Mathematics and Mathematics Education Development in Finland: the impact of curriculum changes on IEA, IMO and PISA results

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Abstract

Mathematics has got roots in Finland in the last quarter of the 19th century and came to flourish in the first quarter of the next century. In the first quarter of the 20th century, mathematicians were involved in teaching mathematics at schools and writing school textbooks. This involvement decreased and came to an end by the launching of the 'New Math' project. Mathematics education for elite was of positive affect to higher education, and this has changed by the spread of education, the decrease of mathematics teaching hours at schools and the changes in school mathematical curricula. The impact of curriculum changes is evident in Finnish students' performance in the IEA comparative studies, PISA and IMO.

1. Brief and incomplete history of mathematics cultivation in Finland

The University of Helsinki of today is a direct continuation of the first Finnish university established in Turku in 1640. This was the only university Finland had till 1917. Relying on one university, mathematics had got a chance to grow, receive strong roots and to flourish. The strong roots of the Theory of Functions had got its beginning of growth through the five years professorship of the eminent Swedish mathematician *Gösta Mittag-Leffler* (1846-1927) in Finland, from 1876 to 1881.

Two eminent Finnish mathematicians continued *Mittag-Leffler* work, first his student *Robert Hjalmar Mellin* (1854-1933) and then Mellin's illustrious student *Ernst Leonard Lindelöf* (1870 – 1946). Our most well known mathematician, *Rolf Herman Nevanlinna* (1895-1980) was a student of Ernest Lindelöf, who was the cousin of his father. In 1919, at the age of 24, Nevanlinna presented his thesis, but the most important work of Nevanlinna was published in 1925. This includes the invention of harmonic measure and developing the theory of value distribution, named after him "Nevanlinna Theory". After Nevanlinna it is difficult to find a Finnish mathematician of the same international standing. But *Lars Valerian Ahlfors* (1907 – 1996) is such one. He was a student of both Lindelöf and Nevanlinna. At the age of 21, Nevanlinna's teaching gave him inspiration to solve the Denjoy's conjecture problem. This achievement gave him 8 years later, to be one of the two recipients of the First Fields Medal, awarded in 1936 (Kaskimies 1947, Elfving 1981, Ahlfors 1982).

2. Mathematics, mathematicians and mathematics education in Finland

2.1. School teaching and school textbooks before reforms

The cultivation of mathematics in Finland wouldn't have happened without cooperation of eminent European mathematicians. But, it is also true that this cultivation wouldn't have happened unless a success in school teaching of mathematics was achieved. In Finland, it was common to find a distinguished mathematician working as a schoolteacher or writing a school textbook. From the above mentioned eminent mathematicians Mellin and also Nevanlinna had worked as schoolteachers in Helsinki (Elfving 1981, Lehto 2001).

Ernst Bonsdorff (1842-1936), the eminent researcher in the Invariant theory temporarily had held the professorship of mathematics. But, in 1976 Mittag-Leffler was choosen to this professorship vacancy, and not Bonsdorff. This has made a turn in the history of mathematics, and also mathematics education in Finland. Bonsdorff then continued his work as a head teacher of Hämeenlinna Normal Lyceum (Elfving 1981, 58-60). Normal Lyceum is a Secondary Teaching Practice School. Besides his high teaching skills, Bonsdorff wrote some of the best, ever written, Finnish mathematics textbooks, among others Geometry textbook of 1889 (Bonsdorff 1889).

From Rolf Nevanlinna's family both his father Doctor *Otto Wilhelm Nevanlinna* (1867-1927) and his uncle Doctor *Lars Theodor Nevanlinna* (1850-1916) were Secondary School mathematics teachers. They had played a major role in the development of mathematics Education in Finland. Lars Nevanlinna was a teacher for more than 20 years and in 1902 he became the superintendent of Mathematical Subject of the National Board of Education (Elfving 1981). Lars Nevanlinna's textbooks have been used for more than half century (Neovius-Nevanlinna1950).

