WORKING GROUP 4 / GROUP DE TRAVAIL 4

Cultural, political, and social issues / Sujets culturels, politiques et sociales
In the first day, we have started with a presentation of some objectives for the group work. Besides that, two papers regarding Culture Constructions (both in History and for students ranging) were presented and discussed. The first, from Samuel Bello and Karin Jelinek, focused on the school selection process and monitoring of gifted students and analysing language games through Wittgenstein’s concept. The second, from David Guillemette, brought us a discussion of the value history of mathematics in mathematics classrooms from a sociocultural point of view, after some conceptual elements of the theory of objectivation.

Second day was allocated for the papers regarding sociocultural factors in mathematics teaching. Three papers were in this slot. The first, from Vasiliki Chrysikou and Charoula Stathopoulou, was dedicated to the issue of teaching mathematics to students with severe intellectual disability. The focus of this paper was on the sociocultural factors that affect teaching and learning mathematics of three students, and the potential of home-school collaboration to promote students’ active involvement during grocery shopping and money dealing. Second paper, from Filipe Sousa, Pedro Palhares and Maria Luisa Oliveras, tried to analyse the knowledge and the critical thinking level of students from two different cultural contexts (one in a fishing community the other in a more urban area) regarding the mathematical topic of symmetries, finding some slight differences between students of the two contexts in these aspects. Third paper, from Nina Bohlmann and Uwe Gellert, concerning students solving word problems, discussed the claim that standardized testing (re)produces the myth of mathematically illiterate students, but they argue the problem may rely on the standardized test and their designers and not on students’ capability itself.

Third day was devoted to culture and language either as obstacles or resources. First paper was from Peter Appelbaum, Charoula Stathopoulou, Christos Govaris, and Eleni Gana and explored aspects of culture, its role as a resource or as an obstacle, discussing, through their experience in a project regarding the education of Roma Children, how it affects mathematics teaching in the classroom, considering that norms and practices in the classroom are mostly political rather than culturally embedded. The following two papers, second and third of the day were both presenting and discussing aspects of a European Comission funded project. The second paper, from Franco Favilli, attempted to describe a teaching unit, which aimed at overcoming the learning obstacle represented by the contrast between the simplicity of classroom language and the complexity of mathematics language. Third paper, from Hana Moravová, Jarmila Novotná and Andreas Ulovec, focused on the issue of coping with the increasing language diversity and presented some points regarding the implementation of a teaching unit, concluding that teachers, instead of detailed teaching units, what they really need is topics with different cultural origins that they can adapt to suit the needs of their particular group of students.

Fourth paper, from Lisa Boistrup and Eva Norén, discussed the issue of Swedish second language learners and their success in the national tests in mathematics in grade 5. They verified that some schools adapted the administration of the test to give second language students a better opportunity and other schools didn’t and discussed it from an institutional perspective.

Fourth day was dedicated first to the issue of the complexity of mathematics teaching and learning through comparative studies and then to the preparation of the group report. First paper, from Benedetto di Paola, tried to understand the reasons why Confucian Heritage students have been performing better in PISA or TIMMS, by interviewing a Chinese teacher and exploring similarities
and differences between East and West didactical approaches. Second paper, from Andreas Moutsios-Rentzos, discussed the role of perceived proximity in mathematics education as a crucial factor in the determination of the relevance of theoretical and empirical tools in mathematics teaching and learning research, by the consideration of a research project on proof.

For all the papers, we chose to limit the presentation to ten minutes, leaving 5 minutes for a reaction prepared in advance by another participant of the group, and ten minutes more for a generalized discussion. The group felt that this allocation of time and roles was extremely productive generating fruitful discussions and great involvement among the members of the group, as it was depicted in the last moment’s video recording.
Culture is “Bricks, stones and tiles randomly thrown”
(Λίθοι, πλίνθοι και κέραμοι ατάκτως ερριμένα )

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Resumé: Nous explorons les aspects de la culture, son rôle comme une ressource et comme un obstacle, et comment tout cela informe ou désinforme l’enseignement des mathématiques dans la classe comme un espace où les normes et les pratiques sont surtout politiquement plutôt que culturellement intégré. Pragmatologique matériel dérivé de notre expérience à travers le projet, «Education des enfants roms dans l’Epire, Iles Ionniennes, la Thessalie et la Grèce occidentale» illustre nos principaux points théoriques.

Abstract: We explore aspects of culture, its role as a resource and as an obstacle, and how all of this informs or misinforms mathematics teaching in the classroom as a space where norms and practices are mostly politically rather than culturally embedded. Pragmatological material derived from our experience through the project, «Education of Roma children in the Epirus, Ionian Islands, Thessaly and Western Greece» illustrates our key theoretical points.

Introduction

In our paper we are exploring aspects of culture, its role as a resource and as an obstacle, and how all of this informs or misinforms mathematics teaching in the classroom as a space where norms and practices are mostly politically rather than culturally embedded. Pragmatological material derived from our experience through a project regarding the Education of Roma Children in Greece illustrates our key theoretical points.

How is “Culture” a resource, and how is “Culture” an obstacle, in mathematics teaching and learning?

Culture as a Resource

On the one hand, Alan Bishop’s perspective from the 1980s offers mathematics educators a useful notion of “Mathematical Enculturation”: Bishop’s (1988a; 1988b) early work offered a broad, universal, set of intellectual practices that could be taken as “mathematical,” independent of the particular social context: counting, measuring, locating, designing, playing, and explaining. The apparent universality of mathematics results, according to Bishop, from the universality of the adaptive, human goals that define these six types of activities, rather than the a priori nature of mathematical principles. Bishop assumed the cultural universality of the activities, but emphasized the diversity found in symbolic mathematical technologies produced by the activities within varying cultural contexts. If we begin thinking about mathematics education with this in mind, then mathematics education would have to be understood more as a subset of cultural practices within a

1 Data for this paper is taken from the project, «Education of Roma children in the Epirus, Ionian Islands, Thessaly and Western Greece», 2010-2013 (E.U. Lifelong Learning, action code: 304263)
broader culture than as a tool independent of such a culture. That is, mathematics education would be one form of enculturation and acculturation within a culture. (See below, in the next paragraph.) Furthermore, if we take enculturation seriously, then we can ground pedagogical decisions in aspects of culture independent of the fundamental properties of mathematical thinking: Cultural variations in the ways that mathematics is practiced might be significant across teacher, student, family and community cultures represented in a school environment. Here culture provides a lens for comprehending what and how to attend to such variations, as well as a theory for what and how to attend to external to mathematics but impacting upon the experiences of teaching and learning mathematics that are taking place in an educational encounter. One consequence is the notion of a “Multicultural Classroom”, a place where a variety of cultures come together and mix, creating hybrid identities, potential epistemological and linguistic conflicts, and a range of ways of valuing or devaluing mathematics as an academic subject. Whether dilemmas, paradoxes, conflicts, or miseducative, any of these confrontations and complexities would be an indicator of culture as a resource, through which better understanding and potentially powerful pedagogical interventions could emerge. (Appelbaum & Stathopoulou 2014)

The example of the experience of Roma students in the Greek educational system can be used to illustrate these ideas. For example, in the new National Curriculum (NC) and the Cross Thematic Curriculum Framework (CTCF) of the Greek Primary Education System, there is a shift from a the concealing of diversity in earlier policy documents toward a position that characterized the older national curriculum, to respecting cultural and linguistic diversity. A further examination of the national curriculum texts nevertheless points out issues related to a thorough network of limitations that continue to entrap multiculturalism as simultaneously important yet crystallized in stereotypical ways. In particular, the direction of the NC and the CTCF within this model of intercultural learning informed by cultural perspectives is founded on a static definition of culture and cultural differences, which works against its broader goals by promoting the reproduction of a stereotyped perception of the ‘other(s)’ (Govaris, 2015).

Roma pupils entering a Greek school can be understood as highlighting common cultural expectations of a 'normal' student through contrasts with the strengths and weaknesses that Roma youth bring with them into the learning experiences. The image of the "normal" student contains and expresses a set of expectations from teachers, and the educational system in general, especially regarding the knowledge capital and skills to bring a student in order to be able to participate successfully in teaching organized learning processes. The expectation of successful participation requires the ability of individual performance, which necessitates ongoing evaluation processes of the learning process of each student. In such a shape of thinking and student responses, a deterministic understanding among teachers, for example, about the strong influence of family background on students' school performance, in combination with the dominant perception that Roma students come from families with deficient cultural and linguistic capital, plays a catalytic importance for Roma students. Because teachers are encouraged to (only) think in cultural terms, they privilege the perception of different cultural backgrounds, inadvertently depriving Roma students the ability to be carriers of a 'normal' knowledge capital, skills and attitudes that a student needs to make a career without any problems at school, at least within the expectations that people have for these students. Teachers appreciate the Roma students' performance skills as lower than their non-Roma peers, therefore cultivating reduced expectations in terms of school success.

If we start from the premise that there are particular and unique ways of being mathematical, then school mathematics experiences are either experiences of enculturation, in which younger members of the culture are enabled to become more sophisticated, grown-up, members of that culture over time; or acculturation, in which there are power relations between those who are more or less sophisticated and experienced, and those whose identity is more closely affiliated with the dominant, school mathematical culture (and thus have advantages in these power relations); or
both. In this sense, the term “learning” might be suitably replaced by enculturation, acculturation, or both. One consequence, or indicator, of this type of cultural circumstance, is that a subculture of the broader community is apparently used informally or formally as the measure of other cultures for the determination of what is accepted as legitimately mathematical or not, “more mathematical” rather than less, “good mathematics”, and so on; it is this subculture that would be named a “dominant” culture.

In the example of Roma children in the Greek school system, the Roma culture has in particular ways been pre-determined by the expectations that teachers have for Roma children, because they try to be ‘good’, multicultural-aware teachers. Students’ culture of origin has already been assessed by the teachers. So the students are faced with specific assumptions (stereotypical or not) about "what" their culture “is”, and how this culture affects their learning potential. Thus, “the” culture –as a resource or as an obstacle –is needed to be seen in relation to the image of teachers about culture, in relation with what teachers are expecting from different students (Roma), in other words, what teachers expect from the concept of culture itself. In the example of mathematics: in school it is widespread among teachers that the Roma culture is probably conducive to mathematics learning, as trade is at the heart of Roma life, etc. How this expectation determines their attitude to teaching is an empirical question. What is important is that teachers at the level of expectations are likely to perceive positive terms of cultural origin, to correlate positively with the objectives of the teaching of mathematics, searching to find points of contact with those who believe that students know.

At the same time, Roma students can be seen to exhibit the results of what is called stereotype threat: the students’ achievement is adversely affected by their perception of themselves as representatives of all Roma, and thus of the potential of any Roma child to learn. In general, Roma youth in this study have incorporated in their self-image stereotypical representations of significant others (non-Roma) that affect their perceptions of their own potential. We could say that culture might be a resource for these learners; however, in this particular case it becomes more of an obstacle for these youth, who tend to create their own sense of self as a learner in general, and as a learner of mathematics in particular, within a particular school and cultural context.

Culture as an Obstacle

On the other hand, “Culture” as a conceptual tool carries with it a legacy of anthropological history, as well as political implications. It becomes confusing to tease out how mathematics in this broadly universal conception is and is not imbricated in a colonialislt enterprise. The expectation of a universality at some level carries with it some elements of the Western, ideological framework of mathematics as neutral and distant from culture, so that an analysis on this macro level creates a continuity with that perspective, rendering local variations across cultures and subcultures less significant or seemingly irrelevant. This is because anthropology, as a European construct, created “other” non-European cultures as alien others, in its early development, and has been deconstructed to demonstrate that these early, simplistic tools, such as concepts of “culture”, inscribe implicit forms of hierarchy and “epistemicide” –the erasure of any potential awareness or existence of alternatives (Parasekeva 2015). Every mathematical knowledge not historically- and culturally-embedded in Western mathematics is measured and defined in such an approach by the Western mathematics. Indeed, it has become central to the study of mathematics and mathematics education to view mathematics itself as the alien culture, into which learners must enter. In more politically nuance terms, mathematics is seen as a particular cultural construct within the Western, European colonialislt enterprise (Barton, 1996, p.9; Appelbaum, 1995; Davis & Hersh, 1986).

With respect to the experience of the learner, culture thus can become an obstacle to educative experience. When speaking of early years of schooling, enculturation is not really apprenticeship within the child’s own culture since a school curriculum has already decided before the arrival of
the child what and how the child will learn. In this sense, no school experiences are possible to actually describe as “enculturation”. We inherit the term “enculturation” from the title of Alan Bishop’s (1988b) book; yet he himself later used the more appropriate concept “acculturation,” given that every learner grapples with cultural conflicts (Bishop, 2002). A psychological approach that contrasts with the socio-cultural orientation discussed here would make an analogous distinction: enculturation can be understood as acquiring the characteristics of a subculture, in this case, mathematics, through being enmeshed within that culture, while acculturation would refer to “fitting in” to a cultural milieu by emulating the characteristics of those who are already members of that milieu (Kirshner, 2004). Again, the cultural perspective emphasizes how intercultural experiences are always bound up in unequal power relations that serve important roles in the experiences of those involved. We might say that school mathematics serves, through acculturation, important functions in social and cultural reproduction, contributing to the development of “reasonable” people who reason in particular ways, and who are also able to be governed by systems of power and established authorities (Cline-Cohen, 1982; Walkerdine, 1987; Appelbaum, 1995). On the other hand, an awareness of the special vocabulary of school mathematics, and the idiosyncratic ways of working as a student of mathematics that help learners succeed in such a context, offers useful ideas for supporting learners of mathematics who are not yet demonstrating mastery of the material. The particular kind of cultural approach discussed in this context, in this paragraph, distinguishes between the subject knowledge of a course in mathematics and the norms and expectations that teachers of mathematics might have for learners in the course. (Appelbaum & Stathopoulou 2014)

It is common in a typical Greek classroom, for example, that the teacher does not know what sorts of knowledge Roma children bring with them from everyday life into school. In this way, teachers cannot legitimate or exploit the funds of knowledge of Roma children through the school curriculum. A cultural analysis of school experience highlights in this respect the ways that school mathematics does not authorize Roma children’s strengths and weaknesses, making them both invisible to those participating in the school curriculum and to the children themselves.

Sometimes described as “academic literacy,” the norms and expectations for how one works and demonstrates learning in a school context have been shown to be teachable and assimilable when made explicit to the students, and when practiced as explicit ways of working (Appelbaum, 2008; Polya, 1945; Mason, et al., 1985; Brown & Walter, 2005; Cotton, 2010). To use Geertz’s (1973) image of webs of signification, the culture of a mathematics classroom is the tangled interweaving of webs of meaning and interpretation brought to the classroom by teachers, learners, broader characteristics of the social milieu of the school and society, etc. The academic literacy expectations structure the potential interactions that occur in educational encounters, constraining and enabling activity, interpretations, expectations, fears and desires.

In our own recent research in the framework of the project on Roma children education we specifically examine the funds of knowledge that these children bring into the school; curriculum designed with these funds of knowledge in mind becomes far more effective in terms of student learning outcomes. Field work identifies funds of knowledge, and the classroom becomes a ‘third space’/ hybrid space (Moje et al. 20040, where students are encouraged to speak about their knowledge of language and mathematics; the children make connections between everyday life experiences and school mathematics concepts, and inform the general understanding by all students, Roma and non-Roma, of mathematical concepts, through their everyday knowledge (Stathopoulou, et al. 2014).

We notice that, in a typical classroom context, Roma youth cultural differences (measured by comparison with the dominant group, by practical definition a problematic distinction) are considered by teachers as an obstacle—sometimes just their presence is an obstacle. The following
quote from a teacher of primary school is characteristic: “My class is good; for good luck, I have no Roma children, so my work comes easier”. Through our contribution, as part of the project «Education of Roma children…», we tried to respond to issues like this; we tried to transform such obstacles. Our main strategies were: a) research on the spot (on their community of origin) in order to access their funds of knowledge and b) to create spaces, hybrid spaces in the classroom, where discourses from the community meet typical classroom discourses, and the students’ knowledge becomes accepted and in turn transformed in formal school knowledge, creating new academic opportunities for students. What emerged here, something stronger than cultural issues, are better characterized as political issues. Curricula, teacher training, and the broader educational parameters depict broader policies regarding education and students of minority backgrounds. (Stathopoulou, et al. 2014)

**Culture as an Analytical Tool**

Approaches to mathematics education and culture establish forms of reality and common sense through the application of distinctions, often without any clear attention to these distinctions. In this way, these approaches create implicit—sometimes explicit—assumptions about dichotomies such as in-school and out-of-school learning, formal and informal education, teaching and learning, mathematics and culture, student or teacher identity and mathematics, and so on. For example, if we carry out a project or teach a school mathematics lesson trying to make it more meaningful and relevant to some students in the classroom by noting that they are members of a non-mainstream subculture, we are reducing the uniqueness of each individual to a set of stereotypical assumptions from a generic caricature of this subculture. Each individual may or may not fit this set of assumptions. Indeed, most of the learners in this situation are members of multiple subcultures at the same time, and are in any given moment having experiences that resonate with cultural habits and dispositions from more than one of these subcultures. As researchers, mathematics educators wish to use categories based on cultural distinctions to analyze situations, because this seems like the only reasonable, common-sense way for us to make sense of the setting and the people in it. Yet, as soon as we use these distinctions, we are already aware of the variations within any given group that seem more extreme than differences between groups. And as soon as we try to take into account the variations within any given group, we are already aware of the ways in which these variations are inadequate to capture the variations within any one individual within that group. That is, borders between categories are permeable, so that, to keep this simple, say, a Catholic, Latina girl in a Chicago classroom may or may not be having an experience consistent with what her teacher might expect of a learner recently relocated from New Jersey with her Cuban-American, Jewish father, working in a small group with her Chicano best friend and a recent immigrant from Albania. In other words, each learner is determined to some extent by the cultural contexts that are part of their life; yet, as individuals, learners have a repertoire of behaviors and ways of making meaning out of experience that are specific to them.

**Conflicts as Analytical Sites**

It is increasingly challenging to exploit all resources available in the interests of mathematics learners, given the myriad of types of resources and locations of these resources; at the same time, the “resources” take on different meanings depending on one’s cultural perspectives, one’s understanding of others’ cultural perspectives on these resources, and one’s analysis of the conflicts that may or may not emerge. In this sense, again, culture is itself a resource for pedagogical theory. Conflicts exist in most discussions of education broadly conceived about the role of mathematics in the lives of children and adults — both in the present and in their futures, in terms of both individual and societal needs. These conflicts and associated confusions regarding the role(s) of mathematics are made more complex by the expectations for mathematics and mathematics learning, more or less culturally determined, that meet each other in educational encounters. Sometimes, mathematics is taken as a culture itself; funds of knowledge pedagogies offer ways that
home cultures can be appropriated as resources; ethnomathematical critiques and approaches to teaching and learning become resources for changing practices to resonate with cultural expectations. These are not separate, analytic categories, but mutually informing strands of interwoven discourse.

