Developing explanatory compentencies in teacher education

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Abstract

When interviewing school students for what constitutes a good mathematics teacher, the first characteristic usually listed is the ability to explain well. Besides well-founded content knowledge most important for classroom episodes of teacher explanations is knowledge about how to present mathematical concepts in a comprehensible way to students. This encompasses competencies in the area of verbal communication as well as the conscious use of means for illustrating and visualising mathematical ideas.

We report about an analysis of explanatory processes in math lessons and about an analysis of prospective teachers' explanatory competencies. As a result we identify improvements in teacher education at university.

1. Introduction

Concerning negotiation processes, teachers everyday have to come up with an appropriate way to make mathematical content understandable for pupils. Permanently they wonder about questions like: How can I explain certain mathematical content to pupils? During the preparation of class teachers have to think of well suited exercises, possibilities on how to make certain mathematical content clear and intelligible to all pupils, and ways of engrossing, diversifying and transferring mathematical thoughts.

These tasks must be accomplished everyday anew, concerning every mathematical content, every course and every pupil, not to forget pupils' different cognitive levels that have to be considered.

It is astonished that despite this knowledge in teachers education - and we are speaking out of our own experience - the competency of "adequate explaining" which teachers need in everyday class preparation, is not being taught.

This dilemma is caused by various reasons, which shall not be discussed in this paper.

Apart from that we emphatically point out, that the deficits of prospective teachers in decision-making and responsibility can not be compensated by in-service teacher training after university due to the complexity of the current everyday life at school.

In our opinion these competencies have to be developed in academic courses, what makes a change in educational content become mandatory.

In the following, theoretical basics of explanation are mentioned. Engaging in these basics, the lack of literature dealing with arrangements of explanatory sequences in class becomes obvious in the form of missing national empirical studies as well as in missing empirical models. Even though some theoretical models do exist, those models can not be adapted to use in class from our point of view. This will be explicated in chapter 2.1.1. The same conclusion can be made regarding the usage of representations within explanatory sequences in math class (see chapter 2.1.2). Although in didactical literature it is always advised to use representations due to their significance in gaining insights, there are only very few statements on whether and how to use these representations in explanatory sequences effectively.

In this paper theoretical basics to explanatory sequences in math class are presented at first. Based on actual literature as well as on our own empirical studies we afterwards present necessary changes in teacher education and concrete approaches towards the arrangement of academic courses.

2. Background

2.1. Theoretical basics

2.1.1. Theories of explaining

In literature different theoretical models for the concept of "Explaining" are mentioned. For instance Hempel and Oppenheim's deductive-nomological model is well-known in the philosophy of science. It characterizes scientific explanations primarily as deductive arguments with at least one natural law statement among its premises (Hempel/Oppenheim, 1988).
In the field of education Kiel's model has to be mentioned (Kiel, 1999). Kiel in contrast to Hempel/Oppenheim does not focus on the explanation as a product but on the process of explaining which is sub-divided in eight different steps. Each of these steps is allocated to three categories of cognitive operation: analysis (recognizing the constituents of an explanation), synthesis (recognizing the function of an explanation) and syncrisis (comparing with other explanatory objects and if necessary integrating in one's cognitive structure) (Kiel, 1999, S. 267 ff.).
As we formerly have adumbrated these models are not or only partially adoptable to teaching math, due to the following points:

- Class room explaining always refers to consignees. In Hempel/Oppenheim's model consignees are irrelevant which leads to our denegation of this model.
  Although Kiel takes the consignees into account, he does not go into the variety of explanatory approaches which teachers have to have available concerning to such a multiple set of consignees.
- Whether an explanation is applicable or not depends on such multifaceted factors as the type of school, the headcount in class, the cognitive level of pupils, etc.
  When deciding for a special way of explanation those factors have been considered. As an exemplification the introduction of negative numbers can be accomplished by using different approaches such as the model of step, the law of permanence, real world model, etc.
- The intention behind an explanation (producing occasions of communication, reflection of one's own or others problem-solving-strategies, defining terms, …) is crucial to the way of explaining.

Although the field of explanation is complex mentioned above, when looking on the current state of mathematical didactical research, there are no national empirical findings in the field of explanatory processes in math class.

