A class practice to improve student's attitude towards mathematics

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Abstract

For many students, mathematics, traditionally thought to be difficult and dull, is often considered inaccessible, generating a negative attitude towards it.

In order to encourage a positive attitude towards mathematics, we propose class practices that, through research activities, will lead the students to experiment a similar path to the one that has given, as a final product, a structured theory, so as to enhance their self-efficacy, give a correct vision of the discipline and stimulate positive emotions. This can be realized, for example, as a "laboratory activity" in which the students compare ideas, intuitions, arguments, and work together to obtain results, using their critical capabilities in a collaborative learning activity.

A team of university professors and high school teachers has developed a laboratory activity that focuses on some properties of quadrilaterals. The activity has at any rate been experimented in different first biennium classes of some high schools and has obtained very good results.

Introduction

Several national and international studies have focused on learning mathematics at secondary school revealing that students lack both proper knowledge of mathematics itself and the capacity of using mathematical tools to interpret reality. In fact, it is more and more common for students to feel uncomfortable towards mathematics, which they consider an abstract science quite far from their experience and interests, of no use for every-day life, a bulk of disconnected theorems just difficult to demonstrate, a mass of rules and formulas just to remember. A cold and fearsome mountain, too rough to climb, or as project to give up before starting.

In order to handle the difficulties of the students in this discipline the Italian Ministry of the University and of the Scientific Research has assigned resources to projects whose intent is to monitor the phenomenon, to circumscribe it and to reduce it. One of these Projects is the Progetto Lauree Scientifiche - Scientific University Degrees Project (PLS), that aims to improve the relationship between students and basic scientific subjects: chemistry, physics, mathematics and material sciences.

The proposal that we present has been developed within the PLS by a team composed of university professors as well as high school teachers. The group has been working with the purpose of developing in students "*correct attitudes towards mathematics, that is also an adequate vision of the discipline, not just a set of rules to memorize and to apply, but recognized and appreciated as a context to face and to investigate significant problems…*" (ministerial guidelines, 2007) .

The attitude towards mathematics consists of three interacting components: an emotional disposition, a view of mathematics and a sense of self-efficacy [5].

In particular:

the *emotional disposition* is the set of emotions (fear, anxiety, frustration, rage, pride, satisfaction, excitement, joy, to cite some) that are awakened by an activity [13];

the *vision of the mathematics* is the set of beliefs that the person has about it;

the *sense of self-efficacy* is defined as people's beliefs about their capabilities to produce designated levels of performance that exercise influence over events that affect their lives. Self-efficacy beliefs determine how people feel, think, motivate themselves and behave [2].

Such beliefs produce these diverse effects through four major processes. They include cognitive, motivational, affective and selection processes. Precisely:

cognitive processes: Much human behavior, being purposive, is regulated by forethought embodying valued goals. Personal goal setting is influenced by self-appraisal of capabilities. The stronger the perceived self-efficacy, the higher the goal challenges people set for themselves and the firmer is their commitment to them.

motivational processes: Self-beliefs of efficacy play a key role in the self-regulation of motivation. Most human motivation is cognitively generated. People motivate themselves and guide their actions anticipatorily by the exercise of forethought. They form beliefs about what they can do. They anticipate likely outcomes of prospective actions. They set goals for themselves and plan courses of action designed to realize valued futures.

affective processes: The stronger the sense of auto-efficacy the more vigorous people are in facing stressful difficult situations the more they succeed in modifying them. A low level of self-efficacy can generate anxiety as well as depression. Mood and self-efficacy feed each other reciprocally in a bi-directional manner;

selection processes: People are partly the product of their environment. Therefore, beliefs of personal efficacy can shape the course lives take by influencing the types of activities and environments people choose. People avoid activities and situations that they believe exceed their coping capabilities [2].

If emotional disposition, view of mathematics and sense of auto-efficacy manifest themselves in an improper or negative way, they generate in the student a close-mindedness towards the discipline that is impossible to undo. At this point, all intervention of cognitive nature that can be applied are unsuccessful or produce poor results.

The diagnosis of negative attitude has to be, then, a starting point for a intervention that is finalized to modify those components that did not developed and manifest correctly [13].

It is possible to influence this attitude [5] and the most significant mediating factor of the formation of the attitude is constituted by the rule of the teacher. As a coach can influence the self-efficacy and the emotional disposition of his athletes, favouring the consolidation of the future expectation by programming training in which they experience the success of overcoming an obstacle, so the teacher can influence the attitude towards mathematics of his students arranging paths finalized to reach goals that are concrete and that respect the capabilities of the single students and of the whole class.

