money that the families spend for their children per year in private courses for foreigner languages followed regular distribution N (300, 302). i) Find the percentage of families they spend at least 400e, ii) Find the percentage of families they spend maximum 250e, iii) Find the percentage of families they spend from 280 until 350e.

The fourth and last example of Normal Distribution that reported in the expenses that make the families for the particular courses of foreigner language is an example constitutes a successful manufacture of calculating problem of Probability.

## 5. Results

Results for the experimental group: The results showed differences in the ability to handle every type of given distribution. Table 1a shows the success rates of the experimental group in all types of tasks. It is clear that task V1nn is favoured related to normal distribution, as it presents the greatest success rate. On the other hand, tasks V4pp and V3pp 3 seemed to be the most difficult on the estimation of Poisson distribution parameter λ of the asked Poisson distribution probabilities (4.1% and 12.2% respectively).

**Table 1a**. Total success rates of (a) experimental group in all tasks.

| Tasks | Incorrect | Half correct | Correct |
|---|---|---|---|
| Task 1 (V1nn) | 20 (20.4%) | 25 (25.5%) | 53 (54.1%) |
| Task 2 (V2bb) | 24 (24.5%) | 23 (23.5%) | 51 (52%) |
| Task 3 (V3pp) | 57 (58.2%) | 29 (29.6%) | 12 (12.2%) |
| Task 4 (V4pp) | 69 (70.4%) | 25 (25.5%) | 4 (4.1%) |
| Task 5 (V5bb) | 33 (33.7%) | 51 (52%) | 14 (14.3%) |
| Task 6 (V6nb) | 26 (26.5%) | 51 (52%) | 21 (21.4%) |

**Table 1b**. Total success rates of (b) control group in all tasks.

| Tasks | Incorrect | Half correct | Correct |
|---|---|---|---|
| Task 1 (V1nn) | 86 (65.2%) | 26 (19.7%) | 20 (15.2%) |
| Task 2 (V2bb) | 104 (78.8%) | 18 (13.6%) | 10 (7.6%) |
| Task 3 (V3pp) | 109 (82.6%) | 18 (13.6%) | 5 (3.8%) |
| Task 4 (V4pp) | 119 (90.2%) | 7 (5.3%) | 6 (4.5%) |
| Task 5 (V5bb) | 102 (77.3%) | 13 (9.8%) | 17 (12.9%) |
| Task 6 (V6nb) | 105 (79.5%) | 13 (9.8%) | 14 (10.6%) |

The KMO=0.738>0.60 measure of sampling adequacy (table 2a) showed that the sample data were adequate in order to undergo factor analysis and Bartlett's test of sphericity (sign<0.01) also showed the utility of principal components analysis.

Table 2a shows 2 uncorrelated factors that explain 74.391% of the total data inactivity, and which can be described separately further on.

**Table 2a**. PCA' results of (a) experimental group.

| Experimental group | Factors | | |
|---|---|---|---|
| Tasks-Variables | 1 | | Communality |
| V1nn | 0.881 | | 0.824 |
| V2bb | 0.818 | | 0.676 |
| V6nb | 0.790 | | 0.655 |
| V5bb | 0.680 | | 0.580 |
| V4pp | | 0.914 | 0.868 |
| V3pp | | 0.900 | 0.860 |
| Eigenvalue | 3.302 | 1.116 | |
| Variance Explained % | 43.559% | 30.832% | |
| *Total Variance Explained %* | 74.391% | | |
| Mean score per Factor | 0.546 | 0.219 | |
| Standard Deviation per Factor | 0.391 | 0.321 | |
| Kaiser-Meyer-Olkin Measure of Sampling Adequacy = 0.738 | | | |
| Bartlett's Test of Sphericity: $x^2$=267.737, p=0.000 | | | |

