Chapter-spanning Review: Teaching Method for Networking in Math Lessons

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Abstract
Central to this article is networking in math lessons, whereby concentration is placed on the construction of a student-focused teaching method for the networking of mathematical knowledge in the lower secondary. Firstly, normative standards and descriptive results will be compared. Secondly, several already existing teaching methods for networking in math lessons will be added to the method of „chapter-spanning task variation". Using this method, attention is be placed on the integration of mathematical content and specific social netowrk-form (e.g. teacher led classes, group-work etc.). This paper will be concluded with the presentation of the testing of the method in the school context).

Introduction

With respect to networking in math lessons in secondary education I, there is a gap between the normative standards and the descriptive results. I will present the normative standards according to the concept of so called „basic experience" (Winter, Baptist 2001) versus the results of older and newer empirical studies (Bauer 1988, Baumert, Klieme 2001). Many pupil do not regard math lessons to be a "universe with a maximum level of inner (deductive) networking and openness toward new orders and relationships" (Winter, Baptist 2001) as described in the first of these so called „basic experiences", but as a collection of "incoherent materials neighboring each other" (Bauer 1988) which „are not in a sufficient manner" (Baumert, Klieme 2001) connected with each other. In stead of a „reservoir of models suited to rational interpretation or to the systematic organization of the following of operations" (Winter, Baptist 2001) mentioned in the second of the „basic experiences", many pupil experience mathematics as „a self-sufficient structure which has little contact with other areas of perception" (Bauer 1988). Consequently, difficulties come up for the third „basic experience", in which mathematics appears as the „practice field for heuristic and analytical thinking" (Winter, Baptist 2001) because many pupil are not successful in translating knowledge learned in math lessons into the „processing of complex questions" (Baumert, Klieme 2001).

In order to counteract the problem described, the *chapter-spanning review* will be introduced as a teaching method for the stimulation of networking in math lessons. Based on the third „basic experience" (Winter, Baptist 2001) not only the processing, but also the development of complex questions stands, by way of the pupil, at the centre of this teaching method.

1. Networking Concept

Brinkmann's dissertation (2002) constructs the theoretical basis of the teaching method. Accordingly, networking will be understood as the process and result of the relational situation of mathematical content and application on the level of the teaching materials as well as the cognitive level of the student. According to Brinkmann it is possible to categorize networking as being outer and inner mathematical in the same manner at both levels.

In the frame of this paper the level of the teaching material and the cognitive level of the student will be assessed on the epistemic level according to Brinkmann. The term *epistemic,* introduced by myself here, should serve to emphasize the significance of knowledge in both cases. I will add to this idea of „level" by observation of networking on the social level. The latter should bring the potential of the social structure of the study group for the development of networking in math lessons to fruition.

2. Design of the Teaching Method

There are various suggestions for the stimulation of networking in math lessons to be found in the didactics of mathematics, especially on the epistemic level. For example Vollrath (2001) suggests making the *topic threads (central themes and terms)* of mathematics visible to pupil with the help of tables of contents. He also suggests designing transitions between various textbook chapters through *themes* (inter-mathematical, chapter-spanning contexts) and *groups of themes* (application oriented, chapter-spanning problems). pupil will also be given the possibility of continually working on mathematical problems and studying relationships through self-productions using *study diaries* (Gallin, Ruf 1998). Brinkmann (2003) suggests the usage of *mind-maps* and *concept-maps* in lessons in order to encourage networking.

The social level of networking, and especially the role of social networks in the construction of knowledge, is particularly thematised in general pedagogics (vgl. Fischer 2001). For example, e*xpert groups* and *learn by teaching* are suggested and observed in order to encourage the development of knowledge networking.

