

Bridging the gap between technology design and school practice: a specific experiment within the ReMath Project

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Abstract

This contribution describes an experiment carried out by a team within the ReMath (Representing Mathematics with Digital Media) European Project (<http://remath.cti.gr>). Within this project six digital dynamic artefacts (DDAs) have been developed, thirteen experiments have been planned (Artigue & al., 2007) and carried out, analysis of the collected data are still in progress. In this contribution, we focus on the case of the Aplusix DDA (<http://applusix.imag.fr>), from the point in which the designers deliver their product to the team in charge of planning the experiment, up to the point in which the artefact is experimented within the ReMath project.

Introduction

It is well known that the impact of digital media on mathematical teaching and learning in schools at the European level is weak (Artigue, 2007). Although looking into the reasons of that situation is undoubtedly a complex task, it is possible to look for responsibilities to both in the community of researchers in Mathematics Education and in the community of Mathematics teachers. On the one hand, theoretical frameworks emerging from research on teaching and learning mathematics with digital media are fragmented and usually involve assumptions bound to the specific contexts from which they emerged; on the other hand, teachers are often reluctant to use new technologies in class justifying their behaviours in terms of time constraints as well as curricular constraints. New tools ask teachers spend time to familiarize with them, ask them invest time to plan lessons in a new way, ask them to manage time in a different way in classroom. Obviously all these aspects have to be taken into account each time a new tool is integrated in the school practice, but only if considered in a superficial way, the need to change becomes an obstacle impossible to overcome.

Moreover, deeper reasons of the lack of integration of new technologies within the school practice can be found in analysing the tools themselves. In fact, very often, the potentialities of these media are not clearly understandable by teachers; it seems that the design is not immediately transparent to the users. The ReMath Project, moving from the analysis of the situation both at the design level of a digital medium and at the use level in classroom, proposes a solution ‘shortening the distance’ among tools’ designers, math education researchers and teachers.

A way to make designers, researchers and teachers interacting

According to the assumptions shared by the teams of the ReMath Project, the starting point on which the dialogue between teachers, designers and researchers has to be constructed, consists in identifying the ‘didactical functionalities’ of a given ICT (information and communication technology) in respect to a stated educational goal.

With didactical functionalities we mean those properties (or characteristics) of a given ICT, and/or its (or their) modalities of employment, which may favor or enhance teaching/learning processes according to a specific educational goal. (Cerulli, Robotti, Pedemonte, 2006, p.1390).

But, who is in charge to identify didactical functionalities (DFs)? Since by definition, they can be identified according to a specific educational goal, either designers, or researchers, or teachers, can propose modalities of employment of ICTs to reach a didactical goal.

The designer who has implemented the piece of software, has been motivated by a (some) particular educational goal (goals), and in designing specific components has been guided by modalities of employment who has hypothesized. As a consequence, there are didactical functionalities that the software has embedded from its origin and which can be called ‘design-DFs’.

On the other hand, the ‘design-DFs’, can be not shared by a researcher who decides to face a research problem, i.e. the researcher can use the software either for a different educational goal or, even sharing the educational goal, the modalities of use can differ. We call ‘research-DFs’, the didactical functionalities identified by a researcher facing a specific research problem.

The teacher who decides to use a software in class usually after having read the ‘design-DFs’ described in the manuals of the software, is led to use the software coherently with those. Nevertheless the possibility to attribute new educational goals by a teacher, not belonging to ‘design-DFs’, can not

be neglected. We call ‘implementation-DFs’, the DFs identified by a teacher, where the word ‘implementation’ stands for ‘implementation in classroom’.

This classification of the different ways of approaching the issue of using a software in the school practice, gives the idea of the complex net of relationships between designers and users with respect to specific educational goals.

The experiment with the Aplusix DDA

Within the ReMath Project, the team of University of Siena, developed a pedagogical plan, that is a teaching/learning sequence aiming at reaching a specific educational goal (Earp & Pozzi, 2006), centred on the use of the Aplusix DDA, by means of a close cooperation with a group of teachers. Referring to the terminology we have introduced, starting from the analysis of the ‘design-DFs’ we can say that an attempt to come to an agreement with ‘research-DFs’, identified by the research team, and ‘implementation-DFs’, identified by the group of teachers, has been done.

Before describing the result of that cooperation, which ended in drawing up a pedagogical plan, called ‘Introduction to algebra: structure sense of algebraic expressions’, the main characteristics of the DDA are reported.

The Aplusix DDA

Aplusix is a computer algebra system which allows students to perform both numerical and literal calculations (Nicaud & al., 2006). One of the main characteristics of Aplusix is the feedback that it provides. The feedback is based on the equivalence between two consequent boxes, each of them containing an algebraic expression. More specifically, different signs show whether the current expression is equivalent/not equivalent to the previous one, or whether it is not well-formed (Fig. 1).

