

Moving mathematics and mathematics moving: An exploration of what mathematics moves and what moves mathematics – An MmAmM approach

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This paper reflects on an innovative partnership that Tshwane University of Technology (TUT) has initiated with WallStar (pseudonym), a high school adopted by TUT and located in the neighbourhood of TUT in South Africa. The TUT-WallStar partnership involves TUT's mathematics education unit working with the school to mentor Grade 10 learners and develop their mathematics learning through interactive collective group strategies. In this paper we report on the initial interactions between the students and the school learners, and demonstrate that taking a multi-pronged approach to learners' mathematical learning not only helps to deepen and broaden the experiences and knowledge learners need for learning and understanding of mathematics, but also helps us to understand a myriad of contextual issues related to schooling and teacher education in complex learning systems such as we are typically faced with in township schools in South Africa.

Introduction

In this paper, we describe and discuss an innovative partnership that Tshwane University of Technology (TUT) has initiated with a neighbouring high school in South Africa. The paper begins with comments on what we have come to understand as innovation, what partnership in education and in mathematics education entails from our own contextual perspective, and then frame our discussion on what we believe counts as "innovative partnership". We move to a description of the partnership which has emerged and concentrate our gaze on the unique features of this partnership. Furthermore, because this partnership is centrally about mathematics, we elaborate on the general approach we have taken in this partnership and zoom-in on one activity with learners which exemplifies theoretically and empirically what it means to engage with mathematical concepts from a modeling of realities in which learners.

Partnerships and innovations in education

The literature is replete with a varied range of innovations in education, both government or individually initiated. There is also a whole range of partnerships in education, generally and partnerships in mathematics education, in particular, that have emerged and continue to inform and influence curriculum implementation and delivery, worldwide and specifically in the developing world. However, what counts as innovative partnerships depends on the context and situations involved. Within the South African setting, the uniqueness and innovative nature of the partnership that we are reporting here lies in the fact that TUT's mathematics education unit has started working with a school's Grade 10 learners and will continue mentoring those learners until they reach Grade 12, the end of the Formal high school level. The Unit has also invited and is mentoring a group of 30 mathematics education students to act as tutors to learners. Most importantly, each TUT student is allocated to a group of 4 learners, a leadership dimension which is at the core of the partnership activity. Essentially, the most critical and innovative aspect of this initiative involves this academic leadership dimension, where each TUT student is given academic charge of a small group of learners of mixed abilities. The students are expected to maintain regular interactions, both formally and informally, in order to mentor, assist and build up learners' mathematics knowledge till they write their National Senior Certificate exams in 2011.

Mentoring

At the beginning of the partnership, the key question was: “If we mentor students, what would we be mentoring them on what and how? We also had to make choices on what to mentor school learners on, and how, and most critically, how many learners would be involved and from how many schools. A decision was made to focus on one school and one group of learners, Grade 10, the first grade at the Further Education and Training phase of the South Africa’s new education system. We were also aware that a partnership like this one was only going to become better resourced if it involved a critical mass of people in the university, students in this case, who had mathematics and school mathematics teaching as their core current and future activity. We saw that preparing students for mentoring/tutoring learners would not be the only outcome but key of the achievements we wanted to gain.

We need to say again that the allocation of learners to tutors was the most critical and innovative aspect of this partnership. Each TUT student is given academic charge of a small group of learners. This way of working in teacher education is rare. It is not common to see students being given such a role in typical pre-service teacher education programmes in universities. The closest we have observed relates to scenarios when students are asked to conduct case studies of some learners during Teaching Practice sessions. This way of working is constrained because it is often packaged under the student assessment umbrella. The only contact students have with the learners is during the teaching practice period and rarely afterwards. There may occur informal interactions with learners afterwards, but these interactions are often spontaneous and on an *ad hoc* basis.

In this project, student-mentors will have regular interactions, both formally and informally, in order to continually assist learners’ mathematics knowledge and build up learners’ mathematics knowledge. Proceeding in this way ensures a longitudinal approaches to mentoring.