**Table 2a**. PCA' results of (b) control group.

| Control group | Factors | | |
|---|---|---|---|
| Tasks-Variables | F1 | F2 | Communality |
| V1nn | 0.796 | | 0.643 |
| V2bb | 0.793 | | 0.647 |
| V3pp | 0.669 | | 0.471 |
| V6nb | | 0.746 | 0.561 |
| V4pp | | 0.741 | 0.566 |
| V5bb | | 0.700 | 0.493 |
| Eigenvalue | 1.875 | 1.507 | |
| Variance Explained % | 28.911% | 27.446% | |
| *Total Variance Explained %* | 56.358% | | |
| Mean score per Factor | 0.17 | 0.14 | |
| Standard Deviation per Factor | 0.314 | 0.310 | |
| Kaiser-Meyer-Olkin Measure of Sampling Adequacy = 0.621 | | | |
| Bartlett's Test of Sphericity: $x^2$=91.523, p=0.000 | | | |

More specifically, based on experimental group pre-service students as presented by the factor analysis, variables V1nn, V2bb, V6nb and V5bb are loaded mainly on the first axis-factor F1, which explains, following Varimax rotation, 43.559% of the total dispersion. Factor F1 represents the tasks-variables related to normal and binomial distribution. This factor highlights the student way of handling normal and binomial distribution in a discrete way. Tasks V4pp and V3pp are loaded on the second factor (F2), which explains 30.832% of the total dispersion and consist of the tasks that related to Poisson distribution. None of the tasks of the test have loadings on any other factor except of the one mentioned above, and therefore the factors are not interrelated. This suggests that the tasks-variables related to normal and binomial distribution problem data, affected pre-service teachers in such way that they treated the task differently compared with the rest of the tasks.

Figure 1a illustrates the similarity tree of the test. Experimental group' responses to the tasks (V1nn, V6nb, V2bb, V5bb, V3pp and V4pp), which are responsible for the formation of two similarity clusters (i.e., groups of variables).

The first group (Cluster A) consisted of tasks V1nn, V6nb, V2bb, V5bb, which represents students' efficiency in solving the problem tasks, specifically in estimating the asked probabilities and using both normal and binomial distribution types. The strongest similarity, 0.958602, occurs between variables V1nn and v6nb (Almost 1) in the Cluster A. It is suggested that pre-service teachers' employed similar processes to construct a problem solving strategy estimating a simple normal distribution probability and a combination of normal and binomial probability. The similarity of the tasks V2bb and V5bb is very strong (0.920188) too. The total similarity of cluster A ((V1nn v6nb), (V2bb V5bb)) is also very strong (0.819694). The similarity connection of those tasks reveals pre-service teachers' consistency with regard to their performance in evaluating the normal distributions probabilities and binomial distribution probabilities. On the whole, cluster A suggests a close connection between the answers (successful or not) to the normal and binomial distribution.

