Mathematics education reform:The role of coherence within the complexity of change

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Abstract
This paper draws on data gathered from a large-scale, multi-year research project, Curriculum Implementation in Intermediate Mathematics (CIIM), that examines the implementation of a reform (inquiry-oriented) mathematics curriculum in Grades 7 – 10 in Ontario, Canada. To describe classroom practices and ways that teachers have been challenged and supported in implementing an inquiry-oriented approach, the data included teacher questionnaires (n =1096), focus group interviews with mathematics educators across the province, and nine case studies. While some of our data align with the research of others who show that teacher change is complex and inquiry-oriented pedagogies are slow to emerge (Frykholm, 1999; Jacobs, Hiebert, Givven, Hollingsworth, Garnier, & Wearne, 2006), we also have evidence of teachers engaged in a variety of classroom practices that involve students in inquiry-oriented mathematics learning.

Introduction
The NCTM *Standards* (1989) served as a catalyst to prompt reform in mathematics education. Current thinking in mathematics education and, in many jurisdictions, current mathematics curricula reflect these reform views and recognize that knowing mathematics means more than simply knowing procedures but includes being able to reason and communicate mathematically and to engage in solving mathematical problems (Artelt, Baumert, Julius-McElvany, & Peschar, 2003; Ball 2003; Boaler, 2002; Hiebert, 1997; NCTM, 1989, 2000). However, research suggests that although the catalyst document is now 20 years old, evidence of reform teaching practices is not as prevalent as one might expect (Jacobs et. al., 2006). Facilitating mathematical inquiry is a complex process that involves the posing of problems, the generation of thought-provoking questions, and most importantly listening and responding to student thinking. These are not practices that are easily prescribed and they require a substantive re-orientation not only of teachers' practices but also of their beliefs about mathematical ideas and mathematics teaching and learning (Borasi, Fonzi, Smith, & Rose, 1999; Frykholm, 1999). Where reform mathematics curricula exist, "teachers often transform such new materials in light of their own knowledge, beliefs, and familiar practices; as a result, the 'enacted curriculum' can be quite different from the 'written curriculum'" (Sherin, Mendez & Louis, 2004, p. 210). In this paper we shed insights on the complex process of enacting a reform curriculum by presenting the results of a large-scale study that examines the implementation of a reform mathematics curriculum.

Design of the study
The Curriculum Implementation in Intermediate Math (CIIM) research project is a 3-year study designed to provide information about how the Grade 7 – 10 Ontario mathematics curriculum is understood, taught, and supported. Data were gathered through an analysis of the Ontario mathematics curriculum, focus group interviews with leaders in mathematics education (i.e. mathematics consultants) and mathematics teachers, an extensive questionnaire that was distributed across the province to teachers of Grades 7 – 10 mathematics (n =1096), and nine 1-week case studies of mathematics classrooms where there was evidence of inquiry-oriented classroom practices, or in other words, the written and enacted curriculum were fairly well-aligned. In this paper we will focus on describing our evidence of the emergence of reform-oriented practices and report on ways that such emergence has been facilitated and supported.

Context of the study
Our analysis of the Ontario mathematics curriculum suggests that it indeed reflects a reform curriculum. There is a pronounced emphasis on problem solving and investigation as part of classroom practice. The curriculum states that problem solving "forms the basis of effective mathematics programs and should be the mainstay of mathematical instruction" (OME, 2005a, p. 11 & 2005b, p. 12). It also indicates that using a variety of tools, including concrete materials and technology, is an essential part of classroom practice to help students learn concepts and develop

flexible thinking. Communication is a key element of instructional and assessment practices and classroom strategies that promote student-to-student dialogue about mathematical ideas are encouraged to enhance students' understandings of mathematics. The curriculum also takes the view that assessment is on-going, embedded in instruction and should support student learning.

While a curriculum describes the learning expectations for students, teachers often need resources and professional development (PD) to get a better sense of what a curriculum might look like in a classroom. In Ontario, the production of resources and PD occurred through the collaboration of the Ministry of Education with the provincial mathematics education organization and organization of mathematics coordinators. Thus, leaders in mathematics education teamed up with policy makers and practicing teachers to work together to support new teacher learning. The curriculum was also supported through provincial funding of resource materials such as manipulative kits, software and graphing calculators.

Classroom Practices

One of the items on the teacher questionnaire asked teachers "In this class, how often do the following occur?" and then listed a variety of classroom practices. Table 1 provides a summary of the responses.

