Elementary Mathematics from an Advanced Standpoint and Elementary Views on Advanced Mathematics

Ysette Weiss-Pidstrygach
Mathematical Institute, Georg-August-University, Göttingen, Germany weiss@uni-math.gwdg.de

Abstract
What kind of and how much mathematics should a high school maths teacher know? The experience with a math camp, an innovative form of bringing together high school pupils, university math students and math teacher students as well as university professors in the common aim to teach mathematics sheds new light on this question. Different interests define different positions. The different actors have little common aims since they rarely form a joint community of practice.
Over the seven years of its existence the math camp has evolved from a classical lecture-centred activity for gifted pupils to a much more encompassing experience illustrating the importance of a two way communication between advanced mathematics and elementary mathematics in schools.

What are communities of practice?
The term was introduced by Lave and Wenger (1991). A community of practice is a group of people informally bound together by shared expertise and passion for a joint enterprise. It has an identity defined by a shared domain of interest. Membership therefore implies a commitment to the domain, and therefore a shared competence that distinguishes members from other people. A community of practice is not only a community of interests, it implies sharing information, learning from each other and developing a shared repertoire of resources: experiences, stories, tools, ways of addressing recurring problems ...in short a shared practice (see Etienne Wenger).
Many math departments undertake activities for gifted pupils, where university math professors take care of the next generation by introducing pupils to topics of higher maths. Often this forms a community of practise evolving around a joint enthusiasm for advanced mathematics and operating with modern mathematical notions. Rarely, however, are students involved especially not students wanting to become math teachers. The intention of the math camp project was to broaden the scope of participants, involving a broader set of pupils, as well as students both from mathematics as well as prospective math teachers.

The first math camp- organizational frame
The activity started in 2002 as a programme for mathematically gifted pupils aged 15 to 19. The camp was organized as a week long workshop where pupils were introduced to invariant and knot theory. The topics were covered by lecture series given by two math professors, and eight exercise classes held by prospective math teachers and maths students. Maths students and prospective math teachers were supposed to hold the exercise classes as tutors in pairs. All the materials, the problems for the exercise classes as well as the lecture notes were chosen by the professors and given to the tutors two month before the workshop. The problems were supposed to motivate and illustrate notions and concepts introduced in the lectures, some shaped for exploratory learning, some to repeat techniques explained during the lecture. During the exercise classes the 40 participants were split in groups of eight. Every group was supposed to work out a presentation of a solution to "their" favourite problem. The problem given on the poster advertising the camp showed the expected standards: Given two (concrete) positions of a position game called Schiebefix, is there a move to get from the first to the second position. The pupil could start with trial and error and then develop a theory for certain positions. To find a general solution for any two positions was quite a difficult task without the theory of invariants.

Tutor training
In the preparation of the workshop maths students and prospective teachers jointly started to solve the problems given to them and to prepare themselves for the variety of possible solutions by the participating pupils in the forthcoming workshops. Usually the math students developed solutions and explained them to the prospective teachers who often assumed the role of pupils. Quite a few of these solutions, however, were

based on the theory presented in the lectures. Prospective teachers, who could not rely on a thorough understanding of the theory made clear that this implied conditions that seemed rather artificial and incomprehensible from a school mathematics context. Teaching methods for the exercise classes and the preparation of the presentations were rarely discussed in the preparatory meetings of the tutors. At the first math camp the prepared solutions thus used techniques of invariants given in the lecture notes.

The pupils

Pupils applying for the workshop had to attach their last school report including all marks and a short explanation why they would like to take part. 40 pupils between grade 10 and 13 were chosen: one third with prior experience in out school mathematical activities and two thirds knowing maths only from school. All participants underlined in their application apart from their joy in doing maths that they would like to meet other pupils sharing the same interest. We intentionally chose one third of the participants where the interest in math was not reflected in good marks in school math, hoping to get hold of gifted but underchallenged and therefore no longer motivated pupils.

Workshop implementation

During the workshop the prospective teachers hardly used the prepared solutions. Most pupils came up with their own solutions, some of them working in groups some on their own. During the tutorials another division of labour established itself: prospective teachers organized and supported group work, listening to different approaches, dealing with problems arising from competition, different age, different sex, adolescence. The activity of the math students were normally confined to checking solutions and talking with outstanding (usually single working) pupils about the lecture and related to maths. The final presentations differed considerably: those guided by the math students resembled mathematical lectures, using definitions and generally working in a deductive manner, whereas those accompanied by prospective teachers were much more individual also presenting the process of finding the solution.