One of the few approaches in the didactics of mathematics, in which the epistemic and the social level of mathematical learning are interlocked with concrete exercise examples, is the method of student-centered exercise variation (Schupp 2003).
In the following this method is transformed into the *chapter-spanning review* method with the goal of placing a focus on the networking of teaching material; unlike Schupp, who focused on the discovery of problem-solving strategies. In addition in my opinion, the variation of the exercise should become more strongly connected to the teaching plan and the textbook. On that account the student-centered exercise variation will be synthesized with the above mentioned method for the promotion of networking in lessons taken from the didactics of mathematics and general pedagogics.
The segmenting of the teaching material and a student's mathematical knowledge into categories will be kept as part of the segmenting of the class in the phase of experimental training. As a result the pupil will be able to discover differences and similarities between the chapters of the book as well as connecting central themes and central terms and treads with the help of a content-oriented index. This develops through the modification of the table of contents of the textbook or notebook by placing the chapter and subchapter titles in a left-hand column and placing the exercise names in the header. By doing this, it is possible for the pupil to say which skills are connected to which exercises by ticking of the corresponding content skills.
The individual phases of the method will now be introduced in the following in connection with this design.
Preparation: To begin the pupil solve an introductory exercise with their classmates. This implies a cooperative context. The number of the textbook chapters of the initial exercises of school year are presented to the pupil.
Expert training: Each student chooses an initial exercise. Pupil with the same exercise work together in a group to solve it and prepare the presentation of the exercise. Subsequently, each group determines which field of skill the exercise belongs to by filling in the skill table included in the table of contents of the textbook.
Expert round: The groups are reorganized. Now experts from each initial group will meet together in one group. The goal of this phase is to have each group, using the skills from the initial exercises, create at least one chapter-spanning exercise, write down the solution, and determine the skill field
of the exercise.
Plenum: The exercises and solutions of the groups are summarized in a notebook. The notebook will provide a table, in which the exercises are paired with the respective skills.
Both levels are integrated with one another in the expert round. In addition the realization of the epistemic level and social level are thereby brought together. In doing this the pupil can independently discover themes and groups of themes and formulate chapter-spanning exercises.

3. Testing at a Grammar School

For the testing of the teaching method six initial exercises from various subject areas using the topic tangram-puzzle as a connecting element were developed and applied in an 8^{th} grade class in 2007/08. The skill table and the initial exercises were derived from the teaching exercises, as the lesson was structured according to the textbook. In the beginning phase there were only three hours of class time available to be used. To get started the pupil were challenged along with their classmates to form a square using seven tangram-stones. The pupil were subsequently introduced to the following initial exercises.
Exercise A : „Table“ In order to build a tangram table out of wood, the whole tangram diagram is enlarged. At the same time the longest side of the smallest triangle (ca. 5.5cm) lengthens by **x** cm. How does the area of the whole diagram and of the individual pieces change?
Exercise C: „Functions“ Sketch your whole quadratic solution diagram in a coordinate system. Which function graphs can you find in this sketch? Construct the respective function equation.
Exercise D: „System of Linear Equations“ Sketch the whole solution diagram in a coordinate system. Construct linear systems of equations with two equations and two variables, whose solutions correspond with the vertices of the tangram stones.
Exercise E: „Symmetry“ Sketch the whole solution diagram. Which symmetrical tangram stones do you find in the sketch? If necessary describe the kinds of symmetry and sketch the axis of symmetry and the centers of symmetry. Explain.
Exercise F: „Darts“ There is a new magnetic dart game in the market, that looks exactly like a quadratic tangram puzzle. How great are the chances of hitting a parallelogram or a triangle?
Since difficulty levels are often felt to be subject-dependent and various, the adaption of the difficulty level to the exercises was by-passed in this case in order to counteract the problem by means of a measure of self-assesment. This was achieved by presenting all exercises to the pupil at the same time and allowing them to

choose one. With this method six groups were formed within five minutes. The observed collaboration as well as the thoroughness of the solutions varied from group to group, but all were in a position to prepare a presentation.
In the next lesson the pupil had the chance to ask questions about the exercises and complete the skill table. The skill table was subsequently analyzed doing the discussion of the lesson.
The pupil developed fifteen exercises, of which only two can be presented as examples here. In the following exercise a pupil on a purely mathematical basis combines the two large chapters on symmetry and functions (of Brinkmann's term „inner-mathematical”).