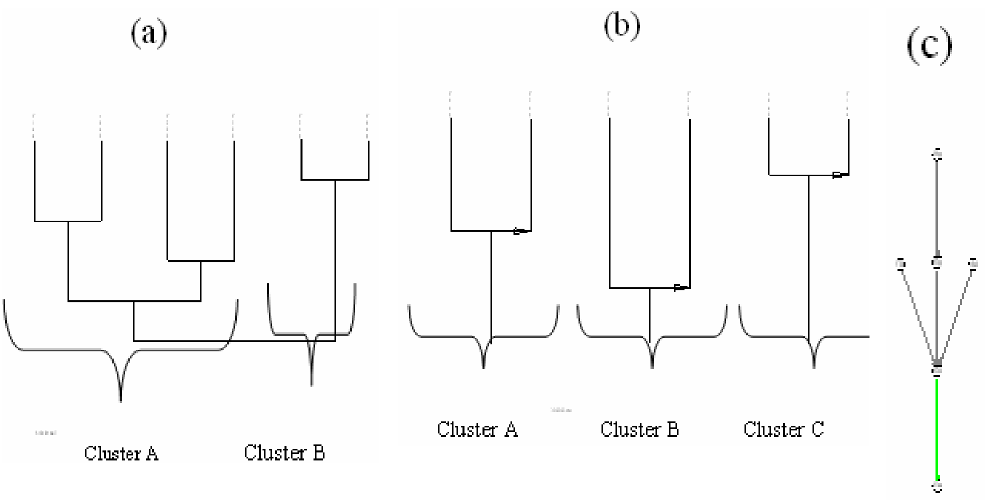

**Figure 1. (a) Similarity tree, (b) hierarchical tree and implicative graph(c) of experimental group.**

The second similarity group (Cluster B) consisted of tasks V3pp and V4pp, it suggests that pre-service teachers’ employed similar processes to estimate the Poisson distributions probabilities. The strongest similarity occurs between variables V3pp and V4pp (0.999564) in the second Cluster that verifiers the above assertion. The similarity Cluster A is disconnected from the other similarity cluster, Cluster B, demonstrating pre-service teachers’ compartmentalized ways of normal and binomial distribution probabilities from Poisson distributions probabilities. The absence of students’ flexibility that means the inability of probability recognition is an indicator of students’ cognitive incompetence. To sum up, the 2 distinct clusters in Figure 1a show that pre-service students carried out conversions by adopting compartmentalised processes based on the kind of the task. This conclusion was resulted with the application both of the analysis to principal components and through implicative statistical analysis. Briefly, the two distinct clusters in Figure 1 show that our pre-service teachers of the experimental group carried out strategies by adopting compartmentalised processes based on the kind of the task (distribution).

In Figure 1b the hierarchical tree shows significant implicative relations between variables of our study, is illustrated. Thus, three groups of implicative relationships are identified. The first group of implicative relations refers to variables concerning Poisson distribution (Cluster A); the second group refers to binomial distribution (Cluster B) while the third one refers to a normal distribution (Cluster C). This result agrees with the findings emerged from the similarity diagram. The formation of these groups of links indicates once again the consistency that characterizes experimental group’ reactions and strategies towards the tasks for different distributions.

The implicative graph in Figure 1c reveals a weak cohesion between the tasks of the test. Nevertheless it is well noticed that all the tasks of the test are related with one or another implication. More concretely, the success in the resolution of task V4pp, that was the most difficult task and had the lowest success rate (4.1%) implies the success of a similar type problem V3pp.

Success in tasks V5bb, V3pp, V6nb involves the success in V2bb. Finally most powerful implicative relation is presented between task V2bb and V1nn, that had also the bigger rates of success, 52% for V2bb and 54.1% for V1nn. In the particular chain the most difficult task is V4pp as it appears also from the table 1a of successs rates while the easier task is V1nn.

The experimental group were asked to fill in a questionnaire in relation to their attitudes towards the difficulty of probability distributions and toward the different teaching methods. The majority of the pre- service teachers of this group, despite the difficulty of the concepts prefer the student-centred teaching environment which includes both lectures and projects in order to have a much greater comprehension. In addition group was delighted to participate to this project because it allowed them experiential active learning rather than passive learning via lectures.

Results for the Control group: Table 1b shows the success rates of the control group in all types of distributions. Control group success varies across the different kind of tasks. Considering the lowest and the highest percentage, this variation decreases with the kind of distribution: normal distribution, 10.6-15.2%; binomial distribution, 7.6-12.9%, Poisson distribution 3.8-5.5%. According to table 1b, tasks connected with Poisson distribution seemed to be the most difficult as it presents the lowest percentages, task V3pp 3.8% and task V4pp 4.5%. Also tasks connected with Binomial distributions, appeared to be difficult as it presents low percentages of 7.6% for task V2bb and 12.9% for task V5bb. Task V6nb which has a combination of a normal and binomial distribution seems that troubles the control group (10.6%). Finally, the task V1nn although it has the higher success rate of 15.2%, still it shows a low percentage.

The KMO=0.621>0.60 measure of sampling adequacy showed that the sample data were adequate in order to undergo factor analysis and Bartlett's test of sphericity (sign<0.01) also showed the utility of principal components analysis (Bartlett) (Table 2b). Through this analysis tasks (V1nn, V2bb, V3pp, V4pp, V5bb and V6nb) were grouped based on the correlation between them.