In July 2009, initial interactions involving student-mentors and learners occurred. As Figure 1 below shows, the first form of interactions involved learners themselves in the context of a group mathematical activity structured as part of a five-day mathematics programme at one of TUT’s campuses.



Figure 1: Learners working on a mathematisation activity on “fouring”

Interactions also occurred between learners and university setting (management and leadership) and with university students (see Figure 2 below).



Figure 2: a TUT student (middle) interacting with learners during winter school activities

A key innovative aspect in the nature of the activities given to learners involved what we called a “Fouring Activity”. The fouring activity was an illustration on modeling with reality. After having dealt with the real number system, we wanted to extend our understanding by asking the question: “what is a number?” What is a 4, and can you see a 4? And what does it look like when we see it? A deeper question with one of the student-mentors arose: “Can a 4 speak to us? Can it say “you are seeing me?”

The Grade 10 learners were given a magazine called “MYWEEK” and asked to:

Look through the magazine. Focusing on page 33, write down how many 4’s they see on that page.

Page 33 (see Figure 3 below) concerned a scenario called “Blind Date”

David
45, ELECTRICIAN,
QUEENSWOOD

Linda
50, PSYCHOLOGIST,
ELDORAIGNE

DAVID, THE NEXT DAY ...
Thanks *myweek*, I had a great time!
Did you find Linda attractive? Linda is very attractive and a very special and warm person.
Were you early or late or right on time for the date? We were both right on time!
What did you speak about? We chatted a lot about our past and the Comrades marathons we've run.
Any funny moments? When the photo was taken – but it helped us relax.
What did you order? We both had fillet steak called Between the Sheets. The food at Butchers Block is very good. We also shared a bottle of KWV Shiraz.
Any embarrassing moments? Being on a blind date! I was a bit nervous to meet someone new but it was great once we relaxed.
Did you and Linda have anything in common? Comrades – we've both run it and Linda is running her ninth marathon this year – good luck, Linda!
How did it end? I walked her to her car and escorted her home.
Do you see a future with Linda? Time will tell ...
Would you recommend a blind date? Yes, it's good to try something new!

LINDA, THE NEXT DAY ...
It was great – I was really impressed with the restaurant.
What was your first thought when you saw David? We were both a little agitated but started to relax after a glass of wine. My first impression of David was of a very friendly person.
Did you get dressed up for the date? I chose not to overdress since I wanted to be in a relaxed mood.
What did you both talk about? We spoke mainly about road running, since we're both Comrades runners.
Was the food good? It was excellent; we both had the same – black mushrooms and fillet steak.
How did you say goodbye? He escorted me home, which I appreciated.
Did you swap numbers? Yes, we did.
Do you see a future with David? To be honest, from my side I don't foresee anything romantic, although I got the impression he'd like to be more than friends ...
Would you recommend a blind date to friends? I think that blind dates are a little risky, but interesting and exciting. But everyone must decide for themselves.

WIN!
Win an evening of pure indulgence with Achiever from Standard Bank. We'll jazz you up, then whisk you off in a Suzuki to a star studded event at the Standard Bank Gallery. POST YOUR PROFILE ON WWW.MYWEEK.CO.ZA

>> Butchers Block
012 665 2084

WWW.MYWEEK.CO.ZA 33

Figure 3: The Blind Date scenario

Initially, and as expected, most of the learners said they could see one 4, the number 4 in the term 0126652084 near the bottom right of Figure 3. Upon reflection, some learners noticed that the number

8 in the term 0126652084 could be broken down to 4+4. This made them see another two 4's. There emerged a deep silence and sense of wonder when we suggested that there could be more than three 4's in the scenario. The silence was broken when one learner said that another 4 could be obtained from the number 6 and the number 2 in 01266... by writing 6-2, giving a 4. The call proceeded in this way to generate many other 4's. They also noticed that we can also see that there are 4 letters in every four-letter words such as "next", "time" and "with", etc, in the scenario. This led to the whole class realizing and seeing that there are many 4's on the page, more 4's than the class was able to see originally. We came up with the term "Fouring" to mean the method we used for "looking for fours" in the way described above. This method led learners to compile the following table (see Figure 4) to indicate how many 4's each group saw on the page.