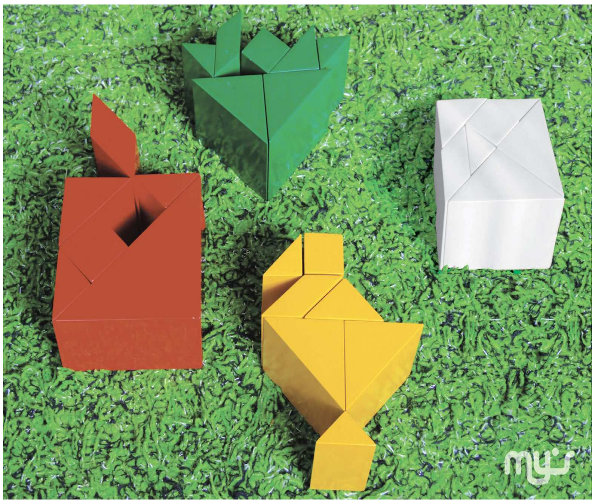
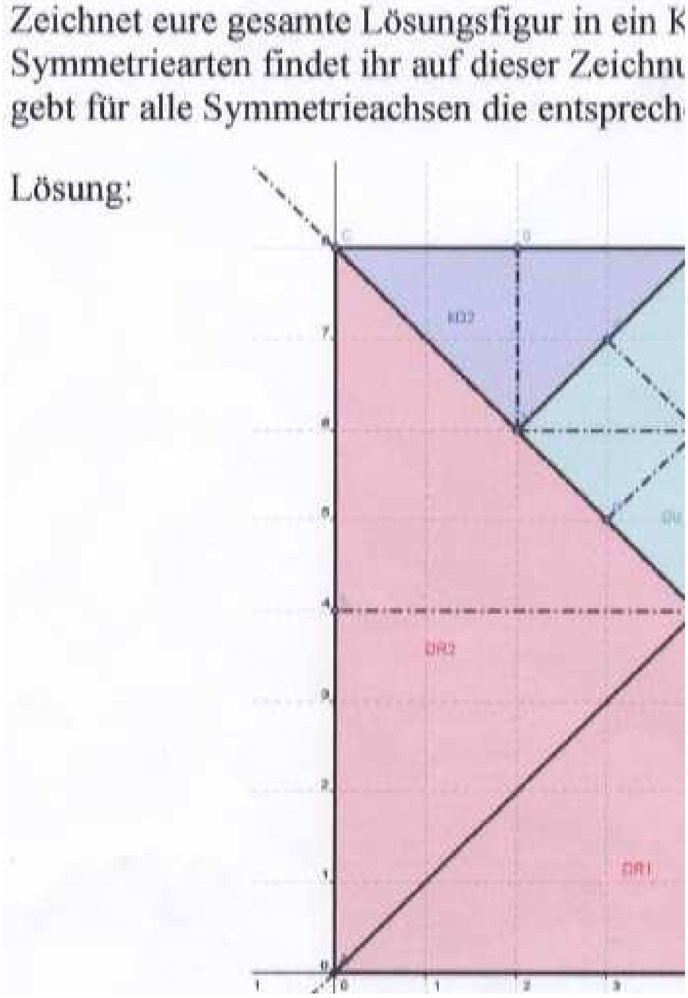