Table 1: Summary of teacher classroom practices

Classroom practices	Never	Some lessons	Most lessons	Every lesson
Students work on practice questions	0%	9%	39%	52%
The teacher explains, demonstrates or provides examples	0%	8%	43%	49%
The teacher provides solutions to problems	1%	32%	42%	25%
Students provide solutions to problems	1%	25%	52%	22%
Students justify their answers and explain their reasoning	0%	34%	47%	19%
Students work on problems with multiple solutions	4%	62%	25%	9%
Students work on investigations to determine relationships or mathematical ideas	2%	62%	29%	7%
Students work with concrete materials or manipulatives	7%	69%	20%	3%
Students use computer software or graphing calculators	14%	73%	11%	2%

In this table we see that practices such as the teacher explaining, demonstrating or providing examples, students working on practice questions and providing solutions are part of most or every classroom lesson. However, we also see that reform oriented practices such as students justifying their answers or explaining their reasoning, working on problems with multiple solutions, working on investigations, or using technology occur in some or most classroom lessons.

Our case study data also provide evidence of these reform practices and help us to describe what these practices look like. During each case study, we interviewed the teacher(s) and school principal and observed and video-recorded the teachers' mathematics lessons over a period of 5 days. For the discussion in this paper, we focus on the classroom practices of providing opportunities for problem solving, encouraging the use of mathematical thinking tools, and facilitating mathematical communication. These categories reflect reform-oriented practices and also align with the practices that appeared as less frequent practices in Table 1.

Opportunities for problem solving. In all of the nine case studies, our video data show teachers posing problems, students moving in and out of groups to work on the problems, the encouragement of multiple solutions and the use of a variety of representations to model problems. Students shared their solutions with the class and through discussion the teacher consolidated understanding. In one case study, the teacher might begin a lesson with a problem to introduce the topic such as in a lesson that introduced partial and direct variation with a problem about selling programs at a baseball game. At

other times, small problems were interjected in the lesson for students to think about, discuss with a partner and then share their ideas in a whole class discussion. In another case study, the class moved very easily in and out of group problem solving activities, as though a culture of problem-solving had been established. Students worked in pairs and were encouraged to investigate, discuss, and seek assistance from each other. The students had access to a variety of manipulatives and the students used these to construct mathematical models, examine their properties, and conjecture connections to other (pictorial, symbolic, and language) representations. All teachers spoke about the importance of problem solving in the current math curriculum. One teacher discussed an opportunity he had to observe a school in Hong Kong that uses the Ontario curriculum and he had been impressed to see the students' willingness to struggle with problems and thus he allows his students time to struggle.

Mathematical thinking tools. We use the expression "mathematical thinking tools" for materials that students use in class that help them create, think about, and discuss mathematical ideas. In the case study classrooms we saw students using a variety of mathematical manipulatives such as linking cubes, algebra tiles, and two-colour counters. In one case, the entire secondary mathematics department focused on including mathematics manipulatives in all their math courses. We observed the teacher assigning each group of students a second difference and asking them to build a quadratic function with that second difference using linking cubes. The students then shared their models and the teacher asked students to use their models to create a table of values and to examine their relationships using a graphing calculator. In later lessons she worked with students on multiplying binomials and factoring trinomials using algebra tiles.

We also saw a range of technologies being used that included graphing calculators, interactive white boards, clickers, motion sensors, and computers, particularly in the Grade 9 and 10 classrooms. In one Grade 9 classroom, the teacher makes extensive use of technology that includes all of the above as well as virtual algebra tiles and even the students' own ipods. In interviews, she suggested that not only does technology help students represent mathematical ideas but it also increases students' motivation. In another Grade 9 classroom several lessons included investigations where graphing calculators and other devices were used to collect data for which students would then create mathematical models.

Mathematical communication. In all case study classrooms, students were observed discussing mathematical ideas with one another in both small group and whole class discussions. In two classes in particular, the teachers were modeling the creation of a math talk community or math congress based on Cathy Fosnot's work (e.g. Fosnot & Dolk, 2001) and in all classes we saw students presenting solutions while other students observed, paraphrased, and asked questions. All of the case study teachers emphasized the importance of students learning from one another's solutions. As one of the Grade 8 teachers stated:

> I believe that children learn constantly from the world around them. They hear each others' voices much louder than adults' voices. Often peers can influence and teach each other quite effectively. In our classroom, you'll usually see students working together to discover new things and explain why to each other. (Angela, interview)

Another Grade 8 teacher revealed that she uses language to create an environment of respect and comfort in her classroom as well as a way of sharing ideas. She talked about using the term 'mathematician' to describe her students, telling them that they are all mathematicians and they need to share their ideas.