**Culture as a Term in Discourse about Mathematics Education**

As a tentative conclusion, we suggest using the term “culture” to refer to aspects of cultural contexts, and more specifically, aspects of culture related to learning and knowledge, rather than to speak of “culture” in general. We do this to avoid the discontinuity that appears at school through dichotomies of formal and informal learning, distinguished by the role of a designer or evaluator of learning experiences not present in the learning context that is necessary for “formal” learning to take place. Culture, more broadly, is both “an historically transmitted pattern of meanings embodied in symbols, a system of inherited conceptions expressed in symbolic forms by means of which men communicate, perpetuate, and develop their knowledge about and their attitudes toward life” (Geertz 1973a, p. 89), and those “webs of significance” people themselves spin” (Geertz, 1973b, p. 5). Culture for mathematics education is a collection of bricks, stones and tiles randomly thrown, so that, after the fact we can see some mosaics and patterns and walls and buildings and surfaces and works of art that seemed to have been created from somewhere, but are, in the sense of Michael Polanyi (1974), a mere happenstance of our human qualities of perception: the tiles, stones and bricks come from the legacies of dominant cultures, colonialism, and local traditions; the magnificent works of art are created by the humans who pick up the pieces and place them in juxtaposition.

**REFERENCES**


Lorsque les hautes compétences en mathématiques ne sont que des formes de vie

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Abstract: This is an issue that introduces a research that aimed to analyze the selection process and the school educational monitoring of students so-called "gifted", by understanding of ways in which “language games” and certain behaviors valued by school are in relation with their life forms. Methodologically, we use the Wittgenstein’s concept of language games as well as “relations of power” and “games of truth” both based on Foucault to create the notion of "games of power-language" from which we make our analyses by highlighting that the subject so-called "Gifted student" is an effect, an invention of discursive practices. We also conclude that process of identification and selection of gifted students are based on strategies of comparison and ranking.

Résumé : Il s’agit d’un texte qui se réfère à une recherche qui a eu pour but d’analyser les processus scolaires de sélection et de renforcer le suivi éducatif des enfants qui sont appelés aujourd’hui au Brésil « porteurs de hautes compétences/surdoués », afin de comprendre les façons dont les jeux de langage et les conduites valorisés par ces processus sont en rapport avec leurs formes de vie. Méthodologiquement, nous utilisons les notions des jeux de langage chez Wittgenstein ainsi que des relations du pouvoir et des jeux de vérité chez Foucault pour créer le concept de « jeux de pouvoir-langue » à partir duquel nous faisons nos analyses en mettant en évidence que le sujet/porteur de hautes compétences n’est que l’effet de pratiques discursives et que les processus d’identification et de sélection sont tout d’abord des processus de comparaison et de classement.

Présentation

L’objet abordé ici se rapporte à la discussion sur « les questions culturelles, politiques et sociales » introduite lors du CIEAEM-67, ce qui concerne les hautes compétences pour l’apprentissage des mathématiques en mettant en question si elles sont effectivement liées au développement cognitif.

Nos idées sont basées sur une recherche de doctorat dont le but était d’analyser les processus scolaires de sélection et de renforcer le suivi éducatif des enfants qui sont appelés aujourd’hui au Brésil « porteurs de hautes compétences », afin de comprendre les façons dont les jeux de langage et les conduites valorisées par ces processus sont en rapport avec leurs formes de vie.

Méthodologiquement, il s’agit d’une recherche qui est appelée au Brésil « poststructuraliste » puisque cette perspective permet que l’on s’interroge sur les idées propres au structuralisme, en particulier sur la fonction du langage et l’essentialité, la rigidité et la fixité de leurs significations, ainsi que l’histoire des différentes façons d’être sujet².

² Il faut souigner que grande partie des études sur le thème des hautes compétences l’exploitent sous une perspective cognitiviste de l’apprentissage et du développement. Cependant, ce travail vise à discuter les hautes compétences en mathématiques en se basant sur une théorie sociale qui les discute sous une perspective de relations de pouvoir et de...
Ainsi, tenant compte des notions « wittgensteiniennes » des jeux de langage en rapport avec les formes de vie et les concepts « foucaudiens » de pratique discursive, de relations de pouvoir et de jeux de vérité, nous créons le concept de « jeux de pouvoir-langage ». Ce concept nous a permis d’analyser comment des enseignants de quelques écoles de la ville de Porto Alegre identifient, comparent et classifient leurs élèves en tant que sujets/porteurs de hautes compétences. Pour ce faire, nous accompagnons les activités de sélection développées par les spécialistes de la SIR/AH puis nous réalisons des entretiens avec non seulement trois élèves dits « porteurs de hautes compétences » en mathématiques mais aussi leurs enseignants.

Une fois les mouvements analytiques achevés, il nous a été possible de vérifier qu’il y a eu un déplacement et une réactualisation de la signification du terme « surdoué » par celui-ci appelé « de hautes compétences ». En même temps, on va confirmer que les sujets de hautes compétences sont produits par l’observation et la comparaison attentive et précise des performances des élèves mises en évidence ainsi et qui sont valorisés par les enseignants et par l’école. (Jelinek, 2013b; Jelinek et Bello 2014). Le développement des ces idées est présenté ci-dessous.

**De surdoués vers porteurs de hautes compétences**

Les hautes compétences seront ici considérées comme des pratiques discursives (Foucault, 1995) car par des règles et régularités qui leur sont propres, elles définissent et modélisent des actions, des conduites en produisant des subjectivités, des identités, c’est à dire des formes-sujets, par lesquelles on est visible, dicible, déchiffrable. Ainsi, la forme-sujet « porteur de hautes compétences » n’est pas seulement une variation du nom de l’anciennement dénommé surdoué, mais un ensemble de pratiques et comportements qui y sont liés. Il convient de noter ici que le langage est plus qu’un simple acte de la parole ou de l’écriture, il implique des façons de penser et d’agir en rapport avec nos formes de vie. Par la création de la notion de « jeux de langage », Wittgenstein (2008) apporte l’idée d’une normativité du langage par laquelle on constitue une réalité et ses significations dans certaines situations d’utilisation (Bello et Régnier, 2014). Le langage est le monde que nous habitons et que nous « pratiquons », à la fois instrument et construction (Paltrinieri, 2011).

De la même façon, les notions de relations de pouvoir-savoir et de jeux de vérité chez Foucault (2013, 2006, 2003) aident à comprendre comment on est devenu forme-sujet à partir des significations linguistiques créées. Il est important de remarquer que pour Foucault (2013, p. 146), le pouvoir ne doit pas être compris en tant que système oppressif, mais comme un ensemble de relations dont le but est d’agir ou de chercher à agir sur la conduite de l’autre. « (…) c’est lorsqu’il y a un rapport entre deux sujets libres et qu’il y a dans ce rapport un déséquilibre tel que l’un peut agir sur l’autre et que l’autre est ‘agi’, ou accepte l’être ». Selon Jelinek (2013a), pendant les années 1960s et 1970s les significations données aux *dits* surdoués étaient en rapport avec les domaines scientifiques (mathématiques et linguistiques) et artistiques (musique et arts visuels) et valorisaient les qualités telles que la performance numérique, la mémoire, le raisonnement logique-mathématique, la vitesse de la pensée, toutes capables d’être mesurées par des tests de QI. Il y avait aussi une caractérisation liée à la vitesse de maturation physique et intellectuelle de l’individu. Au cours des années 1990s, les changements quant à la compréhension d’intelligence comme ceux proposés par Gardner (2001) en rapport avec l’environnement social et culturel des individus nous font croire en l’existence d’habilités et de performances différentes quand l’individu est *porteur* productions identitaires.

Le processus de sélection développé par les spécialistes de la SIR/AH se compose d’une fiche d’auto-désignation devant être remplie par les collègues des élèves de hautes compétences aussi que d’une fiche d’éléments d’observation des activités de classe devant être remplie par leurs enseignants. Ces processus comportent également des entretiens.

SIR/AH est l’abréviation de « Sala de Integração e Recursos para Alunos com Altas Habilidades/Superdotação » - Salon d’intégration et Ressources pour élèves de Hautes compétences/ surdoués.

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d’au moins une de ces habilités. Ce changement de signification apporte au sens de porteur de hautes compétences plusieurs autres qualités qui devront être observées, telles que l’esprit de leadership, d’initiative et de collaboration.

Pour Jelinek (2013b), la compréhension contemporaine de « porteur de hautes compétences » n’est plus seulement liée au domaine des mathématiques, mais bien aux comportements sociaux et économiques souhaitables qui fonctionnent comme normes de classification et de comparaison des sujets scolaires. Les enfants dits de hautes compétences se gênent très facilement et font acte de distraction et de frustration à certains moments, ils résistent à la répétition par cœur, ils possèdent des pensées critiques et sont excessivement actifs, ce que l’on peut comprendre comme hyperactivité. Ce concept de « hautes compétences » devient un discours qui s’oppose à celui des difficultés de l’apprentissage.

Foucault (1995) a appelé discours les pratiques au milieu de relations de pouvoir, elles produisent des énoncés, établissent des vérités et des connaissances à un moment donné historique. Pour le philosophe, les pratiques discursives ne se réfèrent pas à l’activité d’un sujet en soi, mais à l’existence objective et matérielle de certaines règles par lesquelles ce sujet est constitué et/ou produit dans les relations sociales. La pratique discursive des hautes compétences permet non seulement que les identités se fixent, mais aussi que s’établissent des différences entre les individus. L’identité “porteur de hautes compétences” n’est pas seulement une production de sens, mais aussi une manière d’établir des appartenances à des formes-sujet déterminées.

Il existe ainsi, dans une pratique discursive des hautes compétences, un jeu de pouvoir-langage, c’est-à-dire un jeu linguistique qui la désigne, lui donne un sens et un jeu de pouvoir qui l’établit.

**Le sens des hautes compétences à l’école**

D’après le sens des jeux de langage ainsi que les formes-sujet, on peut s’interroger : de quelles manières les pratiques des porteurs de hautes compétences sont-elles en rapport avec leurs formes de vie?

Les chercheurs des hautes compétences sont unanimes en ce qui concerne le rapport entre le contexte dans lequel l’enfant vit et ses pratiques au moment des évaluations (Guenther, 2000, 2006; Silver, 2010; Sim-Sim, 2005; Winner, 1998), tant et si bien qu’un enfant sera porteur de hautes compétences en accord avec un groupe social et à un moment daté et situé. Autrement dit, un enfant possède de hautes compétences dans une forme de vie spécifique et non dans d’autres. Si l’intelligence est culturelle, alors les hautes compétences aussi.

Les pratiques d’identification des porteurs de hautes compétences dans lesquelles les enfants se distinguent vont mettre en évidence des situations, des objets, des savoirs, des conduites qui leur sont familières ou en rapport avec le quotidien. Autrement dit, les situations que les enfants doivent résoudre ou auxquelles ils doivent faire face ont un air de famille avec leurs formes de vie.

A notre avis, la fiche d’observation utilisée par les spécialistes à l’école pour identifier les enfants/porteurs de hautes compétences ne font que valoriser certaines conduites spécifiques qui sont significatives et importantes au point de vue de l’école.

Il est intéressant d’observer que parmi les 27 éléments dont se compose la fiche d’observation du professeur, 20 d’entre eux se basent sur des caractéristiques comportementales, les autres servent à comparer les individus de la classe, ce que nous pouvons identifier par l’expression *meilleurs en*:
**Indiquez pour chaque élément les deux élèves de votre classe, garçon ou fille, qui, à votre avis, présentent les caractéristiques suivantes :**

| 1) | Les meilleurs de la classe dans les domaines du langage, de la communication et de l’expression : |
| 2) | Les meilleurs en mathématiques et sciences : |
| 3) | Les meilleurs dans les domaines de l’art et de l’éducation artistique : |
| 4) | Les meilleurs dans les activités extracurriculaires : |
| 5) | Les plus locaces, causeurs : |
| 6) | Les plus curieux, intéressés, questionneurs : |
| 7) | Ceux qui participent le plus et qui sont toujours présents, à l’intérieur et à l’extérieur de la salle de cours : |
| 8) | Les plus critiques envers les autres et eux-mêmes : |
| 9) | Ceux qui ont la meilleure mémoire, qui apprennent et retiennent facilement : |
| 10) | Les plus persistants, engagés, qui terminent ce qu’ils font : |
| 11) | Les plus indépendants, qui commencent leur travail et le font seuls : |
| 12) | Ceux qui s’ennuent le plus, les plus désintéressés, mais pas nécessairement en retard : |
| 13) | Les plus originaux et créatifs : |
| 14) | Les plus sensibles aux autres et les plus gentils avec les camarades de classe : |
| 15) | Ceux qui s’occupent du bien-être des autres : |
| 16) | Les plus sûrs d’eux-mêmes : |
| 17) | Les plus actifs, perspicaces, observateurs : |
| 18) | Les plus capables de penser et de tirer des conclusions : |
| 19) | Les plus sympathiques et les plus aimés par leurs camarades de classe : |
| 20) | Les plus solitaires et ignorés : |
| 21) | Les espions, drôles, chambrates : |
| 22) | Ceux que vous considérez les plus intelligents : |
| 23) | Ceux qui ont la meilleure performance en sports et exercices physiques : |
| 24) | Ceux qui se distinguent dans les activités manuelles et motrices : |
| 25) | Ceux qui donnent des réponses inattendues et pertinentes : |
| 26) | Ceux qui sont capables de diriger et de transmettre leur énergie pour encourager le groupe : |
| 27) | Y a-t-il dans votre classe un enfant avec d’autres talents spéciaux ? Lesquels ? |

**Table 1. Fiche d’observation utilisée par les spécialistes à l’école.**

“Ceux qui ont une meilleure mémoire, qui apprennent et retiennent le plus facilement, qui sont les plus indépendants, qui commencent leur travail et le font seuls”, ces éléments sont parmi ceux qui identifient un possible **porteur** de haute compétences en mathématiques et qui, bien que nous ayons affaire à un domaine dit exact, se réfèrent aussi à la conduite des individus.

Si nous regardons attentivement les éléments énumérés sur la fiche, nous voyons que les haute compétences en mathématiques ne sont pas attribuées par leurs aspects cognitifs, mais se naturalisent par les aspects comportementaux. Et nous pouvons ajouter : que les vérités à la base de ces pratiques ne sont que des jeux constructeurs d’une objectivité pour une subjectivité – celui dit **porteur** de haute compétences en mathématiques.

Ce que l’on cherche à élucider, c’est que ces conduites, observables dans le cadre des pratiques scolaires, sont des manifestations du respect des règles des **jeux de pouvoir-langage** qui constituent et sont constitués par ces pratiques. Des évidences de ceci ont aussi pu être identifiées à partir des entretiens réalisés dans les écoles tout au long du travail sur le terrain.

En réfléchissant sur certaines formulations des professeurs – comme par exemple “il a toujours fait des observations intéressantes, qui nous ont surpris en classe, on ne savait pas où il allait chercher ça...” – je me risque à dire que le discours est explicite, car il n’a ni significations occultes ni savoirs secrets, bien au contraire, il précise ce qui doit être accepté,– dans ce cas, que les haute capacités sont en rapport avec les conduites exprimées par les autorités scolaires. Et c’est ceci qui doit être tenu pour acquis, c’est-à-dire comment les règlements qui supportent ces pratiques sont

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5 Pour faciliter la réflexion proposée ici, il faut se rappeler que pour être désigné **porteur** de haute compétences en mathématiques, l’individu doit être signalé dans au moins trois des éléments suivants : 2,9,11,18 et 22.
comportementaux et servent de critères pour distinguer les états ou les comportements adéquats ou non.

On ne peut pas s’empêcher de remarquer que, même s’il s’agit de hautes compétences en mathématiques, les caractéristiques relatives à ces pratiques ne sont pas exploitées en profondeur par les professeurs et que les conduites sont explicites à l’extrême. Un exemple de ceci est la valorisation de ces élèves qui “donnent des réponses inattendues et pertinentes” dans le milieu scolaire, comme nous avons vu dans l’exemple donné. En considérant que la perspective de cette recherche a été les pratiques, il est possible de constater que le sujet des hautes compétences devient ce sujet seulement par sa conduite.

En ce sens, il ne vient pas comme un sujet essentielisé, a partir d’une condition innée, mais se produit comme sujet à partir de ses conduites qui fabriquent sa propre identification. Telles conduites agissent à partir de règles, elles aussi constitutives et constituantes des jeux de pouvoir-langage.

L’expression largement utilisée dans la fiche d’observation du professeur, est “meilleurs en” comme nous pouvons le voir à partir des éléments “les meilleurs de la classe dans les domaines du langage, de la communication et de l’expression”, “les meilleurs en mathématiques et sciences”, “les meilleurs dans les domaines de l’art et de l’éducation artistique”, “les meilleurs dans les activités extracurriculaires”, “ceux qui ont la meilleure mémoire”, “ceux qui ont la meilleure performance en sports”. Tels éléments associés aux caractéristiques énoncées par les professeurs, comme “c’est un élève distingué” ou “il était déjà alphabétisé quand il est arrivé en seconde, ce qui est une rareté à l’école”, ou encore, “il s’est toujours distingué”, nous fait penser que la sélection des porteurs de hautes compétences se base fortement sur une norme comparative.

Bien que des caractéristiques d’individualisation soient valorisées et constituent en quelque sorte la plupart des questions utilisées pour définir et orienter un sujet porteur de hautes compétences, il ne suffit pas seulement de dire ceci ou cela. Il faut établir des moyens et des formes de comparaison, sous la forme de relations d’altérité, où interviennent des jeux de pouvoir-langage mêlés à des pratiques de sélection qui mettent en évidence des mesures et des postures d’ordres et de règles méritocratiques.

D’ailleurs, si auparavant un test de QI était employé pour comparer la performance d’un individu avec une « norme » de caractère théorique, aujourd’hui la fiche d’auto-désignation ne fait que comparer les individus les uns avec les autres.

Si cette opération de comparaison entre les sujet est une condition vitale pour que les jeux d’identification des conduites puissent fontionner, cette même fiche établira aussi une pratique de comparaison du sujet avec lui-même, puisqu’il devra réfléchir sur les différentes caractéristiques qui lui sont propres. Le sujet élève, dans ce cas, est amené à se reconnaître comme un sujet bon ou très bon, au moins dans une des catégories : mathématiques, arts, gymnastique, théâtre, sciences, créer des histoires, danse, leadership, lire, faire des recherches, sports, créativité, écrire, musique, amitié ou dans d’autres domaines que cite l’élève.

Le résultat de l’application de ces deux fiches est l’expression de la façon dont les jeux de pouvoir-langage saisissent des conduites des sujets scolaires en les transformant en conduites propres des dits « porteurs de hautes compétences ».