Image 1

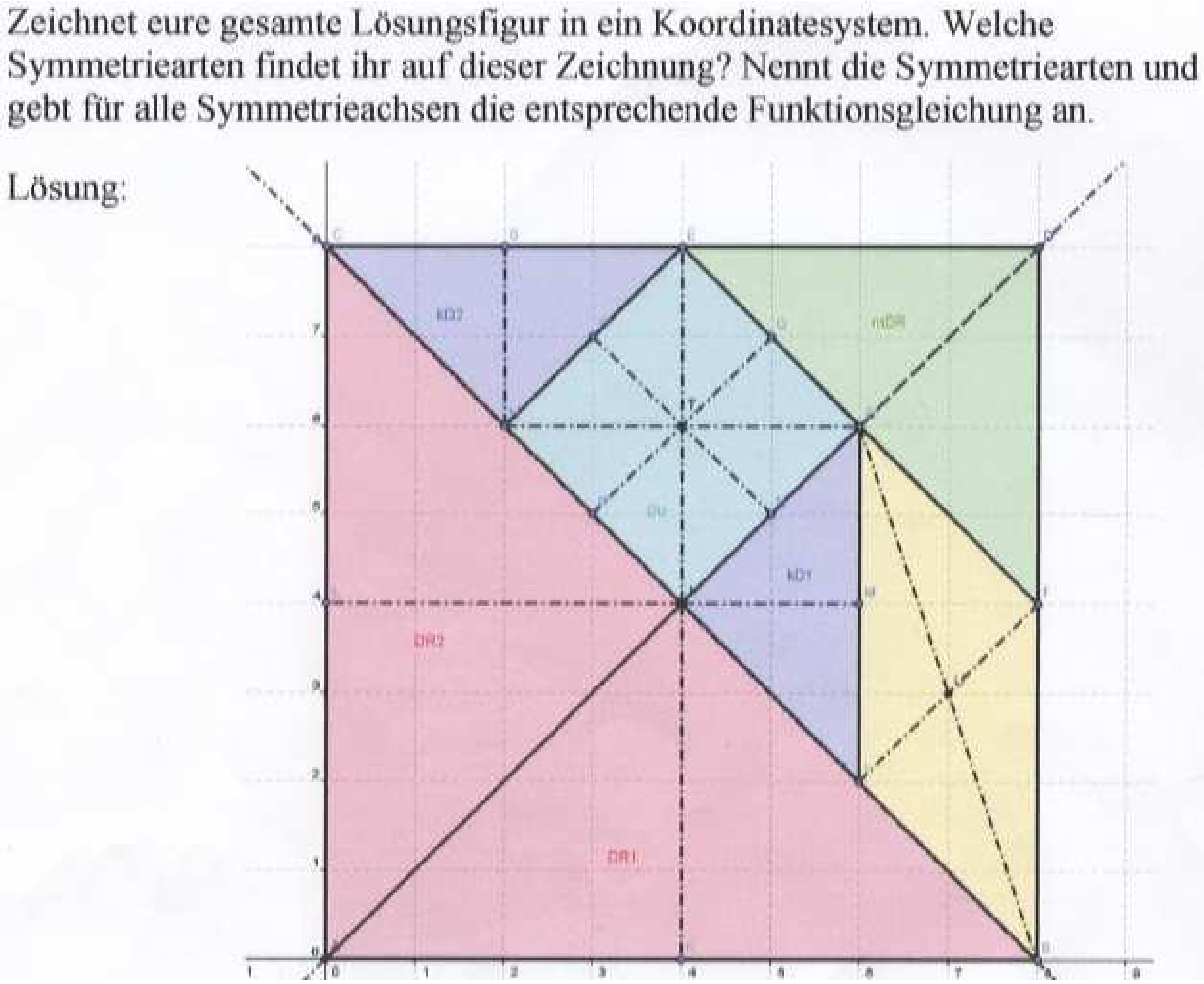
Exercises B: „Cubes“ In the picture you see cube-formed tangram-games. The stones are made of tin and are hollow inside. How many



do you need for a cube-formed tangram-game if you use the dimensions of the wood tangram?

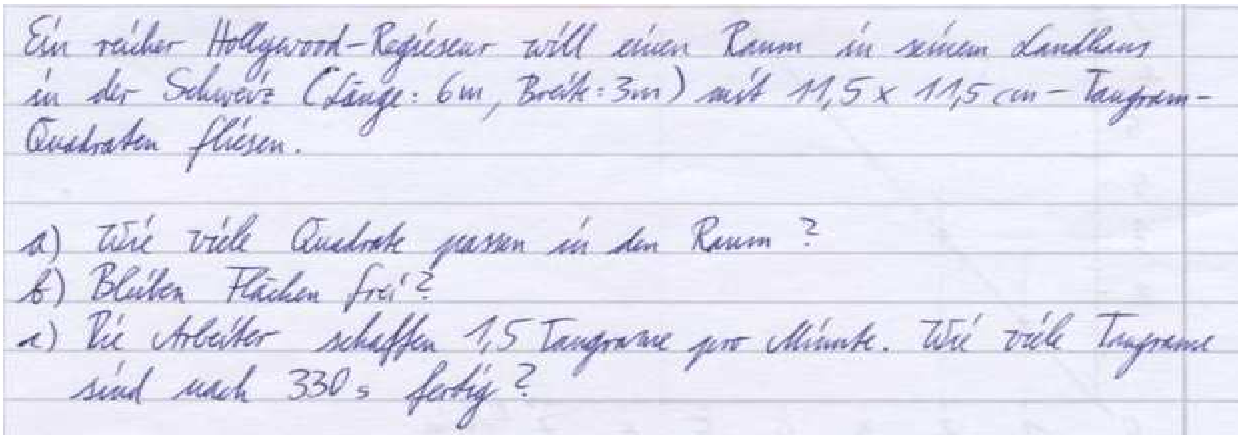
The axis of symmetry is not only to be sketched, but also described with function equations. The main exercise here is to discover the that connect the parts. In this way terms from different themes and exercises appear together meaningfully in one and the same sentence. With minimal change to the mathematical references as well as to the grammatical structure of the initial exercise a theme appears, which can serve as a bridge between the taught units of symmetry and linear functions (Vollrath 2001).