	David	Linda
ongani+Co	28 + 2 + 1 ←	31 + 9 + 2
Peter+Co	37	→ 27
erato+Co	24 + 6	→ 28
olebogang	40	→ 39
itumelo+Co	38	→ 28
	27	31

Figure 4: Number of 4's on a magazine page

Figure 4 shows that depending on how one sees, there will be differences in the number of 4's one is able to record. This ability to see, way of seeing (Brown, Hewitt & Mason, 1994), determines the kind of connections one makes with the situation or world context they are interacting with.

Connections, awareness and learning

The concept of connections lies at the heart of key deliberations concerned with mathematics (see for example, Forgasz, Jones, Leder, Lynch, Maguire & Pearn, 1996). Given their importance, connections need to be made an object of study and understanding (Mwakapenda, 2008). As can be seen in the data below, gathered from a questionnaire completed by student-mentors and learners during the winter school, connections are critically important because they make awareness and learning possible.

As can be expected, participation in the winter school made it possible for student-mentors to gain academic knowledge (mathematics) and general knowledge about learners. A student mentor, SM11, observed:

Since the programme have started, as a tutor I have learned many things academically and generally about learners... It really helps me as a tutor to recognize how much potential I have...

As a tutor I enjoyed ... to be with learners doing mathematics and I recognize many things about the learners, the way they think. In fact the insight of the learners...

Some of the student-mentors saw the programe as contributing to their preparation for their future teaching. SM15 observed: "So I know that winter school is a preparation of my future I have to be in time everyday when I come to winter school because it is a foundation of my future as a teacher". In

agreement, SM22 noted: “this programme gave me a good way of where, how to start when I go in teaching Mathematics in schools”.

Participation in the programme contributed to a realization of their identities, who they are, in mathematics and mathematics teaching. In this regard, SM19 observed:

I have realized the passion that lies beneath teaching and mathematics. I have always assumed that maths is about knowledge and understanding, but during this programme I have realized that I have gained the love and passion for it.

For learners, the programme led to an expansion in the range of strategies they used in working with problems in mathematics.

Before I came here in Winter school I knew only one way of finding the formula for number patterns. But since I came here I think I have many ways of finding the formula for number patterns. For example using a table to find the formula, and I didn't know anything about a straight line graph, how to plot and how to find the equation/the rule but now I have an idea and that makes me to thank myself for taking my time to come in Winter school and to thank also the [tutors] for giving me such strange skills. (Learner L5)

Through the partnership, we introduced and connected learners to a new community, which has enabled them to learn mathematics and to think about possibilities for their future education and careers in mathematics and beyond.

I have known TUT but I didn't know it very well and I have known about the tutors who have educated us very well. Still I want to learn mathematics very well and I still want to know exactly well TUT how is it. I also want to know where is the radio station because when I finish school I want to be a media programme[r] (Learner L9).

Something that I know very well, now [that] I have been in the winter school is about how professors teach students in university and different types of number

I feel very sorry for those could not come and attend winter school in Tut because we learned many things about varsity and Grade 10 Maths and we know how is it about Tut classes and we also meet professor and his student (L13).

Introduction to the TUT community made some learners to think about joining TUT after completing high school.

The winter school has made me think that I will finish my matric and I shall be a student on this university. This week we have done things that our teachers would not teach us (L8).

When asked to say how they felt about those learners who did not attend the winter school, L8 said:

They have missed some interesting maths and they will never know how to make maths simple for themselves... they have missed some opportunities and they have missed to know some new interesting people (L8).