Table 2b shows 2 uncorrelated factors, that explain 56.358% of the total data inactivity, and can be described separately further on. More specifically, based on Control group’ responses as presented by the factor analysis, items V1nn, V2bb and V3pp are loaded mainly on the first factor F1, which explains, via Varimax rotation, 28.911% of the total dispersion. Factor F1 represents control group’ responses to the tasks V1nn and V2bb. Finally, last on the significance scale for this factor lays the task V3pp. Questions V6nb, V4pp and V5bb are loaded on the second factor (F2), which explains 27.446% of the total dispersion.

In Figure 2a the similarity tree shows how tasks are grouped according to the similarity of pre-service teachers’ solutions. This fact allows the arrangement of Control group’s answers (V1nn, V2bb, V3pp, V4pp, V5bb and V6nb) to tasks into groups according to their homogeneity. Figure 2a also shows that two groups are clearly distinguished. The first group consists of V1nn, V2bb and V3pp variables **(**Cluster A) and represents control group pre-service teachers’ efficiency in solving the problem tasks related to all types of the questioned distributions. The second group includes variables V4pp, V6nb and V5bb variables **(**Cluster B) and concerns the case of solving the problem tasks connected with normal, binomial and Poisson distribution. The strongest similarity occurs between variables V1nn and V2bb (0.999018) (see Figure 2a). Also, the answers of Control group’ to task V3pp come along with variables V1nn and V2bb from Cluster A, indicating their’ coherence in dealing with the corresponding tasks, irrespectively of distribution types. In addition, the similarity of Cluster A is also strong (0.967745).

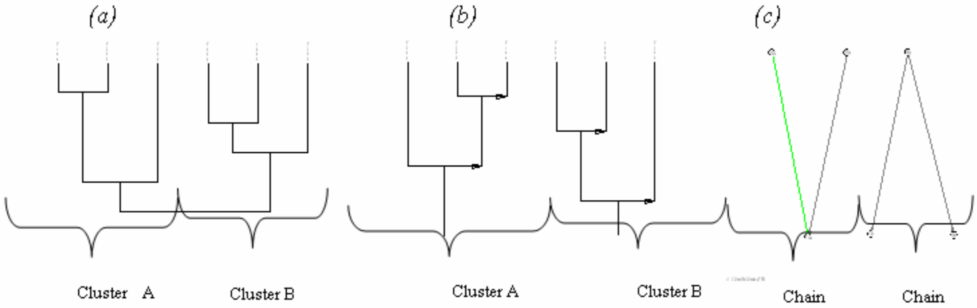


**Figure 2. (a) Similarity tree, (b) hierarchical tree and (c) implicative graph of control group.**

In Cluster B, the strongest similarity occurs between variables V4pp and V6nb (0.99814) and suggests that pre-service teachers of the control group employed similar processes to construct a problem solving strategy in regard with different kinds of tasks. Also, the similarity of the second group is very strong too (0.981403). The hierarchical tree of the control group' answers accords with the similarity diagram as it shown in Figure 2a.

The implicative graph difficult adjustment shows the implicative relations between the variables (Figure 2c). According to this diagram, not all the tasks of the test are connected by implicative relations. The implicative relations indicate whether success of a specific task implies success at another task related to the previous one. The implications represent relations significant at levels of 95% and 70%. This diagram establishes two distinct chains of implicative relations, among the variables based on the type of the task or the type of conversion, namely Chain A and Chain B. The two distinct chains of variables may be seen from the implicative diagram in Figure 2c are (V3pp,V2bb $\rightarrow$ V1nn) (Chain A) and (V4pp $\rightarrow$ V5bb,V6nb) (Chain B). The implicative diagram of the control group' responses is exactly in accordance to the right similarity diagram, and hierarchical tree as shown in Figure 2a and Figure 2b. Pre-service students that succeeded in answering correctly that task V2bb and those answering correctly task V3pp gave in most cases also the right answer to task V1nn (Chain A). Additionally, Pre-service students that succeeded in answering correctly that task V4pp; also succeeded in giving the right answer to the tasks V5bb and V6nb (Chain B).