Par ailleurs, il est important de remarquer que d’autres formes de vie sont négligées ou soumises car les jeux de pouvoir langage les classent ; en fin de compte la valorisation donnée aux conduites des élèves considère des normes, des modèles et des pratiques qu’une société pense être les formes de vie scolaire adéquates et de hautes compétences. Pour conclure, nous pouvons dire que le
sujet/porteur de hautes compétences est le sujet des jeux de pouvoir-langage. C’est la forme-sujet qui résulte de la soumission d’un individu aux règles du langage, du pouvoir, des vérités et des savoirs scolaires et par lesquelles il entre en mouvement et se rapporte à lui-même et aux autres, par lesquelles il construit une subjectivité pleine de désirs, d’intentions, de connaissances et de valeurs qui sont propres à sa forme de vie.

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Word Problems: Resources for the Classroom?

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Abstract: In recent discussions on students’ practices for solving word problems, some researchers argue that the many students who fail on these problems do not understand the nature of the problem situation. Other researchers, on the contrary, point to the requirement of utilizing the appropriate degree of realistic considerations which is related to the difficult decision of how much and what part of the reality seems to be relevant according to the problem designer and the responses coding scheme. Apparently, the issue of the relevance of the problem situation is a main obstacle for generating the expected solution. In the specific context of standardised testing, the student’s achievement of translating the word problem text correctly into a mathematical operation and of performing the operation correctly is often devalued by narrow expectations. Therefore, standardised testing produces a myth: the mathematically illiterate student. In this paper, we use classroom video data to substantiate the claim that the issue of the students’ failure, or difficulty, of taking the problem situations into account ‘correctly’ might be more relevant for those researchers and assessment designers occupied with standardised testing than for students solving word problems in everyday mathematics classrooms.

Résumé: Au cours des dernières discussions sur les pratiques des élèves pour résoudre les problèmes de contexte, certains chercheurs soutiennent que les nombreux étudiants qui échouent sur ces problèmes ne comprennent pas la nature de la situation du problème. D'autres chercheurs, au contraire, indiquent l'obligation d'utiliser le degré approprié de considérations réalistes. Ce degré est lié à la décision difficile de combien et quelle partie de la réalité semble être pertinente selon le conceputeur du problème et le système de codification des réponses. Apparemment, la question de la pertinence de la situation du problème est un obstacle principal pour générer la solution attendue. Dans le contexte spécifique des tests standardisés, le rendement de l'élève de traduire le problème de contexte correctement en opérations mathématiques et d'exécuter l'opération correctement est souvent dévalué par les attentes étroites. Par conséquent, les tests standardisés produisent un mythe: l'élève sans culture mathématique. Dans cet article, nous utilisons les données vidéo pour étayer l'affirmation selon laquelle la difficulté des étudiants de prendre les situations des problèmes «correctement» en compte pourrait être plus pertinente pour les chercheurs et les concepteurs d'évaluation occupées avec des tests normalisés que pour les étudiants à résoudre des problèmes de contexte dans les classes de mathématiques de tous les jours.

Introduction

In recent discussions on students’ practices for solving word problems, one mathematical task has been used paradigmatically to exemplify the intricacies and complexities of this kind of didactic material for assessing the students’ mathematical knowledge (Cooper 1992, 2004, Gates and Vistro-Yu 2003, Gellert 2009, Murphy 1995, Palm 2009). The task has originally been used by the British Schools Examinations and Assessment Council (SEAC 1992) and it reads as follows: “This is the sign in a lift in an office block: This lift can carry up to 14 people. In the morning rush, 269 people want to get up in this lift. How many times must it go up?” This word problem is structurally similar, although less militaristic, to older assessment tasks used in the National Assessment of Educational Progress in the U.S.A., for instance, as discussed in Carpenter et al. (1983): “An army bus holds 36 soldiers. If 1128 soldiers are being bussed to their training site, how many buses are needed?” While some researchers argue that the many students who fail the assessments by providing answers like 19.21 to the lift item, or 31.33 to the bus item, or ignoring the remainder, do
not understand the nature of the problem situation, others point to the requirement of utilizing the appropriate degree of realistic considerations, which Gates and Vistro-Yu (2003) describe as “the goldilocks principle of the reality behind mathematical problems – not too much, not too little – just enough” (p. 53). In this second perspective, for many students the complexity of the word problem is related to the difficult decision of how much and what part of the reality of queuing and transport seems to be relevant according to the problem designer and the responses coding scheme. Apparently, the issue of the relevance of the problem situation is a main obstacle for generating the expected solution.

In this paper, we use classroom video data to substantiate the claim that the issue of the students’ failure, or difficulty, of taking the problem situations into account ‘correctly’ might be more relevant for those researchers and assessment designers occupied with standardised testing than for students solving word problems in everyday mathematics classrooms. We argue that recent decisions in many countries to react on the issue of the ignored problem context—which is an issue of standardised testing schemes—by including more “realistic type” word problems in the mathematics curriculum does a disservice to the teachers and learners in mathematics classrooms who intend to teach and learn mathematics, and not how to correctly respond to dysfunctional assessments of mathematical literacy. Rather than being empirical or theoretical, our paper is related to educational policy and the mathematics curriculum.

The Myth of the Mathematically Illiterate Student

In a teaching experiment, which we describe in Bohlmann, Straehler-Pohl and Gellert (2015), a class of sixth-graders and their mathematics teacher engage with the lift problem and with context variations of the same mathematical operation $269 \div 14$. While the numerical facts are maintained, the context variations put the task in situations (road crossing, cable car), in which it is more or less probable that “real” people in “reality” would wait until it is their turn to cross the road, to enter the lift, or to take the cable car. The students solve the word problems and produce posters, which present their solution. The teacher then discusses with the students the varying degrees of fictitious behaviour that the tasks assume of the people involved. The students seem more and more confident to add what they consider as distortions of reality. At the end of the lesson, the teacher confronts the students’ criticism of unreal problem situations with their mathematical solutions presented on posters. Note that Luke is a special student. It could be reconstructed from the video data that, on several occasions, the teacher positions him as the best student in class.

Teacher: You have now mentioned quite a lot of things that do not fit to the task when you say, this is unrealistic. And yet all of you have made the poster. And found solutions. (3 sec.) Why didn’t you consider all these things? (2 sec.) Tony?

Tony: Because we never thought about it before?

Teacher: Luke?

Luke: Because we are supposed to solve the task and are not supposed to think about it.

Teacher: Again, louder!

Luke: We are supposed to solve the task like always, and we are not supposed to think about the actual situation, but simply find out the maths problem and solve it.

The teacher then reconfirms Luke’s final statement. Apparently, successful participation in this mathematics class requires the students to act according to the strategy described by Luke: When
confronted with a word problem, find out the expected mathematical operation and calculate the result. Don’t distract yourself by thinking of what could happen if the problem situation were “real”.

There is sufficient evidence to assume that people are able to deal with numbers correctly in everyday situations, independent of their proficiency of coping with school mathematical word problems (Lave 1988, Nunes, Schliemann and Carraher 1993, Rogoff and Lave 1984). Thus, as long as the students’ preparation for everyday life matters is concerned, Luke’s strategy does not seem to be too harmful. But what happens in forms of assessment in the classroom? Isn’t this strategy inappropriate because a final answer of 19.21 is not fully meeting the teacher’s expectations? It can be argued that the deviation of the answer “19.21” from the expected result, “20”, is remarkable. However, in a context of everyday teaching, including the typical tests that teachers prepare for their students, “19.21” is rather close to what the teacher might expect: In order to reach “19.21”, a student needs to correctly decide on the mathematical operation (division) that the problem situation requires—which is not trivial a task—and to correctly calculate $269 \div 14$. In an everyday class context, “19.21” is probably credited with, say, three out of four points. Thus “19.21” is a quite good response to the word problem.

The situation turns out to be completely different under a standardised testing regime, where a solution “19.21” would normally be credited with zero points. In the specific context of standardised testing, the student’s achievement of translating the word problem text correctly into a mathematical operation and of performing the division correctly is fully devalued. In this way, standardised testing produces a myth: students’ incompetence to model “real-world” situations mathematically; or, in terms of the Programme for International Student Assessment (PISA), the mathematically illiterate student. As our classroom data illustrate, if asked to relate the textual representation of a problem situation to a “real-world” situation, the students refuse to uncritically carry out mathematical operations with the given numbers. They immediately recognise the crude simplification of problem situations generated by its textual representations in mathematics word problems.

In the aftermath of the first PISA and TIMSS results, mathematics curricula experienced conceptual shifts in many countries. In the case of Germany, “problem solving”, “application of knowledge” and “mathematical modelling” have become the key concepts of the new mathematics curricula, thus promoting a re-orientation of the curriculum towards mathematics-in-the-real-world. Attempts are made to change the nature of the word problems from simply disguised mathematical problems to “more realistic”, open tasks, taking the context outside the classroom and the students’ experiences more seriously. Eventually, these claims perpetuate the myth of reference and the myth of participation (Dowling 1998) rather than enabling students to model “real-world” situations, and additionally still mask the hierarchy of forms of knowledge in the mathematics classroom.

The myth of students’ incompetence to model “real-world” situations is one starting point of recent curriculum reforms. The reform pressures teachers and students to conceive of word problems as reifications of the usefulness of mathematics in the “real world”. Teachers and students can detour this pressure by considering word problems a “genre which carries within its form echoes of related genres (for example, riddles, parables, puzzles and competitions of wit), none of which bears any simple, necessary relationship to a presumed practical ‘reality’” (Gerofsky 2010, p. 62), and develop more sophisticated educational practices in the mathematics classroom that take students’ meta-knowledge about word problems—as a resource for classroom interaction—into account.
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A school for all? Political and social issues regarding second language learners in mathematics education

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Résumé: Pour étudier un aspect de l'équité en ce qui concerne l'apprentissage des mathématiques dans « une école pour tous », nous avons étudié comment les professeurs décrivent l'organisation de l'épreuve nationale de mathématiques. Les étudiants de cet étude sont en 5ème année scolaire (étudiants de onze à douze ans) et sont des élèves qui ont le suédois comme leur deuxième langue (Second Language Learners : SLL). Avec les données d’une enquête parmi les professeurs, aussi avec des profils de compétences pour les étudiants de 5ème année scolaire, nous avons effectué une analyse thématique. Les résultats indiquent qu'il y avait des écoles où les professeurs ont travaillé en conformité avec les instructions de l’épreuve, et, par conséquence, adaptés un organisation de l’épreuve qu’améliore les possibilités pour les étudiants SLL de montrer leur savoir en mathématiques. Ceci est cohérent avec l’intention exprimée dans les documents de réglement. Il y avait aussi des écoles où les professeurs décrivent plutôt des justifications de l'exclusion des étudiants SLL du test, qu'une adaptation de l'organisation du test selon les instructions. La, des mauvais résultats des étudiants SLL sont expliqués par problèmes de langue. Dans ces écoles, les étudiants SLL n’ont pas été invités à montrer leur savoir en mathématiques. Nous discutons ces résultats dans une perspective institutionnelle.

Abstract: To investigate one equity aspect regarding mathematics learning in “a school for all” we have investigated how teachers comment on their arrangements for Swedish second language learners (SLL) to succeed on the National Test in mathematics in grade 5 (students are 11–12 years old). With data from a teacher survey and competency profiles for students in grade 5 we have performed a thematic analysis. The findings indicate that there were schools where the teachers worked in line with the instructions of the test and, therefore, adapted the administration of the test to enable SLL students better opportunities to display knowing in mathematics. This is coherent with a view expressed in policy documents. There were also schools where the teachers did not write about how to adapt the test administration but rather justified the exclusion of SLL students from the test or explained SLL students’ poor results due to language issues. In these schools the SLL students were not invited to display mathematics. We discuss these findings from an institutional perspective.

Introduction

This paper is relevant for Subtheme 4, Cultural, political, and social issues, and its purpose is to illuminate one aspect of equity issues in “a school for all”, namely second language learners’ equal access to a compulsory National Test (NT) in mathematics, which provides possibilities for students to show the mathematical knowing outlined in the national syllabus for mathematics.

Due to school reforms in Sweden (Skolöverstyrelsen, 1962) the compulsory education is an integrated school where everyone is welcome. “A school for all” has been commensurate with the view that the education should have a countervailing affect to help pupils who do not have “enough” prerequisites from home to gain equal access to education in terms of gender or social, cultural or economical factors. Practically “a school for all” means that all students are held together from grade 1 to grade 9, and hardly any ability grouping occurs. The education should be adapted to each student's individual potential. In the increasingly heterogeneous society, however, this type of school is challenged (Tallberg Broman, Rubinstein Reich & Hägerström, 2002).

One group of students that is in need of counteracting measures is Swedish second language learners (SLL) and this is clear in policy documents. One aspect here is that SLL pupils are entitled to instruction in Swedish as a second language and also get graded for that subject (Utbildningsdepartementet, 2011). They also have the right to study their mother tongue as a subject.
and to get instruction in their mother tongue at least two hours a week. The compensatory measures are also significant in relation to socio-economic status and parental education background.

To investigate one equity aspect regarding mathematics learning in “a school for all” we have investigated how teachers comment on their arrangements for Swedish second language learners to succeed on the National Test in mathematics in grade 5 (students are 11–12 years old).

Political issues and mathematics education

Although the political ambitions mentioned above are good, it turns out in practice that many students with less educated parents and many students with non-Swedish background do not do so well in school. Immigrant students learn to see themselves as an “inferior” kind of students. Based on the ideas that Swedish students are well-behaved and have a bright future, immigrant boys refer to themselves as “immigrant boys in the suburbs with poor grades” (León Rosales, 2010). They are affected by contextual factors including media that categorizes immigrant students with a deficient rationale. Teachers’ expectations and requirements to work with students individually, as well as local conditions, segregation, poverty and socio-economic status restrict student achievement (León Rosales, 2010). According to Klapp Lekholm (2008), 3-5% of the grades achieved in grade 9 in Swedish, English and mathematics are based on elements such as interest, motivation and parental involvement. Regarding mathematics teaching it is characterized by educational segregation, where teachers use different teaching methods according to their perception of student groups’ social and linguistic composition. This leads to lower expectations, which in turn leads to lower performance for children with lower socio-economic background or special immigrant groups (Hansson, 2011).

In recent years it has been shown that the socio-economic gap in mathematical performances has widened between students with high-and low-educated parents (Skolverket, 2013). Reports show that the gap is increasing and that multilingual pupils do worse than students with Swedish as their first language. Social selection to higher education remains and educational patterns are reproduced (SOU 2004: 47). Statistics show that there are differences between municipalities and schools in terms of pupils’ performance in National Tests in grade 6 and 9. It may depend on school organization and how teaching is conducted, for example, different ways of working, teaching efforts and schools pupil composition. Parents with knowledge of the education system are enculturated in the ways of education in Sweden and can therefore enculture their children accordingly.

Immigrant students’ difficulties in school is seen as a result of aspects of the students’ background, not as the result of the teaching situation or environment. Mathematics teachers seem to treat students differently depending on whether they are boys or girls, and if they have Swedish or non-Swedish background (Moschkovich, 2007; Parszyk, 1999; Stathopoulou & Kalabasis, 2007).

Data collection and analysis

Swedish students in grade 5 and 9 have completed a National Test in mathematics from 1996 until 2010 and since then National Tests are given in grade 3, 6 and 9. The tests consist of different item formats such as short answer questions, questions which need more elaboration from the student and group tasks. The teachers assess the students’ performances drawing on assessment instructions included in the test material and they can also complete a competency profile for each student. The teachers are asked to answer a survey in order to give the test designers feedback. Here they can comment on test samples and on observations from the test situation in the classroom and the like. For this paper we have analysed 26 of the competency profiles for students from year 2005, and 16 from 2002. We have also examined 155 teachers’ surveys from 2005 as well as the teacher.
instructions for the test.

A theoretical frame in this paper is the concept of institution (Douglas, 1986) and we view the teachers’ answers as representative of the teacher perspective within the institution of school. In elaborating on the presence of institutions, it can be argued that mathematical assessments are situated in a context characterised by dominant (mathematics) education discourses, the use of artefacts developed over time, framings in terms of specific resources for learning, division of time, structures within and between schools, classification of students into schools and learning groups, established routines, and authoritative rules (Selander, 2008, drawing on Douglas, 1986).

We performed a thematic analysis in line with Braun and Clarke (2006). The process required a decision on whether to perform an inductive or a theoretical thematic analysis. We have performed a mainly inductive thematic analysis within the theoretical framing of viewing school as an institution where national tests are one part. In this paper we connect to the concept of institution in the discussion whereas the actual analysis was the interplay between the aim of the study and different phases of a thematic analysis, such as familiarizing with data, searching for themes, and defining and naming themes. Another decision, drawing on Braun and Clarke (2006), was that we adopted a semantic approach where “the themes are identified within the explicit or surface meanings of the data, and the analyst is not looking for anything beyond what a participant has said or what has been written” (p. 84).

Second language learning students opportunities to take the National Test

In the analysis, themes of teacher comments on second language learners in relation to the national test were construed. Three of these themes are outlined below. We also describe what is written in the teacher instruction in relation to the themes.

1. Second language learners did not get the opportunity to take the test:
   One theme was teachers writing that some second language learners were excluded from taking the test. One justification could be that “Pupils attending the preparatory class did not take the test” or that “newly arrived pupils did not take the test”. The reason for this was that the teachers saw language issues as impediments to the student taking the test. Since this was a test in mathematics and not Swedish, this would not have to be a reason for excluding students from taking the test. In the instructions for the test, there was advice for how to enable all students to take part in the test, for example that items could be explained or translated for students as long as the mathematics that was tested was not revealed. It was also clear from the instructions that students had the opportunity to display mathematical knowing in a variety of forms of expressions.

2. Some students did take the test but made low results due to language issues.
   A second theme was that teachers wrote about how students’ results were low due to language issues. Reasons that teachers mentioned were: “limited vocabulary”; “students’ lack of comprehension”; and “did not understand Swedish”. This theme can be related to the same part of the teacher instructions as described above and the heading for this information was Adaption of the test.

3. Second language learners get help in various ways when taking the test.
   The third theme is more in line with the information in the teacher instructions on how to adapt the test for students in need. In this theme the teachers described how they and colleagues went about to adapt the test for second language students. It could be more frequent teacher-student interaction: “Someone reading for the student” or “Smaller group instruction”. It could also be support from teachers with other competence than the regular mathematics teacher: “Mother tongue teacher in Arabic did translations” or “Swedish second language learning pupils did test with Swedish second language teachers”. Some teachers described the adaptation as being about facilitating the language: “Minimizes texts – he/she is immigrant student”, “SLL students received simplified words”; and
“SLL teacher explained words”.