The activity that we present wants to give the teacher an instrument to favour a positive attitude towards mathematics through a "research activity" proposed to the students in order to enhance their sense of self-efficacy, give a correct vision of the discipline and stimulate positive emotions.

In this paper, after describing the "didactic laboratory of mathematics" and explaining its characterising elements, we propose a laboratory activity on a didactic trail of plane geometry and we describe the results obtained in its experimentation.

The laboratory of mathematics

If it is true that "If I listen I forget, if I see I remember, if I do I understand" (Confuscius) then, in order to influence on the negative attitude towards mathematics from the students, it is opportune to support traditional teaching methodology with new ones that make students builders of their own knowledge.

This may be included, for example, in a *math teaching laboratory*, intended as *"a phenomenological space to teach and learn mathematics developed by means of specific technological tools and structured negotiation processes in which math knowledge is subjected to a new representative, operative and social order to become object of investigation again and be efficaciously taught and learnt"* [4].

The laboratory as a mathematics teaching and learning environment is today often used [1, 6, 7, 8, 11] and also the Italian Mathematics Union, in writing the new curricula, suggests [10]: "*We can imagine the laboratory environment as a Renaissance workshop, in which the apprentices learned by doing, seeing, imitating, communicating with each other, in a word: practising. In the laboratory activities, the construction of meanings is strictly bound, on one hand, to the use of tools, and on the other, to the interactions between people working together[…] to the communication and sharing of knowledge in the classroom, either working in small groups in a collaborative and cooperative way, or by using the methodological instrument of the mathematic discussion, conveniently lead by the teacher*".

In particular, among others, important elements that characterize an activity in a mathematics laboratory are [12]:

- *A problem to solve*

The proposed activity is a mathematical research activity on a problematic situation to explore which gives significance to the study of mathematics because it makes the students understand what "to do mathematics" means. The aim of the activity is to give the students a versatile working method rather than some specific knowledge. In order to create an atmosphere of research and discovery, the problematic situations proposed to the students must be "new" for them, do not have to bee too easy or too difficult, and their resolutions must require tools that the students have already acquired. We decided to propose an activity on plane geometry in which the students are faced with known and new properties of quadrilaterals [9].

- *Objects/instruments that can be used/manipulated*

The entire activity is based on the use of dynamic type geometric software (we have referenced *Cabri Geometre II Plus*) which are used as research tools. Following the forms, students build with Cabri the figures that they use in their path, and by means of their manipulation discover and verify conjecture and identify their arguments.

- *Working method (relationship-interaction)*

Our proposal is realized as a "laboratory activity" in which the students compare ideas, intuitions, arguments, and work together to obtain results, using their critical capabilities in a collaborative learning activity: the explore and formulate conjectures, they verify them and then give a proof of them. In this way the pupils emerge themselves in a research atmosphere and use its methods. Awakening in them curiosity and initiative, they are given the opportunity to try the relish of a challenge, the joy of a discovery, and the gratification of obtaining the results [8]. The students become researchers and "builders of their own knowledge". We want, in fact, make the student experiment a path similar to the one that has given, as a final product, a structured theory.

- *The role of the expert-coordinator*

It is the job of the teacher to guide the pupils to attain various results by way of trial and error, to direct the students with appropriate suggestions on the path to follow, to question the proposals that still need to be perfected using counter examples, to encourage them to continue, to praise them for every significant result. Moreover, he beats time and create the right atmosphere.

The proposed activity

After having identified the objectives, methodologies and the outline of the theoretical reference the team of professors has traced the path and has written the teaching forms [1]. The teaching forms are a sequence of reflections for the student and help him go over the difficulties that he may meet in his research work.

We decided to choose a geometry trial because it constitutes, with its inexhaustible wealth of results, a privileged field of research and of learning reasoning.

The proposal starts with the problem of how to extend the definitions of median and of height of a triangle to quadrilaterals. In fact, in a triangle we can define the concepts of angle bisector, axis, median and height and the relative notable points. The notion of axis and angle bisector of a triangle can be easily extended to convex quadrilaterals, but concurrency rules are no longer true. The notions of height and median of a triangle cannot immediately be extended to quadrilaterals because in this case there is not an opposite edge to a vertex.

Then, there are the following problems: give "new" definitions of height and median of a quadrilateral and find concurrency conditions.

