

Individual Approaches in Rich Learning Situations

Material-based Learning with Pinboards

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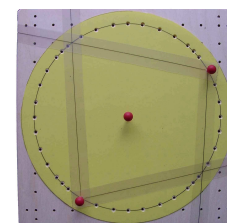
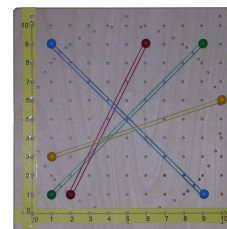
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Abstract

Active Approaches provide chances for individual, comprehension-oriented learning and can facilitate the acquirement of general mathematical competencies.

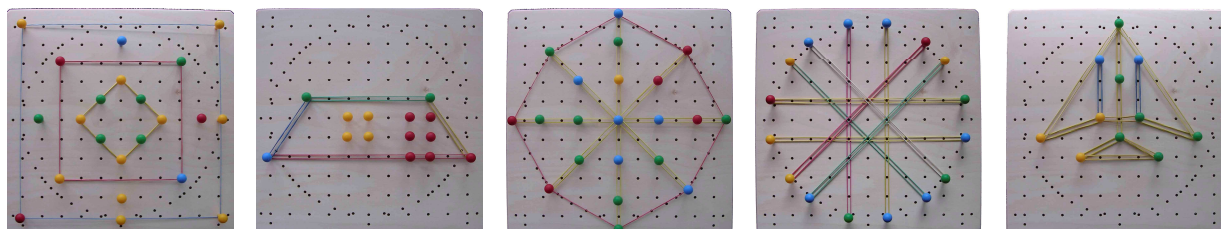
Using the example of pinboards, which were developed for different areas of the secondary level, workshop participants experience, discuss and further develop learning tasks, which can be used for free activities, for material based concept formation, for coping with heterogeneity, for intelligent exercises, as tool for the presentation of students' work and as basis for games. The material also allows some continuous movements and can thus prepare an insightful usage of dynamic geometry programs.

Central Part of the workshop is a work-sharing group work with learning tasks for grades 5 to 8. The workshop will close with a discussion of general aspects of material-based learning.



Introduction

Imagine a classroom situation in 5th grade: Students work with a pinboard for the first time. Their task is nothing else but developing a figure they like, describing their process and explaining the meaning of their figure in the class using the pinboards for presentation. These are typical figures students have developed in such learning situations:



By working with the material and by presenting all figures to the class, the students discover structural elements of the pinboards. While some students mainly use grid points, other students also include some of the 36 points on a circle line in their figures. Students also use the pinboard to present something personal such as making model air planes. The presentation does not need any preparation like drawing the figure on an overhead transparency. The way students are presenting their figure and talking about the figures of other students has a diagnostic value especially if the lesson is a beginning situation in a new learning group.

Students discover basic figures or symmetries. Based on the variety of the figures