Facing dilemmas and uncertainties

While our case study data show teachers who are successfully engaging in reform practices, we also saw them face uncertainties and dilemmas. The practices that are being asked of teachers are often difficult to define, feel unfamiliar, and require a certain level of risk-taking. We observed teachers in the case studies reflecting on their lessons and often questioning whether they did the right thing at particular moments. They discussed the degree to which they needed to adapt moment to moment as the lesson changed direction based on what the students were doing and saying. Time also became an issue and teachers commented that lessons often took longer than expected.

The dilemmas that teachers faced when changing their practice did not always come from within the classroom. Teachers were not only influenced by their own beliefs, knowledge, and practices but were also influenced by student responses, colleague and administrator impressions, and parent concerns. In some cases, teachers were challenged by colleagues or parents who questioned what they were doing in their class as it appeared different from others teaching the same course. Teachers were also worried whether students would be adequately prepared for moving on to the next grade where the teacher expectations may be very different. Grade 9 teachers worried about student performance on the large-scale assessment that is administered to Grade 9 students in June. At times, these teachers felt isolated in their schools. In many cases, they were set apart as they were viewed as leaders in their schools and thus, did not necessarily have someone at the school level with whom they could discuss their concerns and share ideas and resources.

Support for Reform-Oriented Practice

We draw on a variety of data sources to discuss the ways that teachers feel supported in their implementation of new teaching practices. Two items in our questionnaire ask teachers how they learn about new ways of teaching and what resources or learning opportunities have supported their implementation of the curriculum. While teachers report that the top resource to support their teaching is the textbook, their second most valuable resource or learning opportunity comes from dialogue with colleagues. An open-ended item on the questionnaire asked teachers to describe a professional experience that positively influenced the way they teach mathematics. Interestingly, approximately one quarter of the 757 responses to this item mentioned an experience that involved collaboration, such as a professional learning group, a lesson study or dialogue at workshops.

Data from focus group interviews with both teachers and leaders in mathematics education support this view. As one teacher stated:

> The best thing that I got out of the workshop was just having the time to talk about how things work in my classroom in comparison to how things are going in other people's classrooms and having the same concerns. (Teacher Focus Group 2)

The value of collegial support was also seen in case studies. While many of the case study teachers did not necessarily have colleagues in their school with whom they could discuss their ideas, most of them had made strong networks outside of the school through their involvement in district or provincial initiatives. For instance, some had written some of the provincial resource materials that supported the curriculum and others had been involved in district lesson studies or professional learning communities. These initiatives gave them the opportunity to meet and dialogue with colleagues who had ideas similar to theirs and to try out and discuss new ideas in their classrooms. Such opportunities helped to support their changes in practice and provided them with confidence to continue with their work.

Several of the case study teachers were also supported at the school level by the principal and/or the department head. Administrative support was seen as crucial to the confidence and comfort of the teacher in trying out new ideas. This support was realized in a number of ways including scheduling adequate blocks of time for math to allow for problem-solving activities, while also providing time for teachers to meet together to talk about their work. In one case study, the entire department worked together to integrate manipulatives in all of their secondary math classes through the direction of the department head. This department head was, in turn, supported by the principal who provided release time so that the department head could work with new teachers on the integration of manipulatives in their courses. In another case study, teachers had been supported to take part in a lesson study initiative in a family of schools setting. This initiative was led by the mathematics coordinator, a secondary Vice-Principal and two elementary school Principals who not only supplied release time but who also attended and participated in the preparation sessions with the teachers. These administrators remarked on how much they themselves learned through participation.

> As an administrator you get time, while they're having their discussions, you sort of sit back a little bit and listen more than anything else and watch them and listen and you learn . . . (Elementary Vice-principal B, Case study 3, interview)

Conclusion

Fullan (2001) points out the value of coherence and shared meaning in implementing new ideas. Since the development of the curriculum and resources was a collaborative effort, the messages in the curriculum, resource materials and PD were common and reflected current thinking in mathematics education. Teachers appeared to be able to build new meaning and enact the new practices called for in the curriculum as they engaged in a dynamic cycle of discussion with their colleagues and testing out new ideas in their classroom. As Rosenholtz (1989) suggests:

> It is assumed that improvement in teaching is a collective rather than individual enterprise, and that analysis, evaluation, and experimentation in concert with colleagues are conditions under which teachers improve (p. 73).

Our findings support this. We also see that teachers report that they need greater opportunities to work with their colleagues and we saw the important role that administrators and policy makers can play in helping to facilitate such collaboration. The challenge we face at this juncture is to keep the momentum strong so that networks continue to develop.

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