

Tasks for tests and A-levels using CAS

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Abstract

Tasks for different years of the secondary level II are presented on the basis of long lasting experience with computer-assisted mathematics instruction. They include applications of mathematical skills as well as the testing of theoretical knowledge. Finally relevant A-levels tasks are presented that integrate different mathematical contents into every day connections from economy, medical science, sports asf.

0. Explanation

This English text is a short summary from a German paper that was published at the 100th MNU-Congress in Regensburg in April 2009. Main parts concerning the examples of application-oriented test tasks using CAS had to be abbreviated. The whole English version is available at <http://www.acdca.ac.at> (Website of the Austrian Center for Didactics of Computer Algebra)

1. Initial situation

If mathematics instruction would have just stayed the way it always has been. Simplification of fractions, “wild” compilation of formula, more or less realistic survey tasks, curve sketching “according to recipe”, complex partial fraction extension ... Back then everything was just fine. Everywhere you could find a wide range of stereotypical tasks to read, copy or download. These times were over at the latest when the thought achieved to complement this check of automatable skills by a seemingly meaningful approach to mathematics. In the centre of mathematics instruction today should be interpreting, arguing and modelling, whereas the use of computer algebra systems resp. a graphics display pocket calculator seems very helpful in order to reach this aim.

In Austria there have been basic approaches for a certain time as well as efforts by certain groups to support and propagate computer assisted mathematics instruction. As a leader you could name the ACDCA (Austrian Center for Didactics of Computer Algebra).

For the vocational school system – especially in commercial academies, a quinquennial type of school of the secondary level II, comparable with post secondary college with economic focus in Germany – the use of CAS is specified in the curriculum as follows:

MATHEMATICS AND APPLIED MATHEMATICS

Educational and teaching outcomes:

The students are to be

- instructed to independent sense-making
- taking an active studying position
- disputing with economical problems in all years
- gaining insights into the opportunities of the application of mathematical methods in practical operation
- developing a basic understanding for mathematical theories and concepts
- applying mathematical methods to tasks, describing them with appropriate mathematical models, estimating and interpreting solutions
- working independently and in the team
- applying Computer Algebra Systems and/or spreadsheets resp. graphics display pocket calculators in all years and solve mathematical tasks with them

In order not to refuse future trends and break new ground the Commercial Highschool Horn has decided for the introduction of the Computer Algebra System Derive in mathematics instruction more than ten years ago. Other programs like Excel or GeoGebra are used as supporting tools. Meanwhile the HAK Horn only keeps notebook classes whereas the notebooks are bought by the students or their parents in the second semester of the first year. Thanks to this new form of tuition new media can be applied. That proves itself very useful in mathematics when it comes to analyze statistics, check different payment arrangements for internet transactions or as guides for studying in the internet.

2. Tests – a new challenge

The introduction of notebook classes and the resulting change in methods and contents of mathematics instruction we had to face a totally new situation:

- Tasks for tests from CAS-less times had to be adapted resp. canopied totally, hardly one example can be taken one-to-one.

- Real numerical data can be taken. One does not have to manipulate it in order to make the task calculable.
- The elimination of arithmetical difficulties helps to focus on the understanding of mathematical connections.
- Numeral tasks, graphical solutions or mathematical modelling find their place in tests thanks to the use of CAS.
- The preparation of tests takes essentially more time since the teachers have only access to less available material and have to phrase exactly and clearly when it comes to open questions resp. interpretation and argumentation tasks.
- The technical implementation needs an exact plan, a sufficient technical infrastructure, a good teamwork with the appropriate network technicians of the school and permanent control. Nevertheless the risk of a technical breakdown remains.

3. The framework

An important basic consideration about the fact whether the tests should be carried out at the notebooks directly or at other computers in the IT-room is necessary when it comes to written tests in notebook classes.

My personal experiences have shown me that it is much more strenuous to organise an available, working IT-room with a sufficient number of standard computers than to let the students work on their own private machines. Nevertheless it ought to be considered that the control of the students becomes much harder since they have very quick access to their personal files and are able to exchange information at cyberspeed via wireless connection without the teacher realising it. Even if a working monitor-control-software should be at hand, one should consider that the teachers cannot assist the students when it comes to technical breakdowns. Because of this reason I prefer having tests done in an IT-room even it is not always possible due to the schedule.

When preparing tests it has been approved meaningful to think about the following points:

- The functional efficiency of the IT-room has to be checked beforehand in any case.
- Seating arrangements resp. blinds for the neighbour-PCs should be organised (dividers should be used).
- The login at the standard machines is carried out with a particular test profile that should meet the following requirements: no connection to outside, the right to read at one instruction folder, the singular right to write at one solutions folder.
- A “private” collection of Derive-orders (check beforehand!) is allowed in paper.
- Formula worked out together in tuition are being provided in the instruction folder.
- The students each create one local folder on their standard machine wherein they save their tasks.
- Exact instructions concerning the handling ought to be handed out beforehand, maybe even in written form (user name, password, name of the local folder,...)
- The tasks of the test are in any case provided on paper as well.

While executing the test it is beneficial to save the tasks one after the other in the students’ local folders on the standard machine in order to go back to them in case of a technical breakdown.

Headlines with name and time in the singular documents help with the tracking of the printouts. If the students make proof prints during the test it is advisable to have them handed out by the teacher in order to prevent disquietness and not to give the student the opportunity of insight to the solutions. The time of release is the time when the task is being transferred into the solutions folder.