Last well-known Finnish mathematician, who worked as schoolteacher, was *Kalle Väisälä* (1893-1968). As Rolf Nevanlinna, he was a student of Ernest Lindelöf and at the age of 22 he presented his thesis. This was on Algebra and not on the area of most Lindelöf's students: the Theory of Functions. In 1941/42, the mathematics teacher of Väisälä's son's left the school for a military service and no relevant candidate for substitution was available. This event, besides Väisälä's interest in school mathematics, was a reason to make from a professor of mathematics a schoolteacher. School teaching experience was translated into a series of school textbooks (Kaskimies 1947, 202-203). From 1946 to 1970 Väisälä's textbooks were the most used in Finnish Secondary Schools, among others Algebra textbooks (Väisälä 1963).

2.2. School Spread and mathematics education in the 20th century

After Finland's getting its independency in 1917, the decrease of mathematicians involving in school Education was evident. This was an outcome of two new elements: the establishing of new universities and Higher Education institutes, and the spread of Secondary Schools. Finding qualified teachers for the new vacancies wasn't easy, also for the new universities and higher institutes.

The spread of schools in the 1950s, after the end of wars in 1944, was the greatest in Finnish history. In 1938/39 the number of Secondary School Students was 53 000, but this number became 214 000 in 1960/61 (Halonen 1982, 93). The baby boom and the growth of economy after wars were behind this growth, where the compulsory of Primary School, since 1921 Act (Päivänsalo 1973, 10), had made this growth possible. Further, the spread of Secondary School and the growth of economy gave the base for the establishment of Comprehensive School (Grades 1-9), by the School System Act of 1968. Comprehensive School was established to form the compulsory education in a welfare base. This change has led to the increase of students' numbers at Senior Secondary School and as well in Higher Education.

2.3. The place of mathematics in school curriculum in Finland in the 20th century

In the first decade of the 20th century, the number of mathematics teaching hours was significantly higher than that of any other school subject. The other area of special interest was the study of languages. Among others, in Junior Secondary School (Middle School) four languages were provided, including Finnish. The total number of these four languages' teaching hours was more than twice of that of mathematics. But, the number of teaching hours of Art and Physical Education, all together four subjects, was less than mathematics teaching hours (Halonen 1982, 33).

In 1914 a slight drop had happened to the number of teaching hours of mathematics (Halonen 1982, 33, 51), then again similar drop took place in the years 1918, 1941 and 1948. The most significant drop was that of 1972, at the time of Comprehensive School establishment. The decreasing of mathematics teaching hours, beginning in 1914, has changed mathematics place in our schools. Where the number of mathematics teaching hours before 1914 in the five years of Middle school was 23, the corresponding number in 1972 was 18. In 1986, UNESCO published the results of a survey, in which a comparison of mathematics teaching hours worldwide was provided. Among 27 European countries, the number of mathematics teaching hours in Finland was the lowest and one of the lowest among all of the 94 countries participated in the survey: *"… the range varies from an average of 2.6 hours per week in the case of Finland to an average of 6.1 hours per week in Switzerland – and these average are over 12 years of general education"* (UNESCO 1986, 35). An hour in UNESCO report is of 60 minutes. Here to note that the UNESCO survey aimed to investigate the place of Science and Technology in School Curricula. The survey's data collection had taken place in 1981 (UNESCO 1986, 8). In 1985, slight drop of mathematics teaching hours was again made (Kouluhallitus, 1985, 316-317).

3. Mathematics education reforms, Olympiads and comparative studies

Both, comparative studies on students' achievements, and mathematics curriculum reforms, had started at the end of the 1950s as post war plans in Western world. The lunching of Sputnik in 1957 had made the implementation of these plans urgent (Husén 1967, Vol. I, 25). Also, at that time, the former USSR and other 6 former socialistic countries met for the first time in the mathematical competition known as International Mathematical Olympiads (IMO). Finland was one of the 12 western countries, which participated in the First IEA in mathematics (FIMS), started at the end of the 1950s. In 1965, Finland became the first Western country to participate in IMO. In addition, Finland participated in the 'New Math' reform project, started at the beginning of the 1960s. This active participation characterizes a general Finnish well.

In the First IEA (FIMS), for Junior Secondary in the case of Population 1a, Finland got the 3rd place among the 10 countries participated, and for population 1b the 4th place among the 12 countries participated. For Senior Secondary School, Finland got the 6th place out of the 10 participated countries, where mathematics was regarded as complementary part (Short courses). But for Senior Secondary School, where mathematics was substantial part (Long Courses) Finland got the 10th place out of the 12. In addition, the standard deviation of Finnish students was the 3rd lowest for Senior Secondary School and the Lowest for Junior Secondary School. For both schools, the more detailed results show that students' scores are around average level, where high scores were missed completely, and quite the same for low scores (Husén 1967, Vol. II, 22-25). These results characterise the education of mathematics in Finnish Schools, up-to-date.

