Investigating Elementary Teachers' Mathematical Knowledge for Teaching Geometry: The Case of Classification of Quadrilaterals

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Abstract
This paper examines the mathematical knowledge for teaching (MKT) in Indonesia, specifically in school geometry content. A translated and adapted version of the MKT measures developed by the Learning Mathematics for Teaching (LMT) project was administered to 210 Indonesian primary and junior high teachers. Psychometric analyses revealed that items related to classification of quadrilaterals were difficult for these teachers. Further interactions with teachers in a professional development setting confirmed that teachers held a set of exclusive definitions of quadrilaterals.

Introduction

New direction on the study of teacher's knowledge of mathematics has received tremendous attention since the Shulman's (1986) seminal work of identifying pedagogical content knowledge (PCK) as the missing link in the knowledge teachers need to bridge between subject matter knowledge and pedagogical knowledge. Ball and her colleagues (2001) further refine our understanding of this knowledge by introducing mathematical knowledge for teaching (MKT) as a specialized knowledge of the content that is situated in the context of teaching (Figure 1). This construct underscores that the knowledge required for teaching is determined by the mathematical work of teaching (Ball, 1999). Four domains of mathematical knowledge are hypothesized in the U.S. construct of MKT: common content knowledge (CCK), specialized content knowledge (SCK), knowledge of content and students (KCS) and knowledge of content and teaching or KCT (Ball, Thames, & Phelps, 2008).

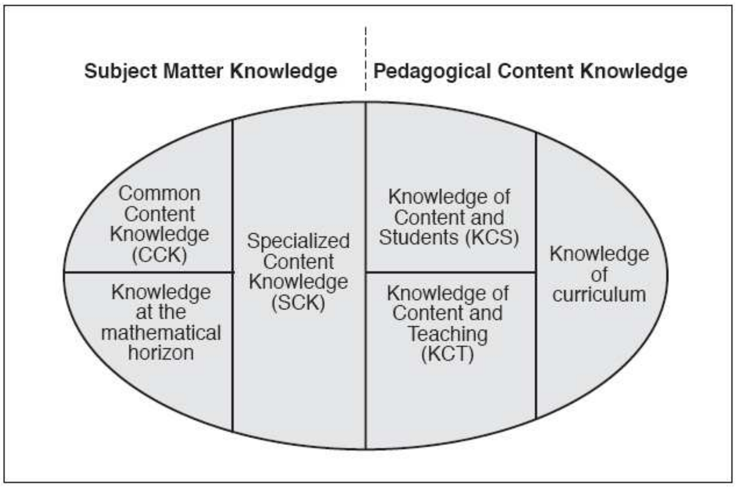

Figure 1. Domains of Mathematical Knowledge for Teaching (Ball, Thames, & Phelps, 2008, p. 403).

Although this construct is developed based on U.S. based teaching practices, recent evidence suggests that it may translate to other cultures (Delaney, Ball, Hill, Schilling, & Zopf, 2008). However, despite commonalities in teaching practices, teaching is a cultural activity (Stigler & Hiebert, 1999), and thus, has to be approached with caution by taking into account differences that may impact understanding of particular topics.

This study examined Indonesian elementary teachers' mathematical knowledge for teaching specifically on school geometry and addressed the following research questions:

- What elementary school geometry topics did Indonesian elementary teachers found to be difficult and why were these topics difficult for them?

Method

Subjects consisted of 210 Indonesian elementary teachers participating in professional development programs focused on mathematics and science between July and November 2007. An adapted version of the Mathematical Knowledge for Teaching (MKT) geometry measures, originally developed by the Learning Mathematics for Teaching project at the University of Michigan, were administered to measure the teachers' mathematical knowledge for teaching geometry. Figure 2 shows a sample from the released items (Hill, Schilling, & Ball, 2004). Detailed analyses of the challenges and issues in translating and adapting the measures are found elsewhere (Ng, 2009).

Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
3 5 x 2 5 ——— 1 2 5 +7 5 ——— 8 7 5	3 5 x2 5 ——— 17 5 +70 0 ——— 87 5	35 x 25 ——— 25 1 50 1 00 +6 00 ——— 8 75

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

Figure 2. Released Item from SII/LMT (Hill, Schilling, & Ball, 2004)

Two psychometric analyses were conducted to assess the performance of the MKT measures: comparing the point-biserial correlation estimates between the U.S. and Indonesian measures and evaluating the relative item difficulties using a one-parameter Item Response Theory (IRT) model between the two countries. The point biserial estimates provide information on how the items are correlated with one another. Higher point biserial correlation indicates stronger relationship among the items and the construct being measured (Delaney et al., 2008). Interactions with the teachers during the professional development program provided further anecdotal evidence on teachers' MKT.