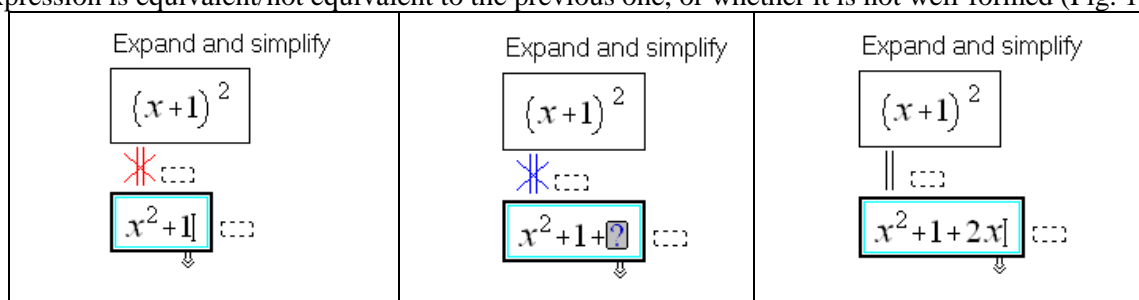


Figure 1. Three different feedbacks provided in Aplusix environment.

The black lines show that the first expression is equivalent to the second, the red crossed lines show that the first expression is not equivalent to the second, and the blue crossed lines show that the expression you are writing is not well-formed (i.e. a plus sign requires an argument).

Another valuable characteristic of the DDA is the possibility to work not only in the usual representation students are used to accomplish calculation in paper and pencil (the standard representation, SR in short), but also in tree representation (TR) (Fig. 2).

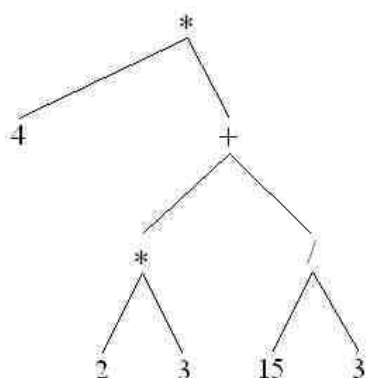


Figure 2. An algebraic expression given in tree representation.

The hypotheses which guided the development of the software are clearly defined by the designers when considering Aplusix a tool able to help students in performing algebraic calculations thanks to the feedback provided. Moreover, the designers state that TR component may constitute an additional support to overcome difficulties in doing algebraic manipulations (Nicaud & al., 2006).

As a consequence, Aplusix seems to be created to help students during training activities. As it will be made explicit in the following, the experiment based on the Aplusix DDA will not be done only in training activities, but in each phase of the mathematical activities in classroom. In this sense, the 'DF-implementation' differ from the 'DFs-design' .

Teaching and learning problems addressed in the experiment

The difficulties encountered by pupils in gaining competencies in algebraic calculation are well-known, and they have been addressed many times and from different point of views (Freudenthal, 1983, Tall & Thomas 1991, Kieran 1992). The introduction of algebra requires the development of a different way of thinking, which cannot be considered as a pure generalization of arithmetic (for an overview of the literature see Mariotti & Cerulli, 2003). Differently from arithmetic, whose main goal is to do calculations so as to have a result, algebra offers an operational language for representing, analyzing and manipulating relations which contain both numbers and letters. This diversity is explained by Sfard (1991) in terms of two different perspectives from which mathematical objects can be conceived: in a structural way (as objects) or in an procedural way (as procedures or processes).

In the school practice, often the rupture between the two different kinds of calculations is not put into evidence as it should be. Many students' difficulties in gaining competencies in algebraic calculations can be explained according to the 'continuity' they established between the two types of calculation.

The pedagogical plan designed with the collaboration of a group of teachers, proposes an introduction to algebraic manipulation starting from the arithmetic field. The school level is the 9th grade, that is the first year of Upper Secondary School in Italy. The teaching/learning sequence has been developed on two basic meanings on which the algebraic calculation is rooted. As a consequence the educational goal is twofold: the equivalence between algebraic expressions and the structure of an expression.

In this contribution we identify DFs related to the first educational goal, that is the equivalence between algebraic expressions. The components of the DDA which are mainly exploited so as to reach this didactical goal consist in the feedback provided and in the TR.

The theoretical framework

The pedagogical plan centred on the use of the Aplusix DDA has been framed by the Theory of Semiotic Mediation (Bartolini Bussi & Mariotti, 2008).

This reference frame, drawing from a Vygotskian paradigm, considers learning processes deeply linked to teaching processes, in a social context. It states that the use of artefacts to accomplish a task leads the individual to the construction of *personal meanings* (Vygotsky, 1978), which are related to the actual use of the artefact. At the same time the use of the artefact can be related to specific mathematical meanings. Such double relationship between the artefact and meanings is called semiotic potential of the artefact. Under the guidance of an expert (typically the teacher), students' personal meanings may evolve towards *mathematical meanings*, i.e. meanings coherent with the mathematical theory.