2.1.2. Using representations within explanatory sequences
To illustrate mathematical terms and operations, in math class often representations are used in explanatory sequences.
Bruner (1966) distinguishes three modes of representation: enactive representation, iconic representation, and symbolic representation. By using the example of tying a knot he explicates these representation modes.
In the enactive mode the knot is represented action-based. "With respect to a particular knot, we learn the act of tying it and, when we "know" the knot, we know it by the habitual pattern of action we have mastered." (Bruner 1966, p. 6)
The iconic representation is comprehended by Bruner in different aspects: the picture of the knot in question, its final phase or some intermediate phase, or, indeed, even a motion picture of the knot being formed (Bruner 1966, p. 6).
On the third mode of representation, the symbolic one, Bruner comments (1966, 7):
„For symbolic representation, whether in natural or mathematical "language", or whatever the medium uses to combine the discrete elements by rule. Note, too, that whatever symbolic code one uses it is also necessary to specify whether one is describing a process of tying a knot or the knot itself (at some stage of being tied). There is, moreover, a choice in the linguistic description of a knot whether to be highly concrete or describe this knot as one of a general class of knots. However one settles these choices, what remains is that a symbolic representation has built-in features that are specialized and distinctive." (Bruner 1966, p. 7)
Zech (1998, S. 106) references Bruner's statements but states certain "levels" of representation. He subdivides Bruner's symbolic representation into a symbol-based representation (according to what Bruner calls the "mathematical language") on the one hand and a language-based (according to Bruner's "natural language") representation on the other hand (Bönig 1995, S. 60).

Lompscher (1972) eminently refers to the different modes of language (besides the modes of visualisation), depending on the different levels of cognition which are similar to Bruner's modes of representation: first, language is the medium of mental action, second, it is a supporting element e.g. to coordinate action or to document results.

Recapitulatory there are only few empirical hints that recommend certain representations in explanatory sequences and state how these could be supported by language. Also, there are no recommendations on the use of certain representations concerning one special explanatory object: What kind of enactive representation is suited e.g. for the introduction of reducing fractions?

Which explanatory model is appropriate for introducing cylinder volume to low-achieved pupils and which one is appropriate for high-achieved pupils?

2.2. Empirical studies

2.2.1. Data

In our empirical study we focus on explanatory sequences comprising pupils working on open ended as well as closed tasks. Our data comes from 45 videographed math lessons from primary school, secondary school with low-, middle and high-achieved pupils. In primary school grade 4 and in secondary school grade 7 was participating. Prior to the video recordings the pupils have slowly been accustomed to the video camera. To avoid disturbing effects triggered by videographing, the cameramen have been instructed to act in an inconspicuous way and not to take part of interaction with pupils. Thereby pupils' interest in the cameramen was reduced to a minimum. The video recordings took place in the pupils' familiar class room. During the recordings, observations made by the researchers and were logged in detail. Furthermore three out of the seven teachers were interviewed promptly after the lessons.

Beside these data additional data were taken out of academic courses: Students at the end of their academic studies were asked to answer explanatory tasks dealing with different mathematical content.

The corpus of our data finally consists of (a) video recordings, (b) pupils' documents, (c) researchers' notices, (d) recall-interviews and (e) academic students' documents.

2.2.2. Results

As a result of the transcripts analysis as well as of the students' documents it appears that explanatory sequences are built up in a certain, mostly unchanged structure.

Furthermore some special phenomena could be realized in explanatory sequences.

a) Structure of explanatory sequences

The structure of explanatory sequences in math class is explicated below:

Cause of explanatory sequences

The cause of explanations is the beginning and therefore the activator of explanatory sequences. Depending on the teacher's know how, complex explanandi are determined already during class preparation. In this case, explanations are well-planned and prepared. Beside this unplanned explanatory causes are the result of unexpected or spontaneous situations of cognitive disequilibrium (Kiel 1991, S.74), followed up by adhoc-explanations (Schmidt-Thieme & Wagner, 2007).

Initiation of explanatory sequences

When a cause of an explanatory sequence is identified, it is of interest to see, who explicitly calls for an explanation and in which way this calling is uttered. This incident shall be denoted as the "initiation of the explanatory sequence".

Process of explanatory sequences

The Explanandum comes to the fore in the process of explanatory sequences. It should constitute the core in this part. However, often the explanatory process does not proceed in a straight-forward and linear way, but there are interposed explanations broaching the issue of something else. (Schmidt-Thieme & Wagner, 2007). In both cases explanatory processes can fulfill different didactical functions.