In the proposal, the students

'**invent**' and define the concepts of *bimedian* and *maltitude* of a quadrilateral – Forms 1-2;

'**discover**', **verify**, **conjecture** and **prove:** *Varignon Theorem* – Form 3; some properties of the maltitudes relative first to trapezium and second to quadrilaterals in general, and then discover and prove a *formula for calculating the area of any quadrilateral* – Forms 4-9; that the axes of a quadrilateral are concurrent if and only if the quadrilateral is cyclic; that the maltitudes of a quadrilateral are concurrent if and only if the quadrilateral is cyclic; that circumcentre and the anticentre of a quadrilateral are symmetric with respect to the centroid of the quadrilateral – Forms 10-13;

reformulate the Brahmagupta theorem in terms of maltitudes and prove it – Form 14.

The experimentation and the results

The experimentation, that has involved 108 students, was done in five classes of four high schools in Eastern Sicily and has been carried out in some cases both during school hours and after-hours, in others only during after-hours, for a total of 20 hours; it was performed in some schools by the teacher of the class in others by an outside teacher. In some classes there were also outside observers (student teachers). The use of the forms has anyway given uniformity to the experimentation.

Students decided to take part of the experimentation themselves on a voluntary basis; they were aware that it was an activity done together with the University and this, rather than discourage them, made them curious and proud to take part in it.

The students were informed of the experimental nature of the activity and of the fact that they would have not been evaluated by the teacher. This permitted even the shyest students, as well as those less capable, to be involved: wrong answers were not negatively evaluated, but actually served as the basis for resulting discussions that would clarify the problem.

In one class, even though it was the first time the students utilized *Cabri*, they were immediately interested.

At the end of the experimentation we thoroughly analysed the teachers' reports and those by outside observers, the student's forms and their final remarks and we found excellent results. Students have worked with enthusiasm and interest, "*stating conjectures and verifying whenever it was asked, posing questions and personal remarks always with more interest, showing maturity and creativity that otherwise would have not come out*" (outside observer).

It is opportune to emphasize that:

- Several times more students gave different proofs for the same theorem, appreciating then their own capabilities, and showing a new enthusiasm and curiosity towards the discipline. This has positively affected their sense of auto-efficacy, their emotional disposition and their vision of the discipline: "*Since I started this* *activity I see geometry in a different way and when I build a figure and I study a figure, I think a lot more and I see all of its aspects"*;

- This positive experience was even more extraordinary for the involvement of some students that, generally, used to attend classes as listeners and did not like to be involved in the educational dialogue: they produced good results and they made their own the method used: "*Since I started this activity, I like geometry and I also work at home*";
- Some difficulties came up in students that were not used to giving proofs and "writing of mathematics". Lots of time was dedicated to the proofs and to writing them. Proofs were first written on the Cabri working area and rewritten on the form only in their final form to which students got helped by the teacher;
- lots of time has been given to the group activity and to the mathematic discussion as an opportunity for comparison both in constructing the knowledge and in presenting the results. These ways of learning, that have created a positive environment of "collaborative competition", were appreciated by the students. "*Group work is enjoyable is beautiful and if you work together you learn better and more easily*"; "*We have learned from this experience that in doing research your individual effort, as well as that of the group is very important*". Every body could say his own opinion without being judged and everyone helped in reaching the final result. The teacher played a fundamental role, checking up on the work done, at time acting confused in order to point out errors or inaccuracies, and praising them for the results obtained. Furthermore, the teacher acted as a moderator when doubts came up;
- the initial skepticism of some teachers was replaced with a great enthusiasm for the activity itself. This helped improve the relationship between the teacher and the students and has favoured a constructive dialogue between them. Also students noticed that "*the activity helped me in the relationship with the professors. I felt important and appreciated*";
- The students appreciated this way of doing mathematics using Cabri, which was new for some of them and was fundamental in the whole activity. In fact Cabri, by simplifying figures construction and allowing their manipulation, helped students to discover and verify conjectures, which was indispensable for their research activity. Cabri was shown to be a versatile and valuable teaching tool for our pupils' mathematic formation: "*You never forget what you have learned with Cabri*"; "*Cabri was really important because perfect figures are helpful*"; "*Moreover we better understood the difference between 'verify' and 'prove', by finding out that not all conjectures were true*".

Mathematics has changed in the student's eyes and it has become an interesting discipline to discover and to investigate: "*It was interesting to know that mathematics is not a dead subject , that not every thing has been written in stone, and to verify personally that still new objects can be investigated, new definitions can be given and new properties can be discovered. Mathematics is alive!* "

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