It appears that learners began to see themselves as belonging to a special community with special knowledge and attributes. Commenting on learners who were absent from the winter school, L17 said: “they don't know what *we* do and they don't ON the pattern graph and they don't ‘no’ the whole number and the integers”. L25 added:

I came here not knowing everything but now I understand a lot about numbers and how they work. I can also name them and also tell how they work, also if the numbers are the same or not the same. I understand a lot about natural numbers and *how it is like to know what they do*. I can tell how they are the set of numbers everyone can count... I need to know as much as I can because I really want to be a better person of tomorrow and to also understand the nature of life

that numbers are everywhere... I am very sad because they don't know everything *we* did here but I understand that maybe they are having family matters or something.

What makes connections possible?

There are several elements that are key to making connections possible. Critical are initiators (staff and student-mentors) and core participants of mathematical activity, the learners, whose presence at the winter school made a difference. As one of the students indicated, if all learners had attended, “their *presence* would have made a great *difference*” (SM11). However, for authentic connections to become a possibility there is a need for teachers and teacher-educators to use activities that go beyond the ordinary. We have in mind here activities that involve “reading the world with mathematics” (Gutstein, 2003), i.e. those that not only “help learners on how to solve *problems in mathematics*” but also those that are concerned with teaching “how to *think critical and mathematically*” (SM11). We make this point because we have observed that there is a lot that is changing in the nature of mathematics activities in the current classroom. However, we ask the question about the extent to which those activities promote critical thinking and thinking mathematically.

Types of connections observed

In our examination of the responses student-mentors and learners gave in the questionnaire, we noticed four types of connections in relation to mathematics and participation in mathematical activities. The first form involved “Connections involving people” (i.e. *People-people* connections). “All of us (tutors, learners) were able to *connect* with everyone in the classroom” (SM6). As Figure 5 below shows, this winter school, created an opportunity for learners to connect not only with other learners but also with their teacher and principal in an academic context different from the one they experienced at their school.

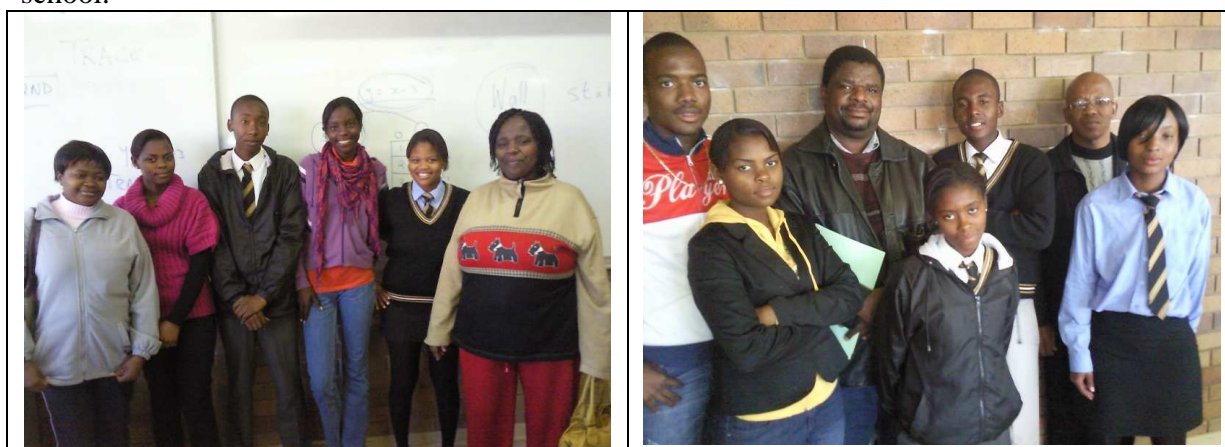


Figure 5: Learners with School Principal, TUT HoD, student-mentors and mathematics teacher

The possibly most useful and pedagogically powerful connections learners formed involved those that concerned their direct relationship with student-mentors, as Figure 6 below shows:



Figure 6: A student-mentor with her learners

The pedagogical connection noted above is linked to another form of connections that was more of a technological nature given that we made a provision for student-mentors and learners to interact with graphics calculators, another innovative dimension of the partnership project. In this connection, SM7 remarked:

I am now able to use the graphic calculator to *play* around with different problems or challenges that I learnt from the winter school... I know different strategies of conveying teaching-and-learning situation to learners using different teaching *media* and aids. Am able to teach educators and other students on the usage of the calculator especially on the chapters that we covered during the winter school lessons.