Here to mention to a unique type of consistency in the FIMS Finnish students results. Not only the total sample of Senior Secondary School with 'Long Courses' got the 10th place, but also the upper 4% of this sample got the 10th place among the corresponding samples of other countries (Husén 1967, Vol. 2, 122). The 10th place of the 4% mentioned of 1964 testing of FIMS, and the missing of high scores explains why Finland in 1965 IMO got again the 10th place out of 10 participated countries.

Despite the all-round type of the established Comprehensive School in 1970, and despite the decrease of mathematics teaching hours accompanied, the implementation of the 'New Math' curriculum to all schools, started in the same year, had been of a positive affect on the results of IEA II (SIMS), and as well on IMO results. SIMS testing was in 1981, after 11 years of the beginning of the 'New Math' curriculum implementation. According to the micro study of D.F. Robitaille and A.R. Taylor, Finnish students of both Junior and Senior Secondary Schools had made better in SIMS, more than in FIMS, with the exception of the case of arithmetic test (Robitaille & Taylor 1989, 174, 160-176).

Regards 1981 IMO, which was hold in the year of SIMS testing, Finland made a remarkable progress by getting the 12th place among 27 countries. In the second year 1982, Finland became the 8th among 30 countries, and this is the best result ever for Finland's teams in IMOs. The participated students in IMO 1982 had started schooling in 1970, the year of 'New Math' curriculum implementation for all the 12 years of General Education. In 1983, Finland got the 12th place, among 32 countries, but in the next year, in 1984, Finland got the 29th place among 34 countries. The beginning of the 1980s was the time of replacing the 'New Math' curriculum by the 'Back-to-Basics' one, in all Finnish Schools.

4. Mathematics education changes since 1980

The 'Back-to-Basics' curriculum changes didn't bring back Euclidean structured Geometry and its deduction to Schools. In the 1980s and 1990s emphasis was putting on mastering mathematical skills, especially arithmetical ones. In comprehensive school, arithmetic education became based on drilling. Even rhythmic cassettes were made for learning multiplication tables by heart in the 1980s. This agreed with the emphasis of the 'Guaranteed calculation kills' in the National curriculum of 1985 (Opetushallitus 1985, 147). For Comprehensive School, the study of Algebra became in practice additional arithmetical one (see Malaty 2007). In the case of Senior Secondary School, ready formulas and algorithms were used to solve algebraic problems in mechanical way. Here we present two different examples. The first is for Algebraic teaching (Short Courses), and it shows how mechanical approaches had moved aside logical thinking and its elegancy. To solve the equation $(x+1)^2 = 9$, the only solution provided by a textbook is based on the use of Quadratic Equation formula. In addition, the formula itself is giving as a ready rule (Mäkinen, Sivonen and Rahikka 1995, 199-200). The second example is related to Senior Secondary School matriculation examinations (Long Courses). To solve the equation, $|2x-1| = |3x+2|$, the model solution published in the main Finnish newspaper was starting by writing the equivalency $|2x-1| = |3x+2| \Leftrightarrow (2x-1)^2 = (3x+2)^2$ (Helsingin Sanomat 1994). This model does not only represent the use of unneeded long mechanical performance, but how the use of illogical approach had been accepted. The understanding of the concept of absolute value has to lead to the conclusion $2x-1 = 3x+2 \text{ or } 2x-1 = -(3x+2)$, upon which the presented equivalency can be obtained. The proposed model is doing the opposite.

Geometry in Comprehensive School became a calculation of perimeters, areas and volumes, and as well the using of instruments to make constructions with the help of given steps. No justifications for such steps are provided. In Senior Secondary School, teaching Geometry became also related to calculation of areas and volumes with more using of trigonometry. In textbooks, it became difficult to find a model for writing mathematical text, where pure mathematical mistakes became not rare.