Evaluation of the first workshop

At the end of the math camp pupils as well as tutors were asked to fill out a feedback sheet that showed that pupils as well as tutors enjoyed the work shop enormously. Amongst others teachers appreciated being backed by specialists and the math student valued the social capabilities and engagement shown by the prospective teachers. Pupils even suggested subjects for the next math camp.

The prospective math teacher were satisfied with their performance during the work shop, however they did not assume that the materials they had worked through during the camp could be reasonably used in schools.

The formation of a community of practice

The experiences of the first camp triggered the following adjustments:

- the theory taught in the lectures has to be developed based on the problems to be addressed in the exercise class; these problems should be formulated without notions from higher maths;
- the lecturing math professors take part in the preparation of the tutors and adjust the propaedeutic and introductive problems to a level the prospective teachers feel confident with;
- in the preparatory meetings the lecture drafts are discussed with the aim to find formulations that are informal, close to common speech or school math notions and descriptions;
- the “favourite problem presentation” should encourage to reflect the process of finding the solution and should preferably be derived in group work
- based on the feedback of the preparatory meetings and the progress during the work shop the lecturing math professors suggests suitable subjects for bachelor and master theses related to the problems

Supported by the existence of an enthusiastic kernel of students and staff members organising and running the camp repeatedly the activity became more and more a community of practice based on the shared interest to bridge the gap between conceptual mathematics and problem solving example oriented school mathematics.

Experiences intensifying the self conception of the prospective teacher as a mathematicians were:

- teachers’ solid mathematical background knowledge and interest in modern maths supports acceptance and respect from the pupils;
- conceptual approaches and knowledge of general methods and techniques in problem solving can often cover missing immediate creativity;
- complex mathematical concepts like classification are universal and if recognised and used intentionally prove extremely useful in everyday life;

- the prepared elementary approach to a subject in higher maths can be used for project work and activities for gifted children in school;
- the joy of approaching mathematics with the spirit of a researcher and problem solver.

As a result some of the prospective math teachers wrote master theses about motivations to notions or elementary introductions to areas of higher maths. The evaluation showed another important experience coming from the mentoring during the workshop: gifted but not motivated teenagers behave very differently when with like-minded people and can find in this environment a new starting point.

Changes in attitude and in the sense of responsibility for the next generation were also noticeable with the mathematicians:

- some math students continued teaching children outside school in math clubs;
- the elaborated elementary introductions to some notions in higher maths found their way into the lectures in algebra, analysis, topology and geometry;
- links to questions explored in school mathematics were used in introductory courses at the University;
- non formal discussions of well understood mathematics (not only related to the math camp) subjects became routine

There is perhaps no universal answer to the question: What kind of and how much mathematics should a high school maths teacher know? Indeed, decisions about which subjects of higher maths are becoming "elementary" now, how much conceptual background is needed to feel secure in the classroom, are individual. However the joint work in this community of practice helped the members to find their own position and a joint frame related to the subjects treated.

The idea to integrate the two approaches "Elementary Mathematics from an Advanced Standpoint" and "Elementary Views on Advanced Mathematics" into teacher training is not new. The new curriculum for the high school teacher training at the ETH Zürich includes such courses (see Urs Kirchgraber).

To have a wide domain of joint experiences the subjects of the math camps are changed annually. So far they were:
- Invariants and knot theory
- Symmetries of Rubik's Cube and different geometries
- Prime numbers and coding theory
- Symmetry and its use in differential equations
- Projective geometry and other geometries
- Invariants in topology
- The projective plane

The Göttinger model to link programs for mathematically gifted children and maths teacher training is easy to adjust to different educational and training systems. In most countries the national programmes for pupils gifted in mathematics are personally linked to, or even run by maths departments. Examples are summer schools, master classes, specialised maths classes and maths schools. These are environments where elementary introductions to modern mathematics and science are permanently developed and informal discussions create the new mathematical culture.

Göttingen counts with a long standing tradition with similar approaches; over a hundred years ago Felix Klein, himself a brilliant mathematician as well as extraordinarily gifted teacher, organized a community of practice of maths researchers and educators in Göttingen in the form of seminars. Felix Klein's books "Elementary Mathematics from an Advanced Standpoint" provides the conceptual background from higher mathematics to the school maths as it is taught until today. Changes in the curriculum (Meraner Reform), participation in DAMNU and IMUK and teacher training reforms were problems which the participants, like R. Schimmack and W. Lietzmann dealt with at the seminars. Today's rapid political, social and cultural changes in schools make it necessary to establish communities of practice, uniting not only researchers and educators but also prospective teachers and pupils.

References

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