**Concluding discussion**

The findings indicate that there were schools where the teachers worked in line with the instructions of the test and, therefore, adapted the administration of the test to enable SLL students better opportunities to display knowing in mathematics. This is coherent with the view expressed in policy documents described in the introduction. There were also schools where the teachers did not write about how to adapt the test administration but rather justified the exclusion of the students from the test or explained SLL students’ poor results due to language matters. In these schools the students were not invited to display mathematics knowing in the same way as the other students. Research has shown the items in National Tests to be more creative than teachers’ own test (Boesen, 2006) so this could, in fact, be a true limitation for the SLL students in these schools.

In order to try to understand these findings we draw on institutional theory (Douglas, 1986). From this point of view the acts of teachers are seen as part of a broader institutional context with different frames and discourses. The teachers that tried to adapt the test taking for the SLL students acted according to the frames in the form of test instructions and this was also in agreement with the dominating discourse in policy documents. The other teachers acted according to other discourses which previous research has revealed (e.g. Moschkovich, 2007). Here the test taking is fair if all students are doing the test in the same way, and in such a discourse adaptations do not come into question (see Norén, 2011). Another institutional aspect refers to framings in terms of number of teachers and the possibilities to actually help students according to the guidelines in the teacher information. In schools where the National Test is made important not only for the teachers in the grade where the test is taken, but for all, there could be possibilities for allocating more teachers to the classes taking the test during the test period. In schools where this is not the case, the teachers may not have any opportunity to help SLL students in the same spirit as theme 3. In these processes, the heads of the schools are important.

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Teaching Mathematics To Students With Severe Intellectual Disability: An Action Research

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Abstract: The present study is part of an educational action research, in which sociocultural factors that affect teaching and learning mathematics of three students with severe intellectual disability are explored. In this study, factors related to family and school environments are further presented. During the first cycle of the research, the teacher-researcher acquires a holistic view of the students’ interests, needs and everyday experiences (funds of knowledge, etc.). As a result of the interaction with the students’ mothers, the emphasis is placed on home-school collaboration and its potential to promote students’ active involvement during grocery shopping and money dealings. The task-based approach used includes simulation, video modeling and community-based instruction. Students appeared eager to repeat grocery shopping and their mothers are motivated to allow students become active participants while making purchases.

Introduction

During the last decades several researches have focused on teaching mathematics to students with intellectual disability. Researchers tend to use experimental or quasi-experimental designs (Browder & Grasso, 1999; Butler, Miller, & Lee, 2001). As Porter and Lacey have argued (2005: 49), this is based on a rather “controlled context” and does not reflect the complexity of real-world classroom settings.

On the other hand, research in mathematics learning has shifted from mastering a predetermined body of knowledge and procedures, to a more sociocultural approach (Goos, Galbraith, & Renshaw, 2004). On this basis, mathematics activities must be meaningful to the students and help them make sense of real world. This is vital for individuals with severe developmental disabilities, because many of them are not adequately prepared to live and work in their community (Xin, Grasso, Dipipi-Hoy, & Jitendra, 2005).

Research Setting

The research methodology is located within action research and, more particularly, falls into the “reflective practitioner” category (Schön, 1983). According to the participants in the National Invitational Seminar on Action Research held at Deakin University in May 1981 “educational action research is a term used to describe a family of activities in curriculum development, professional development, school improvement programs, and systems planning and policy development, These activities have in common the identification of strategies of planned action which are implemented, and then systematically submitted to observation, reflection and change.
Participants in the action being considered are integrally involved in all of these activities.” (Carr & Kemmis, 2004: 164).

The first writer is the teacher-researcher, while the second writer and two more university teachers are the critical friends. The teacher-researcher teaches mathematics to the nineteen students (aged 16 to 36 years old) of a secondary special school located in a rural area of Greece. The participants of the research are three students with severe intellectual disability (Katerina-16 years old, Vangelis-24 years old, Anastasia-29 years old), who constitute one of the six classes of the school. In the research participate, also, their mothers, since these are the persons of the family more involved in students’ education and caring.

While Vangelis is able to count up to thirty-nine and Katerina up to twenty-seven, Anastasia can count up to ten by rote. They can write and recognize numbers up to ten, but they can’t compare numbers at the abstract level (i.e., without using objects or drawings) (Butler et al., 2001). They have difficulty maintaining one-to-one correspondence while counting a collection of up to ten objects, but they can recognize that the last number word used to count a collection has special significance because it represents the total number of objects. Finally, they are able to mentally determine sums up to three (i.e., 1+1, 2+1).

The project proceeds through a spiral of cycles (plan-act-observe-reflect). The main purpose of the research is to study the sociocultural factors that affect teaching and learning mathematics of students with intellectual disability within real-world settings. In this study, factors related to family and school environments are further presented. The data collection methods rely mainly on observations and interviews. The teacher-researcher keeps a self-reflexive research diary, which contains observations inside and outside of the classroom, thoughts, ideas, explanations and interpretations. Some lessons, the interviews of the students’ mothers and the meetings at the end of each cycle with the critical friends, are tape-recorded and analysed by the teacher-researcher.

Findings from the First Cycle

The general idea that the teacher-researcher identified at the beginning of the research is that there is little connection between school mathematics and students’ interests and everyday math experiences. At the first cycle, the teacher-researcher realized that math skills were presented “decontextualized” (Bishop, 1988: 180) and isolated from students’ everyday activities. This is exactly the opposite of what professionals call “criterion of ultimate functioning” (Browder & Cooper-Duffy, 2003). According to this, teachers should consider whether skills are functional (i.e., usable in daily life) and age appropriate (i.e., relevant to students’ chronological age rather than developmental age) for students with severe developmental disabilities.

Since this is the first year the teacher-researcher teaches in this particular school, it was necessary to acquire a holistic view of the sociocultural factors that affect students’ teaching and learning of mathematics. Interviews with mothers showed that they are rather protective with their children and provide them with limited chances of math experiences in everyday life. Both girls have been present many times during grocery shopping, but none mother reported active involvement of their daughters. On the other hand, Vangelis’ mother reported that he has gone to the grocery store alone for several times. During this, Vangelis is able to recall two or three items and an employee at the grocery store would find the items, keep the money and put the change in a plastic bag. All mothers expressed the desire their children acquire purchasing skills necessary for everyday independent living. At the same time, it seemed rather difficult to believe that their children could be able to achieve independence on such skills and tended to lower their expectations.

Observations in the natural environment of the school and the classroom, in combination with interviewing students and their mothers, provided the teacher-researcher with a holistic view of children’s needs and transformed the attention from the mathematics classroom to home-school collaboration as an essential role in students’ learning mathematics.
During the Second Cycle

In order to help students acquire math skills necessary for everyday life, the students, their mothers and the teacher-researcher should work together toward common goals. All mothers agreed that they wanted their children to learn to go grocery shopping and make purchases. As a starting point, the teacher-researcher evaluated students’ ability to go grocery shopping in a simulated setting. In the math classroom there is a corner with empty grocery items including milk, biscuits, toothpaste, etc., and a desk used as a counter where students pay money. Another student had the role of the shopkeeper and each of the three students was asked to role play grocery shopping. At the end of this activity it was remarkable to find out that none of the students had taken money before visiting the store. Also, Anastasia and Vangelis forgot to take the purchased items after leaving the store. At this point, the main teaching objective is students be able to go grocery shopping. The task-based approach used includes simulation in a classroom setting, video modeling and community-based instruction.

The teacher-researcher used a realistic and life-like scenario by informing the students that “the school principal has assigned them to buy what is necessary for the school Carnival party”. All students started brainstorming and agreed to visit a local supermarket a day before the party. Firstly students had to make their own grocery list by cutting the desired pictures of items from leaflets (an adaptation used because students have difficulty reading or writing) (see Fig. 1). This activity was generalized with their mothers and making family’s grocery list.

For the next lesson, the teacher-researcher made a videotape of a model going grocery shopping in community setting. Video modeling was used as an instructional technique. This videotape was also given to the students’ mothers in order to watch it, to interact with their children and to motivate them to allow students take an active role during grocery shopping. As an application of the video modeling instruction, the teacher-researcher asked the students to role play the steps of grocery shopping in simulated classroom setting. The teacher-researcher was the cashier and each student role played the customer.

The next step was community-based instruction, where all three students took their shopping list and a prespecified amount of money (a five euro note) (an adaptation used because students have difficulty giving a specific amount of money) and went grocery shopping. At the grocery store each student fulfilled the shopping steps by having the necessary prompting from the teacher-researcher, according to their needs (see Fig. 2). The whole process was videotaped and these video recordings were used to make three different videotapes, one for each student’s family.
Findings and Discussion

Through the task-based approach students were able to make meaningful connections between their everyday experiences and mathematics activities being engaged in classroom. All students kept their motivation during the whole teaching process and asked the teacher-researcher to repeat the grocery shopping activity. On the other hand, mothers reported that it was the first time their children participated in such activities with the school and they are very pleased and thankful. After families watched the videotapes, mothers stated very proud of their children. The students became the “star” of the family for a few minutes and they described the moment with happiness.

This process managed to make a step towards social inclusion of the three students. Since practicing in the natural environment where skills will be used is essential, the teacher-researcher motivated mothers to engage their children during the grocery shopping process. Students could make their own grocery list and shop independently while mothers could do their grocery shopping. The girls’ mothers reported that after watching the videotape they engaged their daughters during grocery shopping for more than one times. Vangelis’ mother gave him the opportunity to find the location of two items in the grocery store, but did not engage him fully at the shopping procedure. The teacher-researcher explained that it is necessary for Vangelis to participate during the whole process and manage to fulfill independently all the steps. Both Vangelis and his mother were motivated and promised to try it next time.

Although the three students may lack prerequisite skills needed to make purchases, such as counting money or making change, it is important to give them the opportunity to fully participate in the grocery shopping procedure (Mechling, Gast, & Bartholde, 2003). The research is still in progress and next steps include teaching purchase skills to the students by using the next dollar strategy (Browder & Grasso, 1999). Therefore, intense home-school collaboration is important to ensure that the outcome will be relevant to the students’ family routines (Westling, 1996).

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Can we learn from “outside”? A dialogue with a Chinese teacher: the “two basics” as a meaningful approach to mathematics teaching

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Abstract: Since many years Confucian heritage students (Chinese ones in particular), acquire leading positions in numerous international scientific programmes and display excellent performance in international assessments as PISA or TIMMS (OECD, 2013). To understand the “reasons” of this excellence we tried to explore some aspects of the cultural background of teaching practices and classroom life in those countries. With this aim a Chinese teacher was interviewed; we asked him about principles, values and beliefs and their impact on teaching/learning Math in classroom. The paper discusses what emerged from this dialogue and in particular from the idea of the “Two Basics” mathematics teaching approach, typical for the Chinese educational context. Furthermore, this work tries to underline (in a implicit or explicit way) similarities and differences between East and West didactical approaches and to define a sort of integration of these in order to improve a better mathematics education for all students.

Résumé: Depuis de nombreuses années, les élèves de l'héritage confucéen (les chinois en particulier) atteignent des positions de leader dans de nombreux programmes scientifiques internationaux et obtiennent d'excellentes performances dans les évaluations internationales PISA ou TIMMS (OECD, 2013). Pour comprendre les «raisons» de cette excellence, nous avons essayé d'explorer certaines caractéristiques du contexte culturel des pratiques d'enseignement et de la vie scolaire dans ces pays.
Avec cet objectif, nous avons interviewé un professeur chinois; nous lui avons demandé à propos des principes, des valeurs et des croyances et du leur impact sur l'enseignement /apprentissage des mathématiques en classe. L'article traite de ce qui a ressorti de ce dialogue et en particulier de l'idée des "Deux Principes", l'approche à l'enseignement des mathématiques typique du contexte éducatif chinois.
En outre, ce travail veut mettre en évidence (de manière implicite ou explicite) similitudes et différences entre les approches didactiques de l'Est et de l'Ouest, et veut définir une sorte d'intégration entre eux dans le but de parvenir à un meilleur enseignement des mathématiques pour tous les élèves.

Cultural aspects related to Chinese mathematics educational context: an overview of possible values and principles

A general overview of the cultural aspects that could be the linked to education in China is related to the social status of school and the value that School as institution has for teachers, students and families. In this area of remarkable economic development, with the connected social changes, school in fact is seen as an instrument of social promotion. Many expectations of families and students are pinned on the result in English and mathematics. So students study very seriously till late. In the Confucian view school has to be difficult. Teachers must assign “many” homework and require great commitments. The authority of teachers is not discussed and the confidence of
students is granted.

According to Zhang et alii (2004), the didactic aspects related to this and the implication on the way of teaching/learning in classroom could be summarised in five points:

- **Rituals:** in Chinese, Korean and Japanese school, lessons follow a repetitive pattern that always includes welcoming the teacher, summary and review of past lessons, various other activities depending on the grade level, and a final greeting. This habitual structure is heartening for students and makes the lesson a separate space in their lives, where they can concentrate and where specific rules apply.

- **Progressivity:** a good part of the lesson is devoted to summary and review of past lessons. Then you can add some new contents. Knowledge is built with steady rhythm, step by step, with great consistency.

- **Rote learning strategies and significance:** memorization, repetition, repetitive exercises are fundamental in teaching practices in these countries, but it seems that students develop considerable metacognitive skills, which appears like a paradox. In Cheng chu tong Bian ben mo (乘除 通 变本末 “Full Explanation on multiplication and division”, 1274) the Chinese mathematician Yang Hui (杨辉 1238-1298) wrote a list of important skills and knowledge for students, indicating timetables and ways to learn: 260 days in all. According to him, it takes a day to figure out a method (an algorithm or a procedure) and you have to work on it for two months to seize its meaning and applications without forgetting. But, for example, once you understand the method of addition (加法 jiāfǎ), studying subtraction (减 jiǎn) you can reverse all the exercises made by adding the differences to subtrahends. This gives the method its "origins" and reduces the time of study only 5 days. So understanding significance is the target.

- **Relational approach:** in Chinese or Japanese textbooks whenever you find a rule, you find nearby the reverse one; whenever you learn an operation, you learn the reverse one; whenever you see an identity, like in polynomial algebra, you find the same written in reverse form. So you learn together addition and subtraction, polynomial algebra and factorization, and so on… That means that you are not just learning some object and procedure, but their mathematical relations too.

- **Visualization:** schemas, pictures, drawings, blackboard sketch, magnets, stickers, page layouts, graphic arrangement in books, animations and screens, all are strongly used by eastern teachers to represent, organize, and explain mathematical properties and objects at school. Visualization tradition is still alive. Students remember images that mean concepts, not especially words.

In particular, about Mathematics, a cultivated person in China has to know many mathematics and a clever person can do mental calculations quickly and accurately. Mathematics as a subject has big space in school timetable.

If we look at the history of the Discipline, Chinese mathematics has been so important that one of foundational myths of the country is related to magic squares (幻方 huàn fāng, the myth is the Luo river magic square 洛書 luò shū). Mathematics is up to now one of the traditional Six Arts (六藝 liù yì) required in young lords education and played a strategic role in imperial bureaucrats curriculum and examinations (Nicosia, 2010).

Due to historical reasons, in many Eastern countries, including Japan, Korea, Singapore, and China (Mainland China, Taiwan, and Hong Kong), and even Russia, mathematics educators emphasize the importance of foundational training more than is usually seen in the West.

In all the Confucian heritages the didactical consequence of this approach is the principle of the “two basics”, that is the most typical teaching framework in all these eastern countries (Zhang, 2004).

In China the “two basics” principle (basic knowledge and basic skills) in mathematics teaching is a broad and loose concept without a strict definition. According to Zhang (2004), its general meaning is that, between “solid foundation” and “individual development and creativity”, although both are important, foundation is the more important. As a Chinese proverb declares: “although a tower is beautiful its groundwork is more important”.

With the aim to better understand this important principle hidden in Chinese teaching
framework, and its possible link to mathematical competences acquired by Chinese students (as evidenced in international competitions), we met Lai in Guangzhou last year and we interviewed him. With his help we tried to define a general framework of cultural background of teaching practices and classroom life in China. Furthermore, according to his point of view, we tried to define a possible link between these cultural situations, these social values and the excellent training that students showed in international assessments as PISA or TIMMS (OECD, 2013).

The “two basics”: solid foundation and individual development. A dialogue with a Chinese teacher

Lai is a Chinese teacher; he lives/teaches Mathematics in a Upper Secondary School in Guangzhou (廣州) that it is the biggest city in the south of China in the region of Guangdong. He’s teaching since 1978 and he is considering an expert Professor in his school. We met him last year in May, we spent 3 month with him observing his typical way of teaching and its impact in class. After each lesson we discussed with him exploring what happened in class and why.

This paper discusses what emerged from an informal dialogue with him about social and cultural aspects related to the Chinese school and their impact of the teaching/learning phase in mathematics. This dialogue was registered after the first Lai Math lesson at his Upper Secondary School.

In particular we report what we discussed and what Lai put in evidence about the “two basics” principle and its value for the Chinese teaching approaches and its characteristic.

Researcher: What do you mean with Two basics?
Lai: Two basics means “Basic Knowledge” and “Basic Skills”. According to the Chinese prospective, skills can be developed into knowledge and knowledge can induce skills. So ... knowing and practicing. In fact, skills and knowledge overlap at some examples. You have to understand that there are two kinds of knowledge, one is direct and it is possible to acquire it through direct exploration and investigation. The second kind of knowledge is indirect knowledge. It came from imitation, repetition, memorization ... We have two basics that are the slabs, these slabs connected together by ability. The possible success of problem solving in Math but also in the other subject depends on whether such slabs are connected.

Researcher: What do you mean with connection?
Lai: Connection of pieces means connection of knowledge. The quality of foundation depends on number of reinforcement and also the quality of the concrete activities ... so we can say that ... the slabs are joined together.