Thus any artefact will be referred to as tool of semiotic mediation as long as it is (or it is conceived to be) intentionally used by the teacher to mediate a mathematical content through a designed didactical intervention. (Bartolini Bussi & Mariotti, 2008, p. 754)

In this perspective, the function of semiotic mediation of an artefact is not automatically activated with the use of the artefact. In order to make meanings emerge it is crucial to identify the relationship, called the *semiotic potential* of the artefact (Bartolini Bussi & Mariotti, 2008), between the use of the artefact and the mathematical knowledge. Awareness of the semiotic potential of an artefact is a requisite for the teacher for developing suitable tasks for making meanings emerge, and for guiding the evolution of students' personal meanings towards mathematical meanings.

The organization of the teaching/learning process according to the Semiotic Mediation Theory is based on three types of activities, differently contributing to make the semiotic potential emerge and develop. These activities are organized in a cyclic structure: the activity with the artefact is followed by the individual semiotic work, the request of a report, which will be discussed and re-elaborated during a collective discussion. This sequence of three types of activities constitutes what is called a 'didactical cycle'.

The teaching-learning sequence

Exploiting the interaction between teachers and researchers, the pedagogical plan, entitled '*Introduction to algebra: structure sense of algebraic expressions*', has been designed. The main interaction between teachers occurs during the planning of the activities to be performed in class.

The pedagogical plan is constituted by a sequence of four didactical cycles. During the activity with the DDA (first type of activity in a didactical cycle) students work in pairs, and as a consequence, they also fill in pairs the worksheets they are asked to complete during the work in Aplusix. Students individually fill the reports they are required to complete after the work in Aplusix (second type of activity in a didactical cycle). Then, under the guide of the teacher, students' productions are collectively discussed (third type of activity in a didactical cycle).

As already said, the pedagogical plan pursues two educational goals: the equivalence between algebraic expressions and the structure of algebraic expressions.

The goal of the first didactical cycle, 'The meaning of equivalence through feedback', consists in making students conscious of the mathematical meanings related to the different signs showing as feedback by the Aplusix DDA.

The second cycle, 'Decomposition of natural numbers', aims at making students re-interpret the validity of the commutative property for some arithmetic operations by means of the 'tree representation'.

As far as concern the third cycle, called 'Syntactical skills', the didactical goal consists in exploiting the potentiality of the tree representation in order to mediate the meaning of structure of an algebraic expression. In fact, on the basis of the comparison between TR and SR, the different affordances of the two different semiotic registers are expected to emerge and become the key point of the discussion

Finally, the last cycle, 'Towards the structure sense', introduces the natural language as an another representation system for algebraic expressions (to put beside SR and TR) to reinforce the stated relationships between the two different representation systems, SR and TR.

The first cycle 'The meaning of equivalence through feedback'

For reasons of brevity in the following we describe only the first didactical cycle which is devoted to the interpretation of the feedback given by Aplusix in respect to the equivalence between algebraic expressions. The duration of the whole cycle should be two hours.

As a first step to plan the cycle, the semiotic potential of the Aplusix feedback has been analysed. Two possible interpretations of the signs given by the DDA between two consequent steps (Fig. 1) are possible (Fig. 3): the first is immediate – correct/incorrect - the second will need the mediation of the teacher to be achieved. To grasp this dichotomy, we called *feedback-signs* the signs and we called *primary interpretation* and *secondary interpretation*, respectively the first and the second interpretation mentioned above.




Inscription			
Primary interpretation	correct	incorrect	not specified
Secondary interpretation	equivalent	non equivalent	not well-formed

Figure 3.The possible interpretations of the feedback-signs provided by Aplusix.

The cycle is designed on the hypothesis that the first interpretation emerging should be the primary, and only exploring the different contexts in which the feedback-signs appear, the primary interpretation should be overcome and the secondary interpretation emerge.

In the first phase of the cycle the teacher briefly presents the DDA. In particular, the teacher shows how to log in the system and how to edit an expression in it. After that, students are invited to log in the DDA.

Then, students are provided with worksheets containing instructions both on how log in the Aplusix system (creating an account and a password to be remembered for the consequent access to the

system) and about the tasks to be performed. They are requested to accomplish easy calculations on rational numbers. While performing the task students have to take note of the different signs that Aplusix produces as feedback (feedback-signs). They are also requested to make hypotheses on the meaning of such signs. The primary interpretation is expected to emerge as the requested calculations are easy to accomplish. As a consequence, the students can concentrate more on the feedback-signs than on the completion of the task.

On the base of students' report, the teacher organizes a collective discussion with the objective of making explicit and clarifying the relationship between the interpretation of the feedback-signs in terms of correctness and the interpretation in terms of algebraic equivalence. At the end of the discussion the primary interpretation will have been evolved towards the secondary.

Conclusions

Although the educational goals pursued in the pedagogical plan are different from which envisaged by the designers, they are compatible with them. In describing the design of the pedagogical plan, it has emerged the crucial role played by the, Semiotic Mediation Theory. For this reason, during the interaction with the teachers in building the teaching/learning sequence, the theoretical aspects have been made explicit little by little. Some issues to be more investigated arise. What and in what level of granularity the theoretical framework adopted has to be made explicit between researchers and teachers? Since a software brings in itself a theoretical framework which has inspired its design, could this affect the consequent theoretical choices made by teachers and researchers?

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