Results

The correlation between the Indonesian and U.S. point-biserial was moderate (Figure 2, r = 0.369). One item in the Indonesian version had negative point-biserial correlations of -0.045 (Item 3c, Table 1), indicating that respondents who scored well on other items in this test were more likely to get this item wrong than right. This item asked the teachers if it was possible for a parallelogram to have congruent diagonals. A possible hypothesis to explain this difference relates to the way parallelograms are represented in Indonesian curriculum (Departemen Pendidikan Nasional, 2003). Parallelograms are depicted as a quadrilateral with two pairs of parallel and congruent sides. However, there is no indication in Indonesian textbooks or curriculum on considering, say, a rectangle as a special type of parallelogram.

Table 1. Point biserial correlations estimates between U.S. and Indonesian measures.

Item	Indonesian point biserial estimates	U.S. point biserial estimates
1a	0.419	0.561
1b	0.100	0.565
1c	0.325	0.634
1d	0.357	0.715
1e	0.186	0.539
2a	0.524	0.590
2b	0.420	0.606
2c	0.400	0.444
2d	0.476	0.696
2e	0.371	0.620
3a	0.179	0.628
3b	0.491	0.689
3c	-0.045	0.592
3d	0.274	0.650
4	0.121	0.502
5	0.325	0.415
6	0.227	0.694
7	0.412	0.683
8	0.305	0.438

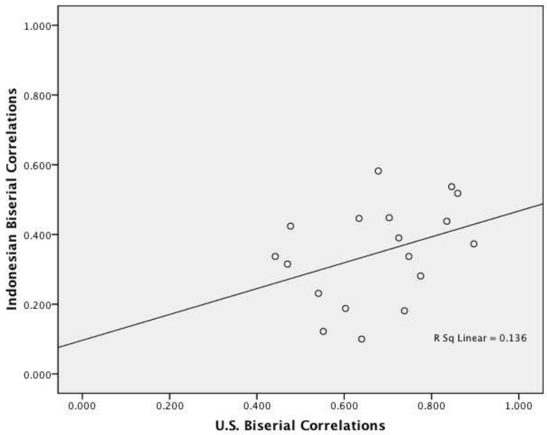

Figure 2. A regression line fitted to a scatter plot of the U.S. and Indonesian biserial correlations.

Such distinct treatment of shapes, in this case quadrilaterals, made items that required the teachers to examine the relationship between classes of shapes, such as item 1b, 3a, 4 had much higher level of difficulties of about two standard deviations compared to U.S. difficulties (Table 2). As mentioned before, shapes such as square, rectangle, and parallelogram are treated as distinct entities in the Indonesian curriculum. Interestingly, the Indonesian term for rectangle is literally “long square”, showing a relationship between the two. However, due to instructional treatment in the textbook, teachers lacked understanding of how these shapes were related. On the other hand, referring to a rectangle as a long square may also prevent students from recognizing that a rectangle can be a square since one pair of sides of a rectangle “has to be” longer than the other pair.

During the professional development program, the teachers worked in groups to explore characteristics of each of the quadrilaterals: square, rectangle, parallelogram, rhombus, kite, and trapezoid. Next, they were asked to compare the shapes in term of their characteristics, looking at their sides, angles, diagonals, reflective, and rotational symmetries. They were then asked to create a hierarchical classification of the quadrilaterals. Only two out of the six groups were able to come up with the correct classification. Even after these activities, teachers had difficulty to evaluate correctly whether the statement such as “No rectangle is a rhombus” is always, sometimes, or never true.