After having finished their work the students can check if they have released correctly via the beamer and that they did not hand in an empty folder (have a look at the memory size!)

From personal experience I find it very important to point out that it would be wise to grant oneself the time of more than one lesson for preparation as well as for the wrap up, since not only the login but also the creation of the standards and the printing takes a lot of time.

At this point I have to refer to difficulties that one has to struggle with in the course of so many years when guiding dozens of tests on the computer. The major problems hereby are concerned with data loss, technical breakdown or the area-wide control that seems to be impossible.

At our school we deal with these problems after an agreement and a demand through the state education authority as follows:

- The teacher can be supportive when it comes to CAS problems at their own discretion.

- Data loss (program got stuck) resp. machine breakdown needs the following action:
 - direct elimination of the problem (regular data saving leads to the loss of not more than one task).
 - The working time for the last task is credited.
- When there are general technical problems the test has to be dismantled totally, the solution of the problem has to be awaited and after that the test has to be continued by all students resp. the time of the breakdown has to be credited.
- It would be wise to ensure the presence of a mathematics teacher or the network operator (mobile phone!).

4. Examples of application-oriented test tasks using CAS

As already mentioned in the beginning the expenditure of time for the composition of a test has become much more. Time and time again one has to find the right measure to test arithmetical skills on the one hand and the understanding of mathematical contents on the other. The composition of a test could be as follows:

- Practical arithmetic problem, divided into some tasks, documentation of the steps of calculation, interpretation of the results.
- Mere theoretical example, comprehension questions, elaborate justification, explanations.
- The more complex the tasks, the more theoretical question or open interpretation of results can be integrated into the arithmetic problem and do not have to represent a singular task any more.

5. A-levels – composition of the final examination

Does not every teacher have a high standard when it comes to compose the final examination? You can not start too early with collection appropriate examples. It is important to focus on the personal interest of the students, current topics resp. on tuition focal points. Like other schools we in the Commercial Highschool Horn have decided some years ago to devote the instruction of the A-levels to one common, throughout topic. Thus after the decision for a topic frame an intensive research for already existing tasks (internet, magazines, colleagues) starts.

Criteria for the tasks are:

- Are there similar questions for the preparation in the tuition?
 - One of the most difficult challenges for teachers of graduating classes is to find similar complex and extensive examples as in the final examination without anticipating the relevant question in a too concrete way.
- Is a clear question possible?
 - Misunderstandings can always happen, since even colleagues teaching the same subject overlook ambiguous phrasing due to their education. It would be beneficial to consult other colleagues (most suitably German teachers) for proofreading.
- Can an open question be weakened by means of canny phrasing?
 - The evaluation of open questions is difficult and needs extremely good preparation. To keep the room for the students on the one hand and to remain clearly arranged with the possible ways of solution on the other, it is meaningful not to phrase too generally.
- Are there sheet anchors for the students in the A-levels instruction?
 - For an A-levels instruction with a balanced degree of difficulty it is necessary to incorporate examples that can be understood quickly and are solvable for average students. Calculation skills and a sufficient understanding of the mathematical connections shall be tested without special demands.
- Are the examples application-oriented and do they include at least four different areas of the curriculum?
 - Especially in their final exam the students should be made aware of the fact that mathematics is present around us in every day life. The different areas are used connected by a common topic (e.g. medicine, sports, stock analysis,...). Independent topics ought to be of equal value, for each of them the same amount of points ought to be reachable.
- Is it possible for the students to use additional material like statistics, newspaper articles, share prices given in the instruction for a more detailed analysis?
 - In the course of the following studies it is very important to be able to separate the essential from the inessential information in newspapers, statistics, graphs, asf. For the calculation or presentation of results it is allowed to use different computer programs.

Notenschlüssel:

sehr gut:	91	bis	100
gut:	80	bis	90
befriedigend:	66	bis	79
genügend:	50	bis	65
nicht genügend:		bis	49

Punkteabzüge:

Angabefehler (AF):	2 – 8 Punkte
Rechenfehler (RF):	2 – 4 Punkte
Denkfehler (DF):	3 – 8 Punkte
Formalfehler (FF):	1 – 2 Punkte
Schreibfehler (SF):	1 – 2 Punkte

picture 1

Submission

The instruction is submitted with the consideration of the allowed use of CAS (resp. other programs). An exact grading key and a completely calculated solution are attached. In the case of tasks that allow more than one result and interpretation, only one is carried out exemplarily with consideration to other possible solutions. A penalty on the scores is submitted as well with a little clearance according to the level of difficulty. One possible variation is presented in picture 1.

Correction

As agreed with the state superintendent of schools only the students' printouts are being corrected. In case of doubt the saved solution files may be consulted as decision support. Answers to open questions or theoretical questions are evaluated concerning the understanding of mathematical concepts regarding the qualification of the chosen mathematical models as well as the independent interpretation of the solutions in preferably practical tasks.

6. My personal résumé

After a couple of years' experience with tests and A-levels one could draw the following conclusion: Students have hardly any problems handling the CAS-programme. They can also deal with the so called arithmetic problems that work as a kind of sheet anchor and they show good results. The documentation of the steps of calculation takes getting used to. In general there are problems mostly with theoretical questions and understanding questions.

Mathematics is the art to spare oneself calculating. (Bruno Buchberger)

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