Beside the elimination of cognitive disequilibrium metacognitive skills can be initiated (Schütte, 2002) as well as opportunities for communication can be created (Bauersfeld, 2002). Here you can consider that different mode of representation of Bruner (1966) were used to adequately support verbal processes.

Coda of explanatory sequences

Irrespective of the success of one explanatory process, this process comes to an end, which is called "Coda".

b) Realized phenomenon's within explanatory sequences

In our data, explanatory sequences are mostly initialized by teachers. In contrast to this, the explanatory processes, themselves part of explanatory sequences, are mainly accomplished by students (Wagner & Wörn, 2009). In this context, teachers' role has changed - they act rather in a supporting than in a teaching way. Altogether it is noticeable that explanatory sequences mostly take place only by verbal interaction and without any further usage of representation. Just in a few cases language is supported by another representation (in the kind of an intermodal transfer), whereas the linking-up of three modes of representation has hardly taken place in the analysis of our data. The few cases that have been recognized all refer to the teachers' acts. In addition to that, it is conspicuous, that in comparison to pupils, teachers use a higher diversity of combining language and other modes of representation.

c) Students' explanatory competencies

In grade 7 math class of middle school the circumcircle of triangles is part of the curriculum. Along with this topic typical pupil questions arise. To figure out whether prospective teachers do have explanatory competencies in this field, the following explanatory task has been given to them:

> Think about a pupil who has realized that the circumcenter can be constructed by the perpendicular bisectors of the sides. His question now is, why you have to use these perpendicular bisectors of the sides and not other construction lines such as for example bisecting lines of angles.

By analyzing students' documents you can notice three categories of an explanation. They deliver insight into the current status of the students' explanatory competency, which will be needed just one year after university at school.

Category 1: No explanatory concept.

Student A's explanation can be used as a representative of this category. He does not seem to have an idea about the task at first. His first note - a question mark - can be used as an evidence for this interpretation. Beyond this, it is conspicuous that the explanation is very short and formulated in key words or rather fragmentary sentences. He for example only figures out that a perpendicular bisectors is a line from whom all points are equally far away for example from point B and C. The student apparently is not able to verbalize an explanation which is adequate and target-aimed to the consignees. In lieu thereof he refers to a definition. Maybe he tries to reduce the complexity of the problem by merely focussing the segment $\overline{BC}$

Category 2: Minimal support (proposal of strategy) devoid of explanation.

Student B’s explanation - as a representative of this category - does not answer the task. He gives a new advice to the pupil instead: "Try to construct the circumcircle by other construction lines like bisecting lines of angles." If we think of a pupil following this recommendation and acting this way (what means constructing the bisectrices which also subtend each other in a single point), he may realize his failure. However his primal question is still not being answered.

Category 3: Detailed explanation.

Student C's explanation stands for this category. In this kind of an explanation it is evident, that the student first specifies the task by identifying the important components of the task (in this example the "circumcircle") and its characteristics (the distance from each point A, B, C to the centre is equal). After that he divides the complex problem into certain sub-aspects. Therefore he first focusses the segment $\overline{AB}$ and explicates the concept of the perpendicular bisector of the beside. This sub-aspect explanation is repeated concerning the segments $\overline{BC}$ and $\overline{AC}$. Finally, by combining these sub-aspects the original task is re-focussed and solved.

3. About possible improvements of teachers education at university

Based on the lack of theoretical basics concerning class room explanation as well as on our own research findings, we advocate the intensely analysis and reflection of explanatory sequences already at university.
Already at university academic students shall acquire knowledge and competencies about

- adequate representations in dependence on the explanatory object
- advantages and disadvantages of respective representation
- intermodal transfer and its importance due to gaining insight
- the importance of internal representations
- the structure of concrete explanatory sequences on the basis of videos or rather transcripts
- specific ways of teachers' behaviour: asking technique or rather impulse technique, interventions including modes of assistance, feedback, usage of representation
- possible "good" explanations by means of adequate tasks
- possible explanatory scenarios in math class
- proper usage of material (by self-experiment)
- development of creative and multifaceted tasks

A lack of knowledge concerning these competencies leads to limitation in structuredness, clearness, comprehensibility and achieving the focussed aims in math class. Thereby essential characteristics of quality of teaching (Helmke, 2008) do not get fulfilled. To avoid this, the explanatory competencies mentioned above have to be acquired necessarily during the teacher education. Only hereby teachers can cope with the complexity of class, at least to a certain extend.

4. Literature

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