Trying to better explain his idea, he had drawn the Model of learning cycles in the “two basics” as follow:

![Fig.1 Model of learning cycles in the “two basics” by Lai](image)

After that Lai explains the drawing:

Lai: If we think to a mathematical content, we can summarise the process in this way. I hope to be clear for you. The first step is imitation, with teacher’s guidance.
Researcher: *Imitation? What do you mean?*
Lai: *Imitation means students need to observe and internalize, personalize. Ok?*
Researcher: *Yes but in which way?*
Lai: *it depends on teacher and students ... The second step in fact is the intervention by teacher. In this phase we have criticism, correction of concepts, strong examples for the concept, and a brief summary of knowledge learned in a trunk and practice.*
Research: *But how to avoid having practice become mechanical repetition?*
Lai: *because ... yes ... only in this way we can have a real understanding by students. The two basics provide the following principles that are the base of a significant learning:*
1. *memory leads to recognition and become intuition,*
2. *good speed of operation provides grounds for efficient thinking,*
3. *using deduction and reasoning to sustain precise logical thinking,*
4. *rising of standard through variation of problems and learning process.*

Lai: *The third step is then abstraction process by students, which is a kind of internalization and self monitoring. By internalization, students with the help of teachers can connect different knowledge, and deduce new knowledge.*

Researcher: *And what about the internalization and self monitoring? How can facilitate it?*
Lai: *it depends on the teacher. On the examples discussed with students, the time allocated to this activities ... It is difficult, I tried to summarize. There are two levels of teaching. The first one is using daily lives context aimed to capture the interest of students. The second level is the learning process through abstraction. Teachers teach mathematics based on the process of “correspondence, variation, induction and deduction”.*

After a brief pause linked to the difficulties of the dialogue Lai argues:

Lai: *Correspondence means to map the concept of mathematical problems to another problem with variation”; it also involves relations of expression, structure, meaning ... We will discuss more about it. It is complex. Tomorrow I will show you some practices in classroom. These practices refer to the “routine” daily practice commonly accepted by Chinese teachers.*

**The model of learning cycles in the “two basics” applied to variation problems: an example form Primary School**

To better discusses his idea of variation and trying to generalize this Chinese practice Lai shws a mathematics book (published in Japan) for Primary grade and in particular discusses the exercise N.1 (pages 62 and 63) reported below:

Fig.3 Use of variation problems in Primary textbook (pages 62 and 63)

According to Lai, the typical approach proposed by the text in this exercise is significant to “facilitate” the students strategy solution of the proposed problematic in order to work on their

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7 In this paper we don’t discuss the typical Chinese variation approach to mathematical problem solving. Some interesting didactical considerations on this subject are in Bartolini Bussi, Sun, Ramploud, (2013); Spagnolo, Di Paola (2010); Di Paola, Ramploud (2013).
memorization procedure leads to recognition and intuition; to favour their efficient and logical thinking and to generalize through variation of problems and learning process.

Lai: *In general in a mathematics book it is easy to note how the use of schemes (as the ones proposed in this exercise) is totally linked to a didactics focused on the research of solving methods, rather than on the presentation of isolated contents. This happen in many western book. Do you agree?*

Researcher: *What do you mean?*

Lai: *If you look for example to these pages (he refers to 1 pages 62 and 63) it is clear ... when observing the associated schemes ... the operation of addition and subtraction and the strong interdependence between them is clear, they are proposed in the same context, their use is evident for students. This approach is typical in Chinese curriculum. I think it is not typical in your country. The operation requested to students are linked to the idea of variation as I said before. All Chinese students are able to solve this problem, all are able to read it with different “lent” and to move inside the mathematics calculation requested by the interdependence between the operation of addition and subtraction together. The representation help also students to conceptualize this strongly relation and to generalize it.*

... 

Researcher: *And what about the model of “two basics”? Why you shows me this exercise?*

Lai: *Do you remember what I said on the “two basics” model? This is an example of connection of pieces of knowledge. I shows it because this is one of the mail point of the Chinese curriculum, an example of the philosophical idea of two basics to practical activities in classes trough the use of variation.*

...

Lai is right, if we look at a typical Italian textbook (we can generalize to the western textbook), the approach is different. The idea of variation is distant from the western culture, it is instead a key point of the curriculum of China and all the Confucian area for all the grades. It is possible to find some example just from the pre-school grades as showed below.

![Fig.3 Use of variation problem in pre-school Chinese textbook](image-url)
Conclusion

Can we learn from “outside”? Can we contaminate (Ramploud, Di Paola, 2013) our typical didactical approaches with something “different” in order to enrich it? We think yes. The tables below try to summarize what emerged from the dialogue with Lai and in particular discusses some possible key aspects stressed by Lai on the typology of mathematical activities that are typically propose in Chinese classroom and the teaching/learning time related to their implementation according to the two basics approach at School.

<table>
<thead>
<tr>
<th>Mathematical activities</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Activities mainly individual or in a big group,</td>
<td>• Students are required to adapt to the pace of progress setted for most students by the teacher,</td>
</tr>
<tr>
<td>• Learning by imitation: “small steps” teaching instead of free discover,</td>
<td>• Teachers present the main mathematics contents as quickly as possible to avoid students spending too much time on winding paths.</td>
</tr>
<tr>
<td>• Great importance to mental calculation and memorization: “practice makes perfect”,</td>
<td></td>
</tr>
<tr>
<td>• Few importance to proofs,</td>
<td></td>
</tr>
<tr>
<td>• Strong importance to the relationship between comprehension and manipulation,</td>
<td></td>
</tr>
<tr>
<td>• Emphasis on the teaching of problem solution schemes.</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Typical didactical approaches in China

Is it possible to integrate in our western didactical practices some different activities and teaching proposals coming out from abroad (for example from China) in a sort of continuous dialogue from distances and cultures crossing? We think yes. The different cultural background in teaching and learning hidden in the Easter context can, according to us, help us to better understand our own background and, on the other side, be pertinent, for example, to better understand the excellent performance in international assessments of Confucian area. To study similarities and differences between our western typical teaching approaches and the Chinese one could represent for teacher and researchers only the starting point to adapt possible new “multicultural approaches” in own classes.

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Abstract: The multicultural nature of modern society constitutes one of the most significant changes to have influenced schools in many European countries, especially at primary and middle school level. The teacher is seldom aware of the need to rethink and if necessary modify his/her methodological and pedagogical approach. This attitude is even more evident in maths teachers who often consider their subject universal and culture-free.

Little has been done in Europe as far as maths teaching in multicultural contexts is concerned. The different languages and cultures present in the classroom make the teaching/learning process even more arduous than it already is, especially for pupils from minority cultures and/or with a migrant background or for Roma pupils.

A teaching unit, designed in a European Commission funded project, is described. Its aim is to provide teachers with a tool to help their pupils to overcome the learning obstacle represented by the contrast between the simplicity of classroom language and the complexity of mathematics language. Teachers have to bear in mind, however, that the language used in class is an element of further complexity for pupils from minority cultures with a different mother tongue.

Résumé: La nature multiculturelle de la société moderne constitue l'un des changements les plus importants à avoir influencé les écoles dans de nombreux pays européens, notamment au niveau de l'école primaire et du collège. L'enseignant est rarement conscients de la nécessité de repenser et si nécessaire modifier sa/son approche méthodologique et pédagogique. Cette attitude est encore plus évident dans professeurs de mathématiques qui considèrent souvent leur sujet universel et sans culture.

Peu a été fait en Europe dans la mesure où l'enseignement des mathématiques dans des contextes multiculturels est concerné. Les différentes langues et cultures présentes dans la salle de classe font le processus d'enseignement/apprentissage encore plus ardu que l'est déjà, en particulier pour les élèves issus de minorités culturelles et/ou issus de l'immigration ou pour les élèves roms.

Une unité d'enseignement, conçu dans un projet financé par la Commission européenne, est décrite. Son objectif est de fournir aux enseignants un outil pour aider leurs élèves à surmonter l'obstacle de l'apprentissage représenté par le contraste entre la simplicité de la langue en classe et la complexité du langage des mathématiques. Les enseignants doivent garder à l'esprit, cependant, que le langage utilisé en classe est un élément de complexité supplémentaire pour les élèves issus de minorités culturelles ayant une langue maternelle différente.

Rationale

Mathematics teachers feel the necessity for training and materials which reflect the needs of their classes in terms of linguistic and cultural differences. Their pupils from minority cultures and/or those with a migrant background encounter even more difficulties than their native classmates in acquiring fundamental mathematics skills.

The above mentioned needs have been identified in several research studies carried out as to multicultural and inclusive education ([1], [4]), the role of the foreign language in mathematics learning ([2], [5], [7]) and the educational approach and methodologies for mathematics education in multicultural classrooms ([3], [6]).

The M³EaL project aims to identify teaching strategies for teachers and activities for pupils who
allow both to approach the challenges and facing them satisfactorily. The methodological tools used, to be considered innovative compared with the standard routine of the mathematics classroom, are the following:

- **Great attention to the language used in order to provide suitable compromise between the simplicity of classroom language and the complexity of mathematics language, bearing in mind, however, that the language used in class is an element of further complexity for pupils from minority cultures with a different mother tongue;**

- **Proposals for didactic units for the mathematics classroom which facilitate interdisciplinary extensions and which are inspired, above all, by practical problems and situations from everyday life and from different cultures.**

These methodological tools should, in general, help to make all pupils more interested and motivated to learn mathematics; in particular, enable pupils with different cultures and languages to overcome some of the difficulties they encounter in maths due to these very differences: the teaching of mathematics by using aids to activate different thought processes and skills which otherwise risk remaining latent because of language shortcomings.

Moreover, the above-mentioned methodological tools facilitate the appreciation of the positive aspects of different cultures and create favourable conditions for intercultural dialogue in the classroom, thus creating an inclusive educational setting.

A further innovative aspect is the contribution from language specialists to the communication and intercultural issues of the teacher training activities.

**A teaching unit from M³EaL project**

The teaching unit aims to provide teachers with a tool to help their pupils to overcome the learning obstacle represented by the contrast between the simplicity of classroom language and the complexity of mathematics language, bearing in mind, however, that the language used in class is an element of further complexity for pupils from minority cultures with a different mother tongue.

The teaching unit has been designed by the M³EaL project coordinator Institution, Centro interdipartimentale per l’Aggiornamento, Formazione e Ricerca Educativa – C.A.F.R.E. of the University of Pisa (Italy), and already piloted in the project participating schools selected by CAFRE and two further project partners: Ecole Supérieure du Professoral et de l'Education – E.S.P.E. of the University of Paris-Est Créteil (France) and the University of Thessaly (Greece).

Its primary target group are mathematics teachers in primary and lower secondary schools in socio-culturally diverse areas, the secondary target group consequently consisting of students from cultural minorities and/or culturally deprived groups.

The educational aims of the teaching unit can be roughly divided into general and mathematical aims.

Among the general aims we can consider:

- The appreciation of the positive aspects of different cultures.

- The creation of favourable conditions for intercultural dialogue in the classroom, and, therefore, an inclusive educational setting.

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The development of awareness and critical attitudes towards the use of language and its interpretation.

The awareness of how important it is to use specific and unambiguous language.

The capacity to state the reason for the choices made and used during the activity.

Among the mathematical aims we can consider:

The increase of the learners’ capacity to understand and to elaborate the mathematical discourse.

The improvement of the ability of reading and understanding mathematics textbooks and word problems.

The improved usage of mathematical language.

The reinforcement of the knowledge of mathematical glossary.

The development of the ability to find a proper balance between the natural language and the mathematical language.

The teaching unit should lead teachers to reflect upon a number of aspects:

• Difficulty in using mathematical language correctly: uncertainties, doubts and mistakes shown in understanding the written texts express the need to favour, in teaching, the verbalisation process, which induces students not only to make their ideas explicit but also to try and make it in a clear and correct way to make them understood.

• The need to use the linguistic instrument appropriately, its use is a fundamental step towards the construction of knowledge, although it requires a considerable time for maturation.

• The need to develop activities like this one, because they offer information about pupils’ knowledge, the conceptualisation level they have reached, possible gaps, and misconceptions. This information is fundamental to be able to intervene in the classroom with appropriate and well planned teaching actions.

The teaching unit consists of five main activities. If possible, all the activities should be carried out in small groups, each of which including a minority pupil at least.

Analysis of a textbook (Reading and Writing)

Pupils are asked to read a chapter of their textbook and, thereafter, to search for and make a list of words and verbs in the vehicular language that are “difficult”, discuss about their meaning and translate them into the foreign languages spoken in the classroom, thus producing a micro-dictionary.

Pupils are then asked to search for and make a list of words and verbs that are proper of the mathematical language, compare them to the same words and verbs in the natural language, discuss about and write their possible different meaning and translate the words and verbs into the foreign languages spoken in the classroom, thus producing a mathematics glossary and a mathematics dictionary.

All groups are asked to re-write the analysed pages of the textbook in the vehicular language and minority pupils are asked to translate the most significant sentences into their own mother tongues.

Analysis of a “word problem” from a National standard assessment test (Reading and Writing)

The teacher chooses a “word problem” from a National standard assessment test that is meaningful as to the language used. Pupils are then given the same tasks as in the first activity.
Pupils are asked to identify possible conflicts originated by different meaning of words and verbs that are common to both the natural and mathematical languages, and to write the two different meaning in their own mother tongues.

- **Writing a “word problem”**

Pupils, still working in groups, are asked to write in the vehicular language a word problem. The problems are presented to the whole class for discussion about their clarity as to the language used and the mathematical notions required. Greater attention is paid to minority students.

- **“Writing a textbook”**

Students, still working in groups, are asked to write in the vehicular language a “page of a textbook” about a mathematical topic chosen by the teacher. The “pages” are presented to the whole class for discussion about their clarity as to the language used and the mathematical notions involved. Greater attention is paid to minority students.

**REFERENCES**


Rôle de l’histoire des mathématiques dans l’enseignement-apprentissage des mathématiques : le point de vue socioculturel

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Abstract: Several authors bring arguments supporting the presence of history of mathematics in mathematics classroom. Their theoretical considerations show various, and sometimes divergent, positions concerning; ontological and epistemological status of mathematical knowledge, philosophical principles for mathematics education and status of the history of the discipline itself. Of course, these positions taint inextricably the content and the orientation of their arguments. This being said, there is now a recurrent problem in the field of study concerning an important “gap” between empirical and speculative researches. In this problematic, the need to reveal the epistemological position of those different theoretical discourses becomes crucial. Indeed, this could lead to a better operationalization of the empirical research. This paper is an attempt to clarify specifically the sociocultural point of view on the potential of history for the learning of mathematics, and this, trough conceptual elements from the theory of objectivation.

Histoire et enseignement-apprentissage des mathématiques
Depuis plusieurs décennies, de nombreux penseurs, chercheurs et enseignants se sont penchés sur le « comment » et le « pourquoi » du recours à l’histoire des mathématiques dans la classe de mathématiques. Dès le début du 20e siècle, pédagogues, philosophes et mathématiciens s’y sont intéressés. Jusqu’à récemment, il semble que tous, enseignants et chercheurs, s’entendaient pour dire que l’histoire est bénéfique et se veut d’emblée un outil cognitif efficace et motivant dans l’apprentissage des mathématiques (Charbonneau 2006). Cet engouement a donné lieu à de nombreuses études concernant l’utilisation de l’histoire des mathématiques dans l’enseignement des mathématiques.

de Jankvist (2009, 2010) en prises davantage pragmatiques sont de vibrants exemples

De plus, la nécessité d’articuler cette recherche spéculative aux études de terrain apparaît cruciale dans ce champ de recherche. Dans un article maintes fois cité, et encore aujourd’hui fortement pertinent, Guliker et Blom (2001) mettaient déjà en évidence, au début des années 2000, le véritable « fossé » qui sépare les recherches de terrain et les recherches théoriques du domaine. En effet, on observe, d’une part, une recherche théorique importante fournissant des conceptualisations profondes, riches et fécondes et, d’autre part, une recherche empirique qui tente de « mettre à l’épreuve » le développement de certains outils d’introduction de l’histoire en classe de mathématiques sans prendre en compte et y articuler les développements théoriques du domaine. Ces deux formes de la recherche marchent côte à côte et ont peine à se stimuler et s’orienter mutuellement. Non seulement le manque d’études de terrain sérieuses se fait sentir, mais apparaît aussi le besoin de mettre en évidence les postures épistémologiques sous-jacentes aux différents discours théoriques, afin de les voir mieux s’opérationnaliser dans la recherche de terrain.


**Un point de vue socioculturel sur l’éducation mathématique**

D’inspiration vygotskienne, la théorie de l’objectivation est une théorie socioculturelle contemporaine de l’enseignement-apprentissage qui plaide pour une conception non mentaliste de la pensée. S’opposant au courant rationaliste et idéaliste, elle propose une conception de la pensée que serait à la fois sensible et historique. D’une part, elle est sensible, car elle s’enracine dans le corps, les sens et l’affectivité, lesquels sont invoqués dans la saisie des objets de la réalité. Le corps, la perception, mais aussi les gestes et les signes sont donc considérés comme des parties constitutives de la pensée elle-même. D’autre part, elle se veut historique, car tout aussi enracinée dans l’histoire et la culture, étant perçue comme une forme sociale de réflexion et d’action historiquement constituée. On parlera ainsi de la pensée comme d’une praxis cogitans (Radford, 2011).

Concernant les objets mathématiques, la position ontologique de la théorie de l’objectivation s’éloigne du discours réaliste qui les considère comme indépendants de l’époque et de la culture, précédant l’activité humaine. L’attraction de cette ontologie réaliste réside dans son pouvoir explicatif du miracle de l’applicabilité des mathématiques au monde phénoménal. Cependant, les penseurs réalistes font un acte de foi en croyant que l’accès aux objets véritables est possible. La théorie de l’objectivation suggère plutôt que les objets mathématiques sont « générés historiquement au cours de l’activité mathématique par les individus » et constituent des « schèmes fixes d’activités réflexives incrustés dans le monde changeant de la pratique sociale » (id., p. 7). Comme tous les objets mathématiques, le concept de cercle par exemple est une réflexion du monde dans la forme de l’activité des individus. Cette dernière forme la racine génétique de l’objet conceptuel, laquelle renferme une dimension expressive variée qui se décline sous des aspects rationnels, esthétiques et fonctionnels liés à la culture.

Pour les penseurs du socioculturel, le problème théorique central est donc d’expliquer comment se réalise l’acquisition du savoir ainsi déposé dans la culture. Avant tout, selon la théorie de l’objectivation, l’apprentissage n’est pas un processus de construction ou de reconstruction personnel de la connaissance. L’apprentissage résulte plutôt de notre contact avec le monde des artefacts culturels de notre environnement (objets, instruments, productions littéraires et scientifiques, etc.) et de l’interaction sociale.

Dans cette perspective, l’apprentissage, entendu comme objectivation, est précisément un processus social de prise de conscience progressive de l’eidos homérique, c’est-à-dire de quelque chose qui se dresse devant nous, une figure, une forme, quelque chose dont nous percevons continuellement la
généralité en même temps que nous lui donnons sens. L’objectivation signifie littéralement la rencontre avec quelque chose, quelque chose qui s’objet, qui se donne à voir, s’affirme en tant qu’altérité et qui se présente à nous petit à petit (Radford, 2002). Elle est « le perçu qui se dévoile dans l’intention qui elle-même s’exprime dans le signe ou dans l’action que médiatise l’artefact au cours de l’activité pratique sensorielle […] quelque chose susceptible de se convertir en une action reproductible, dont le sens vise à ce schème eidétique culturel qui est l’objet conceptuel lui-même » (Radford, 2011, p. 12).

Allons rapidement un peu plus loin dans l’exploration de ce processus d’apprentissage dit d’objectivation, processus au cœur même des approches socioculturelles.