A second type of connections that we identified involved situations where learners considered that everything they can possibly come across was related to mathematics. We termed these connections “Connections with a capital C”. This can be seen in the following remark by SM6 and SM20.

I therefore summate that the maths has been *planted* in people’s mind where I started to realize that ~~o~~ everything is maths – when I walk, I walk maths, e.t.c. ... every holidays in future a project like the one should be done to help the learners revise their maths. (SM6)

I know about number patterns and how to find them in everything I can imagine of. (L20)

We identified a third form of connections involving situations when learners asked broader *Q*uestions about mathematics. We called this form: “Connections with a *Q*, i.e. *Q*onnection”. Learner L22’s question below is illustrative of questions of a rare nature that learners asked. “My question is that who grouped numbers according to their classifications and their similarities, and who identified graphs”.

The fourth form of connections concern what we called: “Connections that are about mathematisation: reading the world with mathematics” (Gutstein, 2003). This form of connection involves creating a new world or reading the world with mathematics. In this regard, we asked learners and student-mentors the following question:

Create a new question which is about maths in Grade 10. Make sure that this question cannot be answered correctly by someone who has not attended this winter school.

SM7 responded as follows:

#. A new question about maths (Grade 10)
 Calculate how many 'two-ing' we have on
 following patterns.

Pattern 1 Pattern 2 Pattern 3

20 20

20 20
 20 20
 20 20

20 20
 20 20 20
 20 20 20
 20 20

The message that we need to clarify here is that it is important how we connect mathematics to the world. The connections need to be made clear. For this particular connection, we see that the question posed by the mentor satisfies our criterion of a “new” question. This is because it fits within the ways of working and thinking we introduced in the session that had to do with the “fouring” activity described earlier. There are clearly challenges and complexities that are involved in connection mathematics that we need to foreground here. There are expectations in the new curriculum that expects teachers to make deliberate connections with the everyday world. How these get implemented is an interesting challenge. We need to follow up these challenges in our future sessions with the student mentors. SM7 was clearly able to formulate a “new” question based on what they had learned from the “Fouring” activity. Other responses that were similar to SM7’s were as follows:

Is there any number patterns in the magazines. (SM13)

Look for the number 4 on this page and write how many 4’s are there (L6)

Find the fours in your maths book and explain what Approximetly mean in ‘Sepedi’ Not in English. (L14)

Do fouring on a magazine and show them. (L20)

How many fousls do you have. (L24)

What I would like to ask is? Do the fouring. (L29)

It is interesting that learners were the ones who formulate a question that had similarities with the “fouring” activity. The way in which these questions were formulated would make it difficult for those learners who had not attended the winter school to respond correctly. As the comment below shows, the knowledge of “fouring” was only accessible to the group of learners who attended winter school.

The[re] was something that we have done that they donnot know things like fouring, threeing, twoing, and oning it was very nice to be *intertained* ...and we know that every where we do we go is maths. L11

From the responses, we were able to see at least four perspectives from which these could be viewed. First, there are responses which show that students were able to or not able to create a new question. Questions such as “How many numbers are in between 1 and 2?” (L5), and “Solve the the equator ($y=0$)($y=1$)($x=0$)($x=1$) you have to solve for this equation and anther” (L30), illustrate this point. L21’s comment shows that some learners were not sure what a question meant and how they would tell whether a question is a question.