Since 1985, besides the 'Back-to-basics' affect mentioned above, special interest arose in 'Problem Solving'. This includes encouraging students from the first Grade of Schooling to solve problems, where the solution is in need of rather common sense, than studying of mathematics. Under the name 'Problem Solving', also puzzles had been offered to children, where they had only to wait until the trick of the solution is presented to them. The so-called 'Ethnomathematics' had been translated into 'Everyday Life Mathematics'. This includes everyday life problems, where among others making diagrams and reading ready ones became a common activity since the beginning of schooling and even in pre-school education (Malaty 2002).

5. The impact of changes since 1980 on TIMSS

The changes we have mentioned are in need for more detailed discussion, but this we leave to another paper. These changes had taken place in our schools in the 1980s and 1990s and this has affected in the results of the two comparative studies, in which Finland took part after SIMS. These studies are the Third IEA study, which turned into TIMSS, and PISA. PISA, by its philosophy is related to Junior Secondary School, but TIMSS is much wider. Finland took part in only the Junior Secondary School part of TIMSS, where testing was in1999.

For Junior Secondary students, where in FIMS and SIMS Finland results in algebra were much better than in arithmetic (Robitaille & Taylor 1989, 174, 160-176), the opposite has happened in TIMSS. Finland had got the 10th place out of 38 countries in the performance of the test on 'Fractions and Number Sense and operations' of TIMSS, but the 20th place in Algebra. Where Finland had got the 18th place in 'Geometry', it got the 15th place in the test of 'Measurement', where calculating areas were common items. The best ordinal rank achieved was the 9th, and this was in the test of 'Representation, Analysis, and Probability', where test items are mainly related to 'Everyday life mathematics' (Mullis et al, 2000).

6. PISA success and the future of Mathematics Education in Finland

PISA tests measure only mathematical literacy, and this is relevant to the Finnish School Curriculum. The over all ordinal rank, for Finland in TIMSS 1999, was the 14th, where performance in 'Representation, Analysis, and Probability' related to everyday life was the best. PISA items are not related to a particular mathematical content, like Algebra or Geometry (OECD 2004, 18). The dispersion of Finnish students' results in PISA is one of the lowest through the participated countries. This was always the case in comparative studies, also before the establishing of Comprehensive Schools. The new in PISA is that high scores are now achieved by reasonable percentage of students. In PISA 2003, where the focus was in mathematics, 6.7% of students achieved the highest level. The percentage of other 6 OECD countries and a Partner Hong Kong-China was higher, leading by Hong Kong (10.5%) and Belgium (9%). But this didn't prevent Finland from getting the First Place among 31 OECD countries, and the second among all the 41 participated countries. The reason is related to the equity principles in Finnish society, according to which special care of students with learning difficulties in mathematics is provided. Therefore, where 26% of all 40 countries participated students were at level on**e** or lower, only 6.8% of Finnish students were of such low achievement. Looking back to PISA tests we find that Finland got the first place in only one test out of four, and this is the test of 'Quantity', where the difference between the mean of Finnish students, in this test, and the mean of each other country has a statistical significance, with the exception of the case of Hong Kong (OECD 2004).

7. Conclusion and future mathematics education development in Finland

Mathematics Education for all after the independency of Finland had met difficulties in offering understanding of mathematics to the level achieved before, where education was for elite and in some cases offered by elite. Therefore in comparative studies, since FIMS, no one from Finnish students got highest scores and, in general, no success was achieved in IMO. But, Finnish teachers have shown high ability in implementing every reform. Even at the time of establishing Comprehensive School, with low number of mathematics teaching hours, SIMS results were better than in FIMS. Success also was achieved in IMO as the 'New Math' had put emphasis in the structure of mathematics. The implementation of 'Back-to-Basics', 'Problem Solving' and 'Everyday Life mathematics' has brought some success in TIMSS and more success in PISA. Here to notice that TIMSS tests are closer to PISA than to FIMS and SIMS. The problem is that we are not now able to get success in IMO and the mathematical level of Secondary School graduated is not satisfactory for higher education. Since 1995 changes are taking place in developing mathematical curricula, where some mathematicians had become involved to some extent in school mathematics education. Also efforts have been made to take care of gifted students. Taking care of gifted students in Physical Education and Art is accepted, but why not in mathematics? This can raise a question about the success of making mathematics understandable and making its beauty and elegance seen. In other words, it is a question of mathematics education success.

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