L’intelligence des artefacts et la dimension sociale : deux sources de l’apprentissage

En premier lieu, la théorie de l’objectivation est sensible à l’influence des artefacts chez l’être humain. Au contact du monde des objets et des signes, l’être humain restructure ses actions (Baudrillard, 1968/1990) et forme des capacités d’actions et des capacités intellectuelles nouvelles. Dans ce sens, les travaux de Vygotsky et Luria (1994) ont montré leur influence, entre autres, sur les capacités liées à la perception et à l’anticipation. Certes, les artefacts ont une importance considérable dans le processus d’apprentissage, mais ils ne se suffisent pas à eux-mêmes. La rencontre avec les objets et leur contenu historique, symbolique et signifiant ne peut se rendre claire d’un seul tenant. L’intelligence des artefacts se doit d’être mise en œuvre dans des activités partagées avec d’autres personnes qui savent déchiffrer ces contenus intellectuels. Par exemple, l’objet que constitue le langage de l’algèbre ne peut être porteur de sens pour l’élève que lorsqu’il est identifié par l’autre au travers de l’activité sociale ayant cours à l’école. Sans l’apport d’un autre, le code symbolico-algébrique serait réduit à des hiéroglyphes dont le sens serait totalement à élaborer.

C’est pourquoi la dimension sociale constitue, pour la théorie de l’objectivation, la seconde source essentielle de l’apprentissage. Encore là, il ne faut pas réduire la dimension sociale de l’apprentissage à la négociation de signifié (arrière-plan socioconstructiviste) ou comme une simple ambiance qui offre des possibilités d’adaptation supportant le développement cognitif des apprenants (point de vue cognitivist ou behavioriste). La classe ne peut plus être perçue comme un espace neutre et fermé, un monde clos à l’intérieur duquel se négocient les normes, les formes et les valeurs du savoir, car, comme expliqué précédemment, les modes d’activité qui y prennent place sont médiatisés par les objets et la culture, lesquels sont imprégnés de valeurs scientifiques, esthétiques, éthiques, etc., et ces modalités sont, bien entendu, partout présentes dans l’activité de classe. La classe n’est donc pas fondamentalement un lieu neutre dans lequel les apprenants agissent selon des mécanismes invariables d’adaptation générale. En effet, l’interaction sociale ne remplit pas une fonction adaptative, catalytique ou facilitante, elle est plutôt, et c’est là la radicalité du propos, « consubstantielle de l’apprentissage » (Radford, 2011, p. 10).

Ainsi, l’apprentissage comme objectivation culturelle du savoir est à la fois une prise de conscience, entendue comme une (re)connaissance, d’éléments culturels et un processus de développement de nos capacités humaines. Autrement dit, apprendre des mathématiques n’est pas simplement apprendre à « faire » des mathématiques (encore moins à résoudre des problèmes) dits


9 Sans enlever le mérite aux problèmes dans l’acquisition de connaissances mathématiques, la théorie de l’objectivation ne considère simplement pas la résolution de problème comme une fin en soi, mais un moyen pour atteindre ce type de praxis cognitans appelé la pensée mathématique.

Avant de retourner à l’histoire des mathématiques et à son potentiel pour l’enseignement-apprentissage de la discipline, abordons rapidement cette thématique de l’être-en-mathématique dont les réflexions théoriques et philosophiques qui en émergent se fondent sur une perspective éthique fondamentale.

**La classe et le concept du je-communautaire**

La classe est le lieu de la rencontre entre le sujet et l’objet de savoir et l’objectivation qui permet cette rencontre est un processus éminemment social. Cependant, cette dimension sociale ne peut être réduite à un marché de la connaissance ou le savoir est transmis, partagé ou négocié dans une optique pragmatique de recherche de satisfaction personnelle, de jeu entre adversaires où chacun s’investit dans l’espoir d’obtenir une plus value, dans le repli de la sphère privée. Même s’il nous faut ramener quelque chose de la classe vers un chez-soi, cela n’implique pas nécessairement de faire de l’Autre un Même pour soi, la possession étant la forme par excellence sous laquelle l’Autre devient un Même (Levinas, 1971/2010). La relation au monde « qui se joue avec l’être, qui se joue comme ontologie, consiste à neutraliser l’étant pour le comprendre ou pour le saisir […] elle n’est donc pas une relation avec l’Autre comme tel, mais la réduction de l’Autre au Même. » (id., p. 36-37.) Dans ce cadre où règne l’objet et où s’exalte la souveraineté des pouvoirs techniques, la liberté consiste à « se maintenir contre l’Autre, malgré toute relation avec l’Autre, assurer l’autarcie d’un moi » (ibid.).

*A contrario*, la socialité du processus d’apprentissage signifie la formation et la transformation de la conscience, qui est justement (con)science, c’est-à-dire « savoir en commun » ou « savoir-avec-d’autres » (Radford, 2011, p. 12). Transformation des consciences qui est subjectivation, formation de la subjectivité, c’est-à-dire, d’un devenir. Devenir, puisque justement l’apprenant est individu (qui est indivisible, ne peut être réduit, réifié, chosifié par la question du « qu’est-ce que? »). C’est dans cet ordre d’idées que s’inscrit le concept du je-communautaire et se développe celle d’autonomie dans la théorie de l’objectivation. Ici, la théorie de l’objectivation s’éloigne de la conception d’un sujet autorégulé et autoéquilibrant, replié dans un moi-carapace dont la perméabilité se règle aux détours de logiques internes, et à travers laquelle le sujet serait doté des capacités de réfléchir à l’instar d’un scientifique ou d’un enquêteur méticuleux, autrement dit, du sujet souverain kantien autonome.

La théorie de l’objectivation offre donc une perspective profonde, sensible et fort cohérente de l’enseignement-apprentissage des mathématiques. Ses thématiques et ancrages épistémologiques, d’ailleurs très clairement détaillés et mis en évidences, la place de manière indubitale à l’intérieur de ce qu’on peut appeler le point de vue « socioculturel ». Cependant, la thématique de l’altérité qui fonde le concept de je-communautaire, thématique centrale à la perspective radfordienne, apporte une dimension particulière importante, et complète un ensemble dont la profondeur et la cohésion frappe et étonne. Mais ad rem, ayant maintenant exploré l’arrière-plan épistémologique de la théorie de l’objectivation, tâchons de mettre en relief, de ce point de vue, les arguments qui appuient la présence de l’histoire des mathématiques dans la classe de mathématiques du primaire et du secondaire.
Un point de vue sur le rôle de l'histoire des mathématiques

Dans cette perspective, le sens particulier attribué aux objets mathématiques est circonscrit aux limites de notre propre expérience. Cette limite ne peut être franchie que par la rencontre avec une forme étrangère de compréhension, car « le sens ne s’approfondit véritablement que par la rencontre et le contact avec un autre sens, une culture étrangère. Il s’engage alors une forme de dialogue qui surmonte la fermeture et la partialité » (Bakhtine, 1986, cité dans Radford, Furinghetti et Katz, 2007, p. 108, traduction libre).

Dans ce sens, l’histoire des mathématiques se veut un possible endroit où il est possible de surmonter la particularité de notre propre compréhension des objets mathématiques limitée à nos expériences personnelles. Elle « fournit les instruments de dialogues avec d’autres compréhensions […] avec celles de ceux qui nous ont précédés » (Radford, Furinghetti et Katz, 2007, p. 109, traduction libre).

L’histoire apparait ici comme la toile de fond ou le lieu rendant possibles l’introspection, la confrontation et la réflexion critique autour de ses propres conceptions et connaissances. Radford et al. (2000) soulignent que l’histoire des mathématiques est « un endroit merveilleux, où il est possible de reconstruire et de réinterpréter le passé dans le but d’ouvrir de nouvelles possibilités pour les futurs enseignants » (p. 165, traduction libre).

Or, notons que le regard est ici porté non pas sur un individu rencontrant des possibilités d’émancipation personnelles, dans un mouvement plus ou moins appuyé d’autosuffisance et d’autoréférence, mais vers la possibilité pour les apprenants de découvrir de nouvelles manières d’être-en-mathématiques, d’ouvrir, avec les autres, l’espace des possibles de l’activité mathématique. En effet, la classe de mathématique est ici perçue comme un espace communautaire, politique et éthique, ouvert à la nouveauté et à la subversion (Radford, 2006, 2008, 2011).


Dans cette perspective, la rencontre avec l’histoire des mathématiques offre des expériences particulières de l’altérité en mathématique, expériences dont les enjeux pour l’apprentissage et le rapport aux savoirs mathématiques des apprenants restent à mettre en évidence. En effet, quelles manières d’être-en-mathématiques peuvent survenir lors de cette rencontre-événement avec l’histoire ou certains éléments de nature historique en mathématiques? Et quelles manières d’être-avec-les-autres-en-mathématiques peuvent survenir? Quelles sont les modalités d’êtres associées à ces expériences particulières? Ces questions se doublent lorsqu’on se questionne sur les manières de rendre compte en recherche de ces expériences. La prochaine section proposera quelques pistes de réflexion à partir d’une étude menée récemment dans le contexte de la formation des maîtres en mathématiques au Canada.

Avant cela tâchons d’entrée plus en profondeur sur ce que peut vouloir dire manière d’être en mathématique. La pensée étant considérée comme « une réflexion médiatisée du monde en accord avec le mode de l’activité des individus » (Radford, 2011, p. 4), elle a d’abord une nature réflexive, c’est-à-dire en mouvement dialectique entre une réalité construite historiquement et culturellement et un individu qui la réfléchit et la modifie selon ses interprétations et sa propre subjectivité. La pensée n’est donc pas une simple assimilation de la réalité externe (point de vue empiriste) ou la construction ex nihilo de cette dernière (point de vue du constructivisme radical). Autrement dit, c’est l’individu qui crée la pensée et ses objets, mais tout individu est inséré dans sa réalité.
La pensée étant médiatisée par le corps, des signes et des artefacts et, d’autre part, par des signifiés culturels. Ces deux niveaux de médiations laissent une empreinte sur la forme et le contenu de la pensée elle-même.


En interaction avec l’activité (Leontiev, 1984) des individus (objectifs, actions, opérations, distribution du travail, etc.) et ce qui a été appelé plus haut le territoire des artefacts, ce Système Sémiotique de Signification Culturelle génère des modes d’activités spécifiques et, d’autre part, des modes de savoir ou épistémès (Foucault, 1966/1990). La figure suivante montre l’interaction entre ces trois composantes qui permettent de mieux comprendre l’idée de la pensée comme praxis cogitans :
Dans cette perspective, le sens accordé à l’apprentissage est celui de l’« acquisition ». L’apprentissage est compris comme l’« acquisition par l’apprenant de formes culturelles de réflexions sensibles et d’actions instrumentales qui constituent la pensée » (Radford, 2011, p. 3). Cependant, l’apprentissage ne se réduit pas ici à une simple acculturation, à la réception passive et mystérieuse du savoir contenu dans la culture. En effet, le mot acquisition doit être pris dans son sens étymologique, c’est-à-dire du latin adquaerere qui signifie « chercher ». Le mot acquisition doit donc être entendu en tant que processus d’ouverture, attitude ou manière d’être. L’apprentissage n’est donc pas une soumission à une culture ambiante, encore moins une possession d’un contenu culturel, mais plutôt un mouvement d’ouverture sur le monde et sur les autres, un processus d’élaboration active de signifiés soit le processus d’objectivation.

Avec l’introduction de l’histoire des mathématiques, cette ouverture sur le monde se voit radicaliser et prend un tournant inédit. En effet, les manières de penser et d’agir se multiplient autour des apprenants au contact de l’histoire des mathématiques. Ces éléments divergents de la culture ambiante invitent à l’introspection, à la prise de consciences de ses ancrages historiques, et l’ensemble rend possible la confrontation constructive de ses conceptions avec les autres en mathématiques. L’histoire place les apprenants en mode recherche.

Ouverture et invitation

Cela dit, il nous faut maintenant mettre en évidence, à partir de ce point de vue socioculturel, différentes manières d’opérationnaliser dans la recherche l’investigation de ces considérations théoriques sur le rôle de l’histoire dans la formation des enseignants. En effet, de quelle manière est-il possible d’investiguer sur le terrain le rôle que peut jouer l’histoire des mathématiques ou la présence d’éléments historiques ou culturels dans la classe de mathématiques dans une telle perspective sur l’éducation mathématiques? Quelques éléments de réponse seront proposés à partir de diverses réflexions d’ordre méthodologiques déployées dans le cadre de notre récente étude doctorale (Guillemette, 2015).

Notre étude avait pour objet une idée récurrente dans le champ de recherche, celle du dépaysement épistémologique. En effet, les chercheurs soulignent que l’histoire des mathématiques dépayse et bouleverse les perspectives coutumières des étudiants sur la discipline en mettant en évidence sa dimension historico-culturelle. Globalement, l’étude de l’histoire amènerait un regard critique sur l’aspect social et culturel des mathématiques et pousserait les futurs enseignants à reconsidérer leur rapport à la discipline. Cela dit, bien que commentées par de nombreuses études, ces considérations sur le dépaysement épistémologique ne semblent pas encore avoir fait l’objet de recherches systématiques de terrain qui donneraient véritablement la voix aux acteurs des milieux de formation.

Avant tout, trois points de vue sur le dépaysement épistémologique sont présentés dans la thèse : celui de l’épistémologie historique, de l’humanisme et des approches socioculturelles en éducation mathématique. Habilité par les questions que soulèvent ces positions et appuyée conceptuellement par la théorie de l’objectivation, notre étude s’est donnée pour objectif de décrire le dépaysement épistémologique vécu par les futurs enseignants de mathématiques du secondaire dans le cadre d’activités de formation où intervient l’histoire des mathématiques.

Pour ce faire, une approche phénoménologique a été adoptée et adaptée à la perspective bakhtinienne que porte la théorie de l’objectivation. Concernant l’approche phénoménologique, elle vise à décrire le vécu intime et subjectif des participants et à expliciter le sens de leurs expériences. Quant à la perspective bakhtinienne, elle souligne qu’une œuvre scientifique ou littéraire se doit d’être « polyphonique », c’est-à-dire offrir une pluralité de discours et de compréhensions du monde, afin que la réalité perde de son statisme et de son naturalisme. Habité par cette perspective compréhensive et critique, nous proposons une description du vécu du dépaysement.
épistémologique qui prend la forme d’un roman polyphonique.

La sélection des participants de l’étude a été faite parmi les futurs enseignants du secondaire inscrits à un cours d’histoire des mathématiques offert à l’Université du Québec à Montréal. Sept activités de lecture de textes historiques ont été vécues en classe. Six étudiants ont été recrutés. Des captations vidéo des activités de classe, des entretiens individuels et un entretien de groupe ont été réalisés et ont fourni les données de l’étude. Pour les captations vidéo, une analyse séance par séance a permis de décrire les processus d’apprentissage ayant eu cours en classe. Pour les entretiens individuels, inspirée par les procédures de plusieurs chercheurs phénoménologues en sciences humaines, le traitement et l’analyse des données ont mené à l’obtention d’une description spécifique du vécu du dépaysement épistémologique pour chaque participant. Le roman polyphonique a ensuite été construit à partir d’extraits de l’entretien de groupe, et peaufiné à partir des phases précédentes d’analyse.

La description obtenue fournit plusieurs regards, lesquels, mis en tensions, sont porteurs d’un discours fécond sur le vécu du dépaysement épistémologique. Spécifiquement, l’étude montre que celui-ci amène; la perception des mathématiques comme fragiles, débutantes et précaires, le vécu d’une forte adversité dans l’interprétation des textes et le déploiement d’une empathie envers l’auteur. Le roman polyphonique suggère que cette empathie se déploie aussi vers la classe de mathématique des futurs enseignants, et insuffle une attention et une valorisation plus grande envers les raisonnements marginaux de leurs élèves. Ces éléments invitent à penser que ces activités de formation, par le biais du dépaysement épistémologique qu’elles suscitent, supportent une éducation mathématique non violente.

**RÉFÉRENCES**


Ready-made materials or teachers’ flexibility?
What do we need in culturally and linguistically heterogeneous mathematics classrooms?

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Abstract: The paper focuses on the issue of coping with the increasing cultural and linguistic heterogeneity in mathematics classrooms across Europe. The authors come out of a teaching unit developed by Barbro Grevholm within the project Multiculturalism, Migration, Mathematics Education and Language. The goal of the project was development of teaching units supporting linguistic and cultural diversity in mathematics classrooms. The authors of the paper argue that teachers of mathematics do not need detailed teaching units (although a survey among them shows this is what they are convinced they need as support) but they need topics with different cultural origins which they then adapt to suit the needs of the particular group of learners, their abilities, skills, age and language competence. In the presentation and the final paper the authors will show how the same teaching unit was grasped by different teachers and what the outcomes of their approach was.

Introduzione

The paper focuses on one of the much discussed issues in mathematics education, which is inclusion of multicultural elements into mathematics lessons. The paper presents partial results of the project 526333-LLP-1-2012-1-IT-COMENIUS-CMP Multiculturalism, Migration, Mathematics Education and Language (M³EaL), whose main goal is to make teachers aware that pupils’ culture (including their language) plays a significant role in the teaching/learning processes, also in mathematics. This awareness helps teachers to pay more attention to the different cultures (and languages) in the classroom and give them a higher value in the educational process. Conditions for an intercultural dialogue in the classroom as well as for better inclusion of pupils from different cultures are thus created.

The project aims to design and implement, in each of the partner countries, teaching materials for mathematics, which take into account situations or activities typical for specific (or even a variety of) cultural areas, as well as the role played by language in the teaching/learning of mathematics within multicultural and multilingual classes. The experimental implementation of these modules is expected to lead to the identification of good practices to be exchanged inside and outside of the
partnership. The proposed teaching materials for the maths classroom, inspired by practical problems and situations from everyday life and from different cultures, encourage teachers to take into consideration the different cultures in the classroom, highlighting their positive aspects and establishing intercultural dialogue in the classroom, thus promoting better inclusion of minority pupils.

The presented paper here discusses what form the developed teaching units should have – should they be developed with respect to a particular age group or school level, or should they only outline the possible multicultural content and leave it up to the teacher to elaborate it for the needs of their classroom and curricula?

**Research in multicultural elements in mathematics education**

It is generally accepted (e.g. Barton, Barwell and Setati, 2007; Bishop, 1988; César and Favilli, 2005) that mathematics teachers feel the necessity for training and materials which reflect the needs of their students in terms of linguistic and cultural differences. Their pupils from minority cultures and/or those with a migrant background encounter more difficulties than their native classmates in acquiring fundamental mathematics skills. Moll et al. (1992) claim that these different cultural backgrounds also provide “funds of knowledge” (i.e. “historically accumulated and culturally developed bodies of knowledge and skills”) that can support the learning process. However, these “funds of knowledge” have only limited applications in many European classrooms, since they require a close connection and collaboration between teachers, parents and the minority community, and are mainly applicable if there is only one, fairly homogenous minority culture present, which is not the case in most European school backgrounds. Mostly however, learning a new language and culture at the same time as you learn mathematics places additional burden and challenges on migrant and minority pupils (Norén, 2010; Steinhardt & Ulovec, 2013).