The next exercise (see image 3) was developed by two pupil. With an introductory text the reader is placed in a room in a Hollywood world. The text also contains the most important dimensions of the room. The following tasks are related to the tangram. It is asked how many tangram squares fit onto the surface and if spaces are thereby left uncovered. Also it is asked of how many squares are laid after 330 secs. While solving the problem, the pupil tried to use different means of presentation. They presented their calculations in graphs and tables.


Zeichnet eure gesamte Lösungsfigur in ein Koordinatesystem. Welche Symmetriearten findet ihr auf dieser Zeichnung? Nennt die Symmetriearten und gebt für alle Symmetrieachsen die entsprechende Funktionsgleichung an.

Lösung:

Image 2


Ein reicher Hollywood-Regisseur will einen Raum in seinem Landhaus in der Schweiz (Länge: 6m, Breite: 3m) mit 11,5 x 11,5 cm-Tangram-Quadraten fliesen.

a) Wie viele Quadrate passen in den Raum?
b) Bleiben Flächen frei?
c) Die Arbeiter schaffen 1,5 Tangrame pro Minute. Wie viele Tangrame sind nach 330 s fertig?

Image 3

With this exercise it became obvious that it is not simple to develop an authentic exercise with reference to reality while aiming to combine as much content (here proportional functions and area calculation) as possible together. The connection is extra-mathematical and makes reference to a concrete situation, which is supposed to be modeled through mathematical means.

However, if one looks at the challenge which pupil face with the development of networking exercises in the chapter-spanning review, one notices that the real problem for the pupil is not the modeling of an extra-mathematical situation. The real problem is in presenting mathematical content, which is represented by the headings in the textbook, in context. Consequently, one can term the exercise at hand as an „inverse modeling exercise”. Such exercises are known in the teaching methodology as „pseudo-authentic”-exercises. The neologism by the author „inverse modeling” bases itself on the negative coloration of the term „pseudo” and denotes here an exercise, in which mathematics is consciously translated into the extra-mathematical in order to shed light on mathematical content. By doing this, reality is not modeled through mathematics, rather mathematics is modeled through reality with the goal of networking mathematical issues in the perception of those thinking and learning (compare Jahnke 2001). The question of reality or otherwiseof the Hollywood context appears in a different light with this background.

The diversity and quality of the resulting exercises is reason enough to show that the class time used for the testing of the method was effectively used. The time needed for correcting and feedback could nevertheless be seen by teachers as a problem. It is possible to extend the method over six class sessions in order to shift the correcting and feedback out of the preparation time and into the class time. The correcting can thus be divided up amongst the pupil. The feedback is, as a result, also much quicker.

4. Conclusion

“Mathematics as an ideal practice field for heuristic and analytic thinking that seizes up everyday life and talks it up in a specific way” (Winter, Baptist 2001) is seen as the „basic experience” in math lessons. The complexity of the requirements increases if various mathematical categories need to be used in order to solve a problem. As shown above, at least a share of pupil can, through preparation in expert training, be placed in a position to network mathematical content both inner- and extra-mathematically. There is reason to accept that the self-produced exercises are more appealing to the rest of the class than complex exercises out of a textbook. Thus pupil are at least given the experience of mathematics as an exercise field for heuristic thinking while solving such exercises. It can be assumed that an elaboration of the same and the ascertainment of the suitable method for further school grade will contribute to the issue of the networking of mathematics in the lower secondary.

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