I am having difficulties doing that but I know this one $y = 2x - 4$. I don’t know if it is a question but it is one of the things that I know now how to approach. (L21)

The other category involves students were able to clear a question which was either clear or unclear in terms of what was intended in the question.

Constraints to making connections a possibility

There was no payment required for learners to participate in the winter school. As SM17 noted, “the programme/winter school was *free* and it is not everyday that you find a free winter school at a university”. For learners to be able to connect with student-mentors and others at TUT, there was a need for them to travel from their homes to TUT. This was clearly not possible for some due to a number of reasons. L16 noted:

Just like me I didn't attend for three day because of money. I felt very sad and other learners are at their homes wondering what are they doing wishing with all their heart and soul where could we get money to go and attend just like other learners.

However, language was also a constraint to the ability to make authentic connections, especially those abilities that relate to reflect on their learning. Language fluency makes it difficult for us to see what they are communicating about their experiences in the winter school. Due to the context in which the winter sessions proceeded, opportunities for code-switching in ways in which learners were familiar with at school were not possibly afforded. Nevertheless, some learners “learned a lot” in spite of this constraint.

I've learned a lot in winter school, they have been teaching us mathematics mix with English, And I've learned things that I did not know like types of numbers Patterns. L18

Moving forward: Strengthening the partnership

From the data presented and described so far, we can identify two ways in which we could take the partnership and mentoring project activities further. These concern what concepts of “occasioning” and “connecting to the real world”.

Occasioning

There were moments during which student-mentors and learners felt good about being in the project. These need to be built on and modeled and extended further. In our follow-up meetings, we will be aiming to create situations where we deliberately work towards producing moments similar to or of a similar pedagogical power to those we have experienced in the first winter school.

I felt like I was special to be with Grade 10 learners the respect they have shown to us. The learners treated us like teachers as we are that is our true reflection we are educators... I had the special days through the project with the grade 10's if I could [be] able to rewind future and *turn* back to the past I would turn back each *moment* with Grade 10's. (L13)

Our explanation for why we will proceed with the project in ways like above is informed by the notion of “occasioning” as discussed in Davis and Simmt (2003). Here, learning is viewed as an emergent event. Davis and Simmt note that such events “cannot be caused, but they might be occasioned” (p. 147). The concern therefore becomes one about “how the mathematics teacher might occasion the emergence of a complex collective whose interactions and products are mathematical” (p. 149).

Relating mathematics to the real world

The broader perspective guiding our mathematical work will be concerned with making connections. As the following comments show, student-mentors have recorded that it is one of the productive and usefully ways to proceed in mathematics education.

As more as I have explained and help[ed] learners on the number patterns and functions it has made me to realize that *mathematics has to be related to life* challenges as far as possible then what should be the main focus depends on the approach (method) of obtaining the solution (e.g. answer to equation/value of arranged numbers). SM14

I have learned very well to check how linear equations are drafted/plotted. E.g the example of moving forward and backward in number pattern... Integrate mathematics with real life situation (e.g. *magazine*). These learners learned number patterns in a different way... e.g not knowing that stairs could be a gradient... I have learnt that mathematics is integrated or is found by real life situations. You can make mathematics fun by using *magazines, papers* instead of passive teaching and learning method that usually is used in teaching mathematics. ... [It is] very important not only to *theorise* but also to practicalise what you have learned. SM18

I feel sorry for those learners who did not make their time in attending the programme because learners have been shown that where they *walk* they walk mathematics it is like walk in the steps, there is mathematics happening. SM22

Adler, Pournara and Graven (2000) have identified various levels of integration: “integration of the various components of mathematics, between mathematics and everyday real world knowledge; and where appropriate, across learning areas” (p. 3). They have argued that while integration is desirable, the extent of the demands placed upon teachers makes integration less feasible. However, our initial interactions with learners in this project indicate that there are interesting possibilities for learners to make connections, integrate mathematics with the everyday world, when opportunities to do so are presented and demonstrated such as was the case presented here with the activity involving “Fouring”.

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