Two years ago Ulovec et al. (2013) spoke in their paper of the lack of attention paid to multicultural aspects of teaching mathematics in contrast to a relatively large amount of research relating to multiculturalism in general without really making a distinction between subjects. Only some research focused on the difficulties in relation to the teaching of a particular subject and, moreover, it mainly covered teaching of language or natural sciences (McDermott & Varenne, 1995); research on mathematics teaching was rarer.

However, the changing reality in the classrooms across Europe started to attract attention of mathematics educators and now one can come across a variety of researches focusing directly on the specifics of teaching a culturally and linguistically heterogeneous group of learners in mathematics. A whole working group Multiculturalism and reality at the last CIEAEM conference in Lyon was trying to approach the issue from various perspectives (Aldon, Di Paola and Fazio, 2015). Attention was paid to the difficulties an individual with minority background faces in mathematics classrooms, in problem solving in exams, to the issue of what mathematics actually means to different groups of people and what value they attach to it, to the patterns of parental involvement in their child’s mathematics learning in different sociocultural groups, to stereotypes in mathematics assignments and how these may affect minority pupils, to relations between experiences, languages, culture and power in multilingual mathematics classrooms and study the concepts of discourse and agency. Attention was also paid to the issue of how to make the mathematics curriculum more meaningful to minority (Roma) children and how a more meaningful curriculum could improve their participation and performance. This is in line with Meany and Lange (2013), who discuss the issue of learners’ transition between contexts and warn of the additional difficulties for learners if their experience of home context is very different from contexts they come across at school.
This shows that the number of perspectives is considerable, moving attention from one student and their background and obstacles to discourse and power in general, to meaning of mathematics to different learners, to cultural obstacles and to how to construct more meaningful curricula to allow learners from different sociocultural background to get involved. Santomé (2009) warns that if schools are to contribute to increasing justice and equity, they will have to analyze to what extent the curriculum is respectful of people’s different cultures.

The authors of this paper are convinced that culturally heterogeneous learning environments in mathematics will allow learners to get acquainted with other cultures and their values. Moreover, they offer them novel, innovative ways of solving a problem, can offer new tools and procedures that are used in other countries and cultures and may develop their creativity and originality of methods used. Obviously, inclusion of elements from other cultures in mathematics will be of benefit both to majority and minority learners in the classroom.

A questionnaire survey conducted within the project M³ EaL showed that teachers feel an urgent lack of materials that they could easily use in heterogeneous classrooms. They ask for elaborated teaching units that would be ready for their use. This attitude is understandable, the workload teachers in countries across Europe face is enormous. However, taking account the variety across Europe, this means developing thousands of different teaching units.

The project partners agreed they would develop teaching units ready to be used in mathematics classrooms without any further modifications, i.e. in a way that teachers ask for. However, piloting of these materials has shown an interesting conclusion – considering the variety of classrooms, equipment, needs, abilities and skills of learners and teachers across Europe, it seems what is really needed are topics that can be adapted individually by different teachers for different learners and conditions. Instead of thousands of teaching units, we should look for rich sources of mathematics and alternative solving procedures that can then be adapted by individual teachers.

This paper shows one such multicultural topic (finger multiplication as a method of multiplication of a different cultural origin – though it is not really clear where exactly the method comes from – various sources speak of Russian, Gypsy or Chinese origin), which can easily be adapted for mathematics classrooms at very different levels, primary, upper secondary, teacher training.

A teaching unit with multicultural topic

The original topic was developed by the Norwegian partner of the project (Barbro Grevholm). The area of study is Multiplication from different approaches (history, culture, traditions, use of tools and books), the use of concrete tools in calculations, the use of early algebra for formulation of rules for multiplication and for proving mathematical results, different ways of proving in mathematics and mathematical reasoning. The aim of the unit was to make pupils reflect about the process of multiplication, realise the properties of multiplication and see links between multiplication and other areas of mathematics. Pupils may also reflect upon what they need to know by heart in mathematics and what can be reproduced with different tools or aids and may notice that mathematics is constructed and used by ordinary people in many parts of the world.

This teaching unit was piloted in two very different settings – 3rd grade of primary school in the Czech Republic using CLIL (Content and Language Integrated Learning, i.e. the lessons were conducted in English, thus making the language a “barrier” to everybody in the class), 18 year old upper secondary students in Austria and pre-service teachers of mathematics in the Czech Republic. The teachers studied the Norwegian unit, analysed it and adapted it to suit the needs of their classrooms.
This paper only gives an overview of the piloting process. Its scope does not allow the full description of how the teachers conducted the lessons and to describe the course of the lessons and learners’ activity and involvement; this will be presented in the oral presentation at the conference. It shows that the proposed activity can be successfully used in different settings, for different age groups, for both heterogeneous and homogeneous environment and different languages of instruction (e.g. CLIL – learning content through an additional language).

**Piloting in Austria**

The teaching unit was piloted by a female mathematics teacher with five years teaching experience working in an upper secondary school near Vienna. The Austrian project team sent the material to the teacher approximately 3 weeks before the planned piloting activity. The teacher had a 5th (age 14-15 years), 6th (15-16) and 8th (17-18) grade available for piloting. After a meeting with the project team, she chose to conduct the piloting during a regular mathematics class (50 minutes) in the 8th grade. Eight students (age 17-18), three of which are migrant students, attended the class, which was video recorded and observed by a member of the Austrian project team.

The teacher conducted session 1 as described in the Norwegian material by handing out sheets containing a multiplication table from the year 1601 and started a group discussion about it. This discussion lasted about 12 minutes. The students were particularly interested in the aspects of why there was a need for such tables, whether such tables existed in their own cultures’ history, and (mathematically) why these (shortened) tables were sufficient and contain the same basic data as the traditional, square-matrix shaped full multiplication tables they know. The information about the various aspects was partly given by the teacher, partly the students used internet resources to retrieve additional information.

Session 2 started with the introduction of the method of finger multiplication (5 minutes). Students were then asked to try the method out and find an explanation why it works (15 minutes). Students came up with several explanations and wanted to find out whether the method can be extended for numbers with more than one digit. They also were interested in whether this or other hand calculations were used historically. Two of the migrant students (from Turkey) reported about finger-based calculation methods from their own culture (12 minutes).

After the class, students were asked by the teacher about their experience with this teaching unit. Both the migrant and non-migrant students responded very positively. The migrant students particularly mentioned the chance of giving background information about their own culture that the other students did not know before. The non-migrant students commented positively on the various historical and cultural references that they not usually get during regular mathematics lessons.

Also, an interview was conducted with the teacher after the class. She particularly welcomed the possibility of having various anchor points for cultural references, and the opportunity to have the migrant students not only participate, but being a source of information for the other students. The piloting clearly showed that students are interested in mathematics content from different cultures, and that the active participation of migrant students and the introduction of their cultural backgrounds can enrich the learning situation.

**Piloting in Czech Republic**

The teaching unit was piloted directly by one of the members of the project team and a co-author of this paper who, apart from being involved in research in the field of education, is a teacher. The teaching unit was piloted in the 3rd grade (9 year old pupils) in a primary school in Prague. It was piloted in a sequence of 4 lessons that were taught in the period of four months (about once in 6
Some of the lessons were video recorded and all the lessons were open to other teachers of the school as the school is now experimenting with the potential of CLIL in teaching in general and also in mathematics.

The teaching experiment was conceived as a sequence of lessons over a longer period of time. The multicultural background of the original Norwegian unit seemed to be the perfect environment for introducing the concept of learning mathematics through English to the learners who had had no former experience with it. Because the teaching unit was a CLIL unit and because it was planned for several lessons, the original Norwegian unit was supplemented by other activities – two different kinds of line multiplications whose origin is reported to be Chinese and games and other activities aiming at developing language skills (games with numbers, songs with numbers, What number am I?) or calculation skills (number centipedes). All the activities had two objectives – developing language and mathematical competence.

The lesson on finger multiplication was taught in September, 4 weeks after the beginning of the school year. The advantage at that stage was that the pupils had already mastered multiplication tables up to five but had not learned multiplication tables from 6 to 10. Thus it was an ideal situation for introduction of finger multiplication. Children who have memorized multiplication tables will find finger multiplication unnecessarily too difficult and time demanding. The teacher started by demonstrating the principle. The children were explained what number was represented by which finger and then shown by the teacher how the system works using fingers and whiteboard. The teacher then asked the children to do it but it turned out that at this point only very few understood. The teacher decided to demonstrate two other problems in front of the whiteboards but this time a pupil was always invited to assist and be showing it on their fingers. The whole class was saying the numbers out loud. After this the pupils were asked to work on their own. The teacher was monitoring, assisting individually to those pupils who needed help. One by one the pupils eventually grasped the principle. The teacher could see the “aha” effect when the pupils finally grasped the principle. In the subsequent lesson the teacher came back to the principle and could observe that the pupils found it relatively easy (but also inefficient as they know multiplication tables already). It was time to move to line multiplications of two and three digit numbers. The whole sequence of lessons, its outcomes and pupils’ attitude are described in detail in (Moraová, Novotná, 2015).

It can be concluded that the teaching unit with cultural background proved to be very motivating and suitable for CLIL lesson. The uniqueness of the context contributed to the pupils’ motivation and interest to be working on mathematics in English.

Figure 1. Finger multiplication
Conclusion

The fact is that mathematics classrooms are growing increasingly multicultural and multilingual. This growing diversity can be a chance to increase the quality of teaching (Slavin, 1994), but teachers will have to be trained to handle the situation. The differences in cultures and languages make the maths teaching-learning process harder for migrant and minority pupils than it is for majority pupils.

The presented study of adaptations of one topic (which could be understood as substantial learning environment in Wittmann’s (1995) sense) clearly shows that it is possible to use the same environment in working with learners of different ages, knowledge of mathematics and sociocultural backgrounds. It is not needed to develop hundreds of teaching units in which teachers would painstakingly look for the one they can use without adaptations. On the contrary, pre-service and in-service teacher training should develop mathematics teachers’ ability to work with materials creatively, to look for interesting topics and methods and modify them in such a way that makes the materials tailor-made for the needs of their own classroom.

The teaching experiment also proved that introduction of innovative solving methods whose roots come from different cultures is very motivating for the learners, brings fun into lessons and also develops flexibility of learners’ thinking processes and awareness of the value of other cultures. The 3rd graders in the Czech Republic enjoyed the lesson, were active and involved. The teaching unit was piloted in 3 consecutive lessons and the teacher reported on the pupils’ delight when they were told the teaching experiment would continue and not end after the first lesson. Doing something magic, learning a “trick”, getting introduced to a way of work from other cultures was challenging, entertaining and effective. The Austrian students appreciated the chance to inquire into mathematics and its different cultural backgrounds, to look for the principles behind and for justification of the procedures. Although their mathematics levels were way beyond simple multiplications, they still found it interesting and satisfying to find out why these procedures work and what their limits are. Czech pre-service teachers grew aware of the potential of substantial learning environments as a rich source of teaching materials and experimented with how to adapt a teaching unit to meet different purposes.

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Re-approaching the perceived proximities amongst mathematics education theories and methods

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Abstract: In this paper, we discuss the role of perceived proximity in Mathematics Education as a crucial factor determining the relevance of theoretical and empirical tools in mathematics teaching-learning research. It is posited that by topologically re-defining the proximity of various till now distant research tools, the phenomena may be researched as complex wholes thus deepening our understandings. A research project about proof is discussed as an exemplar of such a perspective.

Résumé : Dans cet article, nous discutons le rôle de la proximité perçue dans l'enseignement des mathématiques comme un facteur déterminant de la pertinence des outils théoriques et empiriques dans la recherche sur l'enseignement et l'apprentissage. Il est posé que par redéfinissant topologiquement la proximité de différents jusqu'à maintenant outils de recherche, les phénomènes peut être étudié comme des ensembles complexes, approfondissant ainsi notre compréhension. Un projet de recherche sur la preuve est discuté comme un exemple d'une telle perspective.

Introduction

Mathematics education research often describes the teaching-learning phenomena through conceptual dipoles, including relational/instrumental understanding (Skemp, 1976), conceptual/procedural knowledge (Hiebert & Lefevre, 1986), deep/surface approaches (Marton & Säljö, 1976), process/object (Dubinsky, 1991, Sfard, 1991). Though the value of these perspectives is well-documented, researchers have acknowledged the need for developing more complex approaches, either by extending the dipoles to n-dimensional models (for example, process/object/precept, Gray & Tall, 1994; deep/surface/achieving, Biggs, 2001) or by conceptually re-visiting the phenomena with a completely new theoretical perspective (for example, a systems theory approach that focuses on the relationships amongst the constituting agents of a phenomenon; Davis & Simmt, 2003; Moutsios-Rentzos & Kalavasis, 2012). At the crux of each of these approaches is the researchers’ perceived proximity amongst the phenomenon, the theoretical approach and the employed methods: each of the researchers’ decisions is determined by whether or not they consider a theory or a method relevant, close, to the under investigation phenomenon. In this paper, we attempt to topologically re-approach the notion of proximity in mathematics education, considering three aspects: a) socio-cultural proximity, b) mind-brain proximity, c) cognitive-affective proximity.

It is posited that by topologically re-defining proximity, multiple perspectives, theories and methods may be considered to be relevant to the phenomenon, thus providing the opportunity to gain deeper understandings of the complexity of the phenomenon. Nevertheless, mathematics education theorists have emphasised the dangers that such attempts entail, as they may lead to a bricolage of theories, lacking meaningful conceptual coherence (see, for example, Wedege, 2010; Bikner-Ahsbahs & Prediger, 2014). Thus, it is argued that the re-defined proximity should draw upon a theory linked with the under investigation phenomenon that may act as a meaningful attractor of the different theoretical perspectives concentrated in the different aspects of the phenomenon, thus allowing the networking (Bikner-Ahsbahs & Prediger, 2014) of different perspectives.
The paper concludes with a brief presentation of the implications of such an approach to research project focussed on proving in goal-oriented exam-type tasks (Moutsios-Rentzos, submitted). In this example, Skemp’s theory of social survival and internal consistency (1979) is utilised as the meaningful attractor that facilitates the appropriate networking of the diverse perspectives.

**Proximities: socio-cultural, mind/brain, cognitive/affective**

In this paper, we discuss the proximities within two research loci in accordance with two of the major strands of educational research: the sociocultural locus and the psychological locus.

**Socio-cultural proximities**

The contemporary societies are characterised by socio-cultural diversity, linked with the socio-cultural permeability offered by the technological and financial feasibility to travel. This is more evident in the big urban areas within a country and/or in the so-called ‘developed’ countries where the opportunities (about, amongst others, career and entainment) appear to be greater and more diverse, thus gathering the interest of larger and more heterogeneous parts of the population.

Mathematics education researchers have noted that socio-cultural proximity has been re-defined within the same country and even the same city, acknowledging the multifaceted challenges that mathematics education encounters in a globalised society in their research reports (Atweh et al, 2007) and in their meetings (see the relevant sub-theme in CIEAEM 66).

Within the same school unit, the time-space is expanded to include virtual social networks, re-defining the traditional power relationships amongst the protagonists (including, amongst others, students, teachers, principals, parents). Though the expanded school time-space maximises the permeability of the school unity system, the protagonists’ constructions of the new reality seem to qualitatively differ with respect to the relevance of the virtual social networks with the in-school mathematics teaching and learning phenomena (Moutsios-Rentzos, Kalavasis & Sofos, 2013).

Moreover, within the same city, the students attending a ‘multi-cultural’ school unit have been found to differ in their perceived parental involvement about mathematics (Kafousi, Moutsios-Rentzos & Chaviaris, 2014). For example, the students attending a ‘multi-cultural’ school appear to experience stronger perceived parental involvement, which appears to be relatively stable with respect to the students’ age and to the different calendar years (Moutsios-Rentzos, Chaviaris & Kafoussi, submitted).

Furthermore, the socio-cultural diversity appear to become less evident as the countries adopt international standards (International Baccalaureate, credit units) and evaluations (PISA, TIMSS). Nevertheless, the attempted convergence of the curricula seems not to be in line with the characteristics of educational systems evaluated as ‘excellent’ (Sahlberg, 2011).

Moreover, in countries that are geographically and socio-culturally ‘distant’, aspects of the ways that mathematics are conceptualised appear to be common. For example, the findings of a comparative study (Moutsios-Rentzos, da Costa, Prado & Kalavasis, 2012) conducted in Brazil and in Greece investigating the views that school principals hold about mathematics (both as a discipline amongst disciplines and as a school course amongst other courses) included convergences in the identified epistemic views, which are also evident in their views about mathematics as a school course.

Consequently, the socio-cultural proximity as a factor with important implications in the teaching and learning mathematics seems to be in need of a topological re-definition in order to regain its analytical and descriptive power. Such a re-definition includes the assignment of not necessarily geographical aspects (including cultural identity, technological abilities, social networking etc) to the traditionally considered as mere geographical characterisations (for example, the classroom, the school unit, a city, a country), thus transforming them in topological characterisation of the
contemporary expanded socio-cultural time-space.

**Brain/mind proximities**

Neuroscience has identified invariant characteristics of the human species, revealing links between brain activity and mathematical thinking, with respect to both its specific and its more general aspects. For example, considering the calculation processes, ‘exact arithmetic’ (such as a single arithmetic operations) appears to be linked with word-associated brain activity, whilst ‘approximate arithmetic’ seems to require bilateral visuo-spatial processing (Dehaene, Spelke, Pinel, Stanescu & Tsivkin, 1999). On the other hand, considering broader aspects of mathematical thinking, left hemisphere activity has been linked with logico-mathematical reasoning and problem-solving (Bear, Connors & Paradiso, 2007; Rayner, 1998).

Furthermore, research evidence appear to support the innate character of various cognitive processes or affective processes. For example, subitising has been suggested to be ‘built-in’ the humans’ mental artillery (Lakoff & Núñez, 2000). Moreover, seven of the human’s affective responses and alertness to a stimulus –identified as the seven basic emotions, namely sadness, anger, contempt, fear, happiness, disgust, surprise– have been found to transcend socio-cultural contexts in their manifestation in the human facial expressions, thus appearing to be universally common to humans (Ekman, 1992).

On the other hand, the reported brain universality seems to be in stark contrast with mathematics education research reporting the situated –cognitive and affective– character of mathematical thinking and learning (Brown, Collins & Duguid, 1989; Hannula, 2012), as well as the diverse conceptualisation of mathematics itself (consider, for example, evidence from the ethnomathematics research paradigm; D’Ambrosio, 1985). Furthermore, considerable research evidence seem to question traditionally conceptualised stereotypical views of innate characteristics (such as gender; Walkerdine, 1998).

At this point, it is emphasised that the contrasted research evidence are usually documented with epistemologically ‘opposed’ methodologies and respective research questions. For example, the reported neuroscience research evidence are based on objectively measured variables, in the sense of a natural magnitude (electricity, movement etc) that can be measured with an instrument, thus condensed to a single number. On the other hand, the aforementioned mathematics education research evidence draw upon softer qualitative techniques, including observations and interviews, the results of which are obtained through interpretative methods and are communicated by diverse means, notably text. These accord with Radford’s (2008) triadic view of theory including “a system, $P$, of basic principles […] a methodology, $M$, which includes techniques of data collection and data-interpretation as supported by $P […] a$ set, $Q$, of paradigmatic research questions” (Radford, 2008, p. 320).