## 6. Conclusions

The present study by using Components Analysis (PCA), Hierarchical Clustering of Variables and Implicative Statistical tries to shed some light on the role of projects in relation to pre-service students’ performance on the learning of probability distributions by comparing their outcomes on the same sample data. PCA is used to define the latent variables (factors) based on the observed variables-tasks (V1nn, V2bb, V3pp, V4pp V5bb, V6nb) both for the experimental group and control group. More specific PCA results show the structural organizations of cognitive abilities and capabilities of the two groups based mainly on the loadings of the observe variables-tasks. These structural organizations were different of the pre-service teachers of the two groups.

For experimental group variables-tasks V1nn, V2bb, V6nb and V5bb are loaded mainly on the first axis-factor F1 with high loadings (0.881, 0.818, 0.790, 0.680) with eigenvalue 3.302 which explains, following Varimax rotation, 43.559% of the total dispersion, and highlights the pre-service students way of handling normal and binomial distribution in a discrete way (table 1a). Tasks V4pp and V3pp are loaded on the second axis-factor F2 with extremely high loadings (0.914, 0.900) with eigenvalue 1.116 which explains 30.832% of the total dispersion and consist of the tasks that related to Poisson distribution (table 1a). It is worth noting that the results of PCA in relation to experimental group the percentage of Total variance explained is 74.391% that is very high (table 1a).

For control group variables-tasks V1nn, V2bb, V3pp are loaded mainly on the first axis-factor F1 with high loadings (0.796, 0.793, 0.669) with eigenvalue 1.875 which explains, following Varimax rotation, 28.911% of the total dispersion (table 1b) is a combination of the tree kinds of the examined distributions (Normal, binomial. Poisson), with the same kind of question in the two parts of the tasks. Tasks V6nb, V4pp and V5bb are loaded on the second axis-factor F2 with high loadings (0.746, 0.741, 0.700) with eigenvalue 1.507, which explains 27.446% of the total dispersion that is also a is a combination of the tree kinds of the examined distributions.

Hierarchical Clustering of Variables supplement and enrich the above conclusions by presenting pre-service teachers consistency with regard t their performance. For experimental group it is suggested that pre-service teachers’ employed similar processes to construct a problem solving strategy estimating a simple normal distribution probability and a combination of normal and binomial probability (figure 1a). In addition they pre-service teachers’ employed similar processes to estimate the Poisson distributions probabilities. This means that that pre-service teachers of the experimental group carried out strategies by adopting compartmentalised processes based on the kind of the task (distribution). The formation of groups of links in the hierarchical tree indicates once again the consistency that characterizes experimental group' reactions and strategies towards the tasks for different distributions (figure 1b).

Hierarchical Clustering of Variables for control group presents the way tasks are grouped according to the similarity of pre-service teachers' solution and allows the arrangement of their answers to tasks into groups according to their homogeneity and represents their efficiency in solving the problem tasks related to all types of the questioned distributions (figure 2a, 2b).

The implicative graph for experimental group presents weak implicative relations between the all tasks of the test. These implicative relations shows the success in the resolution of task V4pp, that was the most difficult task implies the success of a similar type problem V3pp (figure 1c).

The implicative graph for control group does not present implicative relations between all the variables (figure 2c) but establishes two distinct chains of implicative relations, among the variables based on the type of the task or the type of conversion. Pre-service students that succeeded in answering correctly that task V2bb and those answering correctly task V3pp gave in most cases also the right answer to task V1nn. Additionally, Pre-service students that succeeded in answering correctly that task V4pp; also succeeded in giving the right answer to the tasks V5bb and V6nb.

Components Analysis (PCA), Hierarchical Clustering of Variables and Implicative Statistical do not show only stable and similar results but also different aspects of the conclusions of this research, each one has its advantages and different prospective, but all three work supplementary and enrich the research outcomes and give valuable elements in relation to the role of projects on the learning of distributions probabilities.

Moreover this study deals with a students' new learning experience that also establishes a connection between studies and their future working environment. Another significant aspect is that students are allowed to become involved to creation of their own task along with academic demands. Thus, students can get meaningful learning and achieve higher performance (César and Oliveira (2005)). This study was also an opportunity to show students that an instructor does not know everything and that he or she can learn from students as well. Further investigation of this issue will take place in a future work, with the use of a more extended analysis.

Besides, longitudinal research might reveal new insights about how the effectiveness in using the types of projects grows.

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