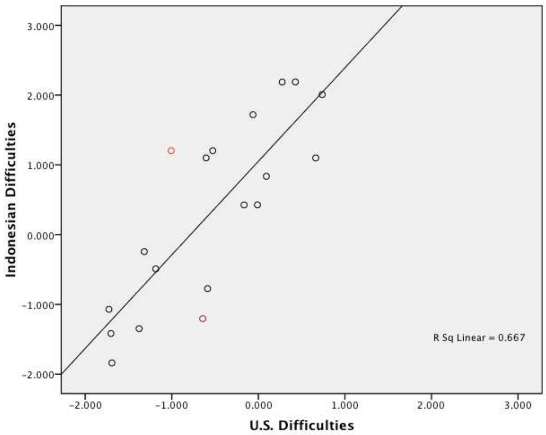

Figure 3. A regression line fitted to a scatter plot of the relative difficulties of items in the Indonesian and U.S. versions of the MKT measures.

Table 2. Comparison of the difficulty estimates for MKT geometry measures between U.S. and Indonesian.

Item	Indonesian difficulties (SE)	U.S. difficulties (SE)
1a	-0.491 (0.236)	-1.185 (0.068)
1b	2.186 (0.277)	0.275 (0.057)
1c	0.836 (0.242)	0.092 (0.057)
1d	0.425 (0.237)	-0.165 (0.056)
1e	1.719 (0.261)	-0.062 (0.056)
2a	-1.347 (0.281)	-1.378 (0.073)
2b	-1.838 (0.316)	-1.692 (0.084)
2c	-0.774 (0.244)	-0.587 (0.059)
2d	1.099 (0.257)	0.663 (0.061)
2e	-1.417 (0.270)	-1.704 (0.084)
3a	1.203 (0.246)	-1.008 (0.064)
3b	-1.070 (0.245)	-1.727 (0.085)
3c	-0.243 (0.241)	-1.318 (0.071)
3d	-1.204 (0.249)	-0.643 (0.059)
4	2.188 (0.291)	0.427 (0.059)
5	2.007 (0.281)	0.737 (0.062)
6	1.203 (0.242)	-0.527 (0.058)
7	0.426 (0.248)	-0.011 (0.056)
8	1.100 (0.258)	-0.604 (0.059)
	0.316 (0.259)	-0.548 (0.065)

Discussion

Results from psychometric analyses revealed that questions that asked the teachers to relate different quadrilaterals were relatively more difficult for the Indonesian teachers compared to the sample from the U.S. teachers. One reason for this difference is the way the topic is treated in the two countries. Delaney and colleagues (2008) point out that school curriculum is an important factor in determining mathematical knowledge. Although curricula from two countries contain the same topics, the treatment of these topics may vary by country. The Principles and Standards for School Mathematics document states the expectation for grades 3-5 students should “identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes; and classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids” (NCTM, 2000, p. 164). In contrast, the Indonesian curriculum document states that one of the objectives for geometry is for students to “identify two- and three-dimensional shapes based on their properties, characteristics, or similarities” (Departemen Pendidikan Nasional, 2003, authors’ translation). The treatment of two- and three-dimensional shapes in the Indonesian curriculum is thus different; shapes are introduced as distinct objects and no efforts to relate them can be found in the standards and textbooks.

Usiskin and Griffin (2008) examined 101 high school geometry textbooks used in the United States, 15 of which were textbooks for preservice elementary school teachers, and found varying definitions of the quadrilaterals. For instance, among these textbooks there were inclusive and exclusive definitions of a trapezoid. The inclusive definition states that “a trapezoid is a four-sided closed figure with a pair of parallel sides” whereas an exclusive one states that “a trapezoid is a four-sided closed figure with a exactly one pair of parallel sides”. Thus, someone who holds an exclusive view will not consider a parallelogram to be a trapezoid. There is possibility that Indonesian teachers held exclusive definitions of quadrilaterals.

This study has many limitations. First, no follow interviews were conducted with the teachers to assess in-depth their understanding of definitions of quadrilaterals and why they found this topic to be difficult. Second, the MKT measures did not cover every geometry topics in the curriculum, for instance there were no questions related to nets of three-dimensional shapes among the MKT measures. Finally, textbooks for preservice teachers were not examined to assess whether indeed exclusive definitions of quadrilaterals were used. This study is only a beginning effort in understanding what specific school geometry topics teachers find difficult and how definitions and treatment of quadrilaterals may affect teachers' understanding. Many questions remain unanswered which require further research:

- How does adhering to exclusive definitions of quadrilaterals differ in terms of instructional practices compared to inclusive definitions?
- How may that in turn affect students' understanding especially when they begin formal study of geometry in high school?

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