Following these, notwithstanding their conceptual differences, we posit that by co-considering the evolutionary derived brain universality along with the socio-culturally derived noetic locality, by acknowledging the fact that mathematical thinking and learning occurs in and emerges from the interaction of both invariant and local characteristics, till now distant and incongruent theories and methodologies may converge to be meaningfully synthesised in a research project.

**Cognitive/affective proximities**

Considering the mental processes, the aforementioned can be localised in the well-documented differentiation between mathematical thinking dispositions (both about the representation and the processing of the information; Burton, 2001; Duffin & Simpson, 2006) and actual, task-specific mathematical thinking. Research evidence (Moutsios-Rentzos, 2009) identify complex links between the two levels, adding a broader third level of general thinking dispositions that go beyond mathematics and extend to cover thinking in general.
In a similar vein, affective dispositions (values, beliefs, attitudes etc) have been identified with respect to mathematics in specific or to education in general (Hannula, 2012), which also appear to be in complex relationships with actual task-specific affective experiences about mathematics (Moutsios-Rentzos & Kalozoumi-Paizi, 2014).

Hence, it is posited that the affective-cognitive proximity could be re-visited in order to identify topological defined loci within the aspects of the six-dimensional space formed by the affective-cognitive and dispositions-actual interactions spectra.

**Instead of a conclusion: re-approaching proximity in proof research**

Re-approaching the theoretical-methodological proximity in proof research produced a multi-levelled methodological-theoretical framework (Moutsios-Rentzos, submitted). The proposed framework incorporates the topological proximity of the affective and the cognitive, of the disposition and the actual, of the subjectively perceived/reported and the objectively bodily expressed/measured in the case of thinking about exam-type university proving questions.

It is posited that the aforementioned ostensible dipoles assume topological proximity through Skemp’s (1979) theoretical framework about both social and inner survival that the learners strive to achieve in goal oriented activities. Skemp (1979) theorised that the learners need to survive socially by meeting the socially accepted, usually externally set criteria of a task, as well as internally in the sense of surviving a task in ways that are consistent with the learners’ internal reality (that may include affective and cognitive aspects).

Following these, the research project focused on the proving processes of mathematics undergraduate students, when attempting to produce exam acceptable proving questions in the form ‘Given that … Prove that…’ (Moutsios-Rentzos, submitted). The multi-level approach brought together different levels and aspects of the phenomenon, along with diverse methods:

1) Actual task-specific cognitive and affective experiences

   a) The students’ cognitive strategies, referring to the students’ actual, task-specific dealing with a proving question. Clinical interviews (in the sense of Ginsburg, 1981) were conducted utilising the A-B-Δ proving strategy classification scheme (Moutsios-Rentzos & Simpson, 2011). The chosen conceptualisation of strategy and methodology were chosen in line with the research question posed: the identification of the qualitatively different thinking strategies employed when the students actually produce an exam-acceptable proof.

   b) The students’ basic emotions (as defined by Ekman, 1992) as they produce and present the solution. The identification of the emotions was based on the students’ video-taped facial muscle movements with a researcher trained to utilise the ‘Emotional Facial Action Coding System’ (EMFACS; Ekman, Irwin & Rosenberg, 1994). The chosen theory implies that in this project we are interested in the students’ universal, objectively measured and evolutionary derived affective reactions during their proving process. These emotions are clearly differentiated from the mentally processed, socially situated, affective reactions towards a proving situation (see Hannula, 2012).

2) Cognitive and affective dispositions.

   a) The students’ reported general thinking dispositions, as conceptualised by Sternberg’s thinking styles and measured by the self-report Thinking Styles Inventory (Sternberg, 1999). With this conceptualisation and methodology, the focus is on the self-reported and experienced cognitive patterns, as identified by each participants.

   b) The students’ affective dispositions (attitudes and beliefs) towards mathematics and exams, as reported by the students in the semi-structured interviews that accompanied the
students’ answering the proving questions. The semi-structure interview was specifically designed for this project and is focussed on mathematics-specific self-reported and experienced affective patterns.

3) The socio-cultural effect as manifested by the students’ country of residence, with respect to the teaching and learning mathematical proof and importantly for this project with respect to the assessment realities that the students experience. For this purpose, students from Germany and Greece participated in the study.

The conducted analyses were quantitative and/or qualitative in line with the topologically attracted to be neighbours yet clearly distinct theoretical considerations of each level (in line with Radford, 2008).

Notwithstanding the important epistemological questions that may be raised (extensively discussed in the literature; Johnson & Onwuegbuzie, 2004; Smith & Heshusius, 1986), it is argued that the proposed research synthesis derives from a theoretical framework appropriately chosen to fit a situation that acts as a meaningful attractor that allows the appropriate networking (Bikner-Ahsbahs & Prediger, 2014) of diverse, seemingly incongruent, theories and methods, crucially avoiding the syncretism pitfall.

Consequently, within the aforementioned framework, it is possible to study the mathematics teaching-learning phenomenon of dealing with exam-type proving questions as complex whole, without losing the analytic capability to investigate its (partial) aspects.

By topologically re-approaching proximity through a theory (specific to the under investigation phenomenon) acting as a meaningful attractor, the phenomenon re-claims its meaningful research wholeness, in ways that go beyond the mere sum of the studies of its parts.

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Symmetry in portuguese fishing communities: students critical sense while solving symmetry tasks

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Abstract: This article analyzes the knowledge and the critical thinking level of students from two schools of different cultural contexts, regarding the mathematical topic of symmetries. The primary goal is to find and compare the critical thinking of students from a fishing community with students of a school of an urban type, when faced with tasks on symmetries involving artefacts / geometric motifs related to fishing activity. With a theoretical foundation based on Ethnomathematics, it gives prominence to the cultural and professional context students are from, so tasks were built taking into account the fishing context. Students revealed difficulties in symmetries identification and particularly in the representation of figures with symmetry. It also found that in tasks completion, students of the fishing context school proved to have a more accurate critical sense than students from the urban type school.

Résumée : Cet article analyse les connaissances et le niveau de la pensée critique des élèves de deux écoles de différents contextes culturels, concernant le sujet mathématique de symétries. L'objectif principal est de trouver et de comparer la pensée critique des élèves d'une communauté de pêche avec des étudiants d'une école de type urbain, lorsqu'ils sont confrontés à des tâches sur les symétries, comprenant des artefacts / motifs géométriques liés à l'activité de pêche. Avec une base théorique basée sur Ethnomathématique, il met en évidence le contexte culturel et professionnel d'où les étudiants viennent, alors les tâches ont été construites en tenant compte du contexte de la pêche. Les étudiants ont révélé des difficultés dans l'identification des symétries et en particulier dans la représentation des figures avec symétrie. On a également constaté que dans l’achèvement des tâches, les élèves de l'école de contexte de pêche ont montré d’avoir un sens critique plus précis que les élèves de l'école de type urbain.

Objectives and main idea  
From the most basic act of drawing a line segment in carpentry, to the more complex geometric reasoning in tracing a large bridge, there is geometry knowledge being applied. Research in the context of professionals with low educational level, demonstrate a strong presence of mathematics and the use of geometry in their everyday professional activities (Sousa, 2006; Lucena, 2002; 2005). In this work we will take into account geometric motifs used in two fishing communities routines, with special attention to symmetries. The focus on symmetry relates to the fact that there are investigations on this subject in professional groups (Sousa, 2006; Vieira, Palhares and Sarmento, 2008), but also because curriculum documents recommend work with isometries and symmetries in elementary schools. Still, students reveal many difficulties when faced with situations involving symmetries. Being this work grounded on ethnomathematics, one of the central concerns (besides investigating fishing context situations revealing the application of symmetry), is to assess students critical sense in performing tasks on symmetries, in two communities, one belonging to a fishing community the other not.

In this research work, we highlight the symmetries of reflection, rotation and translation. These are the ones identified, as a result of previous fieldwork, as used in daily life of fishing communities.
Ethnomathematics

Ethnomathematics is the theoretical foundation that frames this work both from a cultural point of view and from mathematics education. Etymologically, the word Ethnomathematics can be understood as the art or technique (techne = tica) to explain or to understand reality (matheme) within a specific cultural context (ethno) (D'Ambrosio, 2012). Ethnomathematics, rather than an association to ethnic groups (D'Ambrosio, 1998), is the research of mathematical practices and conceptions of a social group, including also an educational work (Oliveira, 2004; Miller, 2004) which develops in order to identify and decode the group's knowledge and draw comparisons between knowledge of everyday life and academic knowledge (Knijnik, 2008).

It is multicultural societies like ours that educators should reflect on which culture should be considered in the classroom. The dominant? Of the minority? Or maybe we can create a new culture, made up of all the cultures of all citizens living in a given territory. Knowledge and culture are the two foundations of mathematics. So we can assume that in a given area where there are many cultures, we are not facing the Ethnomathematics of these cultures, but in the presence of several (ethno) mathematics (Sousa, 2006) developed over years by cultures in that territory. From the epistemological relativism and demarcation of cultures as interpretation systems of the world, maybe we can talk about ethnomathematics or multimathematics (Oliveras, 2006).

In this paper we give special emphasis to the Ethnomathematics within the context of fishing cultures and also within classroom contexts. We look for a didactic transposition, so that Ethnomathematics, seen as the math arising from social groups (D'Ambrosio, 2006), help in this contextualization process, but also in humanizing mathematics (Palhares, 2012). Thus, the position assumed is taken from D'Ambrosio (1993; 1998; 2002); Olive (2004); Monteiro (2004); and Knijnik (2008), who evoke an investigative nature on the mathematics present in social minorities and an educational character, aiming to combine the mathematical knowledge of everyday life with the formal / school mathematical knowledge. The investigative nature of this work concerns the collection of everyday (ethno) mathematical aspect within fishing communities. The educational refers to the context and implementation of these (ethno) mathematics in the classroom, after analysis and cultural and curricular consideration.

Methodology

Qualitative research is our methodological framework because it is based on a holistic view of the particular setting to be researched without isolating it from its natural context (Amado, 2013).

One of the specific methods used was the multiple-cases study in ethnographic context. With it was possible to observe in detail individuals in each of the specific contexts (Lessard-Herbert, Goyette & bouti, 1994).

We also used participant observation, unstructured interviews (with the help of audio and video recordings) and document analysis, mainly documents and small teaching experiences in the classroom context.

It is therefore intended, in accordance with the theme "symmetry", to answer the question:

- What level of critical sense do students of the Caxinas fishing community reveal in comparison with the most urban contexts of other students?

We prepared a set of tasks, taking into account the curricular documents, as well as age and cultural background of the students. Its focus is on symmetry, while the context addresses the fishing everyday. We tried to draw up various tasks on this content, asking students different levels of knowledge regarding the symmetry.

The proposed tasks are contextualized tasks arising from the fieldwork carried out in fishing communities of Câmara de Lobos (Madeira) and Caxinas (Vila do Conde).
The cultural context involves boat construction, objects used in daily fishing and tiles used in the homes of fishermen. The mathematical content is symmetries.

Tasks that were used, adopting the knowledge that fishermen use in their work, were validated with regard to the suitability to the specific context of these fishing communities, but also validated by a panel of mathematics education experts.

Each task was applied in the classroom, in two separate phases. Initially, the tasks were applied before the approach to the subject related to symmetry. After teaching symmetry, the same tasks were reapplied to the same students. The tasks were applied in 2 groups of 5 and 6 years of schooling (10-12 years). The tasks were applied in two schools with different cultural and professional contexts: one belongs to the geographical area of a fishing community, and the other, is embedded in an urban school. The tasks were applied with 45 students of fishing context school, and 39 students from another school.

To apply this set of tasks in the classroom, three 90 minutes periods were dedicated in each class. Students individually solved each task to try to understand the concepts / knowledge that each student had about the symmetries and the critical sense of the students in relation to their own resolutions tasks. Their arguments and strategies were recorded on paper and on video.

**Symmetries and critical sense**
**School of the fishing community of Caxinas**

**Critical sense about the rigour in the construction of artifacts**

In some artifacts of this task (Figures 1 and 2), the overwhelming majority of students feel they have symmetry, but they question the fact that the objects were not built with mathematical rigor necessary to have symmetry. In other words, students admit that the craftsman who built each artifact intended to build with the symmetry and therefore consider themselves to have symmetry, but mathematically think they should be more "perfect". From the close observation of the represented figures (Figure 1 and Figure 2) and the symmetry axis represented by the students, the objects in all the rigor do not have symmetry of reflection due to imperfections in artisanal construction. Students turn out to be critical of this and despite considering the objects have symmetry (because the craftsman have that intention at the time of manufacture), make the repair that the final product should have more harmony. In this situation the combination of knowledge of everyday life and the formal school knowledge is evident, but also the critical sense with regard to the construction of the artifacts and the presence of symmetry in them.

![Figure 1. fishing nets sewing needle](image1)

![Figure 2. Davit](image2)
Critical sense about errors

In this task it is intended that students complete in Figure 3 in accordance with the represented part, obtaining a helix with rotational symmetry. It is found that the students reveal it hard to correctly complete the picture.

Figure 3. Propeller for students to complete

Taking as its starting point the already represented blade (1st quadrant), the overwhelming majority of students can correctly complete the representation of the 4th quadrant blade, but few can represent all blades correctly as in the representation of one of the students (Figure 4).

Figure 4. Correct representation of the propeller after several attempts

Although the student has managed to correctly represent the propeller, there were several unsuccessful attempts as seen in the transcript of the field work (Figure 5). One can also verify that the student recognizes that something is not correct, identifies the part of the picture that is not correct, but does not know why it is inaccurate. His keen critical sense in relation to his representations is salient, erasing immediately each blade that he considers not to be visually in harmony with the rest.

Student - Teacher, I have a doubt here. The propeller leaves, is it like this? (Pointing to the propeller blade already represented in the figure).
Student - Aahhh ... wait. (Deletes the part he feels it is wrong and represents again).
Student - I know. (Correctly representing the blade of the 4th quadrant).
Student - This (the blade of the 3rd quadrant) must be reversed.
Researcher - Reversed how?
Student - This part must be here.

Figure 5. Dialogue on propeller representation
The greatest difficulties arise in the 2nd and 3rd quarters. A considerable part of the students complete the picture according to Figure 6.

In this case the student begins to represent the blade in 2nd quadrant by reflecting the blade of the first quadrant. He watched for a few seconds the representation stating, "The blade is in reverse." The student makes several attempts, but fails to correctly represent the propeller. The final product, is given in Figure 6, however, the student said that the representation is not correct, that is, despite failing to represent the propeller with correction, the student is aware that its representation is not correct and is able to identify the parts of the figure to be changed. There is in this situation, strong evidence that the student is critical of his work because he recognizes that something is not right in the representation. The student delivers the task saying, "Teacher, this is not right." This is just one example of many students who, although can not properly represent the propeller, reveal quite critical sense and assume that the representation is not correct.

![Figure 6. Incorrect propeller representation](image)

**School of Calendário**

**Critical sense about the rigour in the construction of artifacts**

In the school of Calendário a large part of the students indicated that the artifacts shown in Figure 7 have only the vertical axis of reflection symmetry. However a considerable number of students said that the artifact to the right (Davit) has vertical and horizontal axis of reflection symmetries; others indicate 4 symmetry axes (Figure 9) and still others believe that the objects do not have symmetry (Figure 8). Only two students point out that both artifacts were not built with due rigor so that mathematically can be considered to have symmetry. The remaining relate nothing regarding this aspect. They do not reveal a critical keen sense as happens in the fishing environment school. In the city context in which the school is located, students externalize little sensitivity to analyze everyday aspects, at least from fishing everyday. They have difficulties in mobilizing fishing everyday knowledge for math classes and connect this knowledge with mathematics learning on the classroom. In the fishing context of the tasks these students are less critical than students from the fishing community of Caxinas. Although many of the students from both schools recognize that artifacts have symmetry, students of the school of Calendário tend to be more mathematically formal (not more able) which may affect the critical sense in relation to the fishing everyday situations.
Figure 7. The student believes that the artifacts have one or two axes of symmetry.

Figure 8. Students consider the artifacts have no symmetry

Figure 9. Students consider that the artifact has 4 axes of symmetry

4.1.1. Critical sense about errors
At the school of Calendário most students also failed to properly represent the propeller. Most students completed the propeller as shown in Figure 10. The students completed the helix so that it had vertical axis of reflection symmetry, not realizing that the symmetry involved was rotational symmetry.
In the following cases, the propellers presented do not have any symmetry, although students use isometries in their reproductions. In Figure 11 the student properly represent the blades of 3rd and 4th quadrants using rotation, but in the blade of the 2nd quadrant uses reflection. In Figure 12 and Figure 13 students use rotation on all the blades, but represent incorrectly the blades of 2nd and 3rd quarters. In Figure 14 the student tries to represent the helix so it has vertical axis of reflection symmetry, however, the blade of 4th quadrant is not correct. In addition to this failure it is not correct to build the propeller so that it has symmetry of reflection, as in reality it has only rotational symmetry.
Most students make mistakes of this kind in their representations, which reveals something unusual. More curious still is that students consider that their representations are correct as shown in the testimony of this one student (Figure 15).

<table>
<thead>
<tr>
<th>Student</th>
<th>Teacher, I have done.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Researcher</td>
<td>Do you think it is right?</td>
</tr>
<tr>
<td>Student</td>
<td>Yes, it's right. It was a little difficult and I had to delete a few times, but I could do it right.</td>
</tr>
</tbody>
</table>

Figure 15. Student opinion on the representation of the figure 10

In this task there was a huge disparity in the critical sense of students. At Caxinas school students sometimes fail in their representations, but are aware that they are wrong and even warn the teacher that something is not right in their representations. In the school of Calendário students find it difficult to complete the propeller, fail in their representation, however they consider their representations correct and do not realize they are wrong. There is a number of situations in this school where the propellers represented not even have symmetry, yet the students feel they have symmetry. These facts reveal that the level of critical sense of students in these two schools are very different realities.

**Conclusion**

The implementation of the tasks in two schools of different cultural contexts and at different times (before and after teaching of symmetries), allow us to assess the level of critical sense that students have with regard to mistakes and to results in their resolutions. Given that the tasks were built within fishing contexts, the discrepancies in levels of critical thinking among students from both schools are well visible. Students from the fishing community of Caxinas reveal a more refined critical than students of the other school. Students of the fishing community turn out to be more able to detect and censor errors both in resolutions and in obtained results. There has also been no significant changes regarding the performance and the level of judgment when tasks are applied before or after teaching of symmetries. Thus, one can also conclude that students immersion in school/formal mathematics had no influence on their critical thinking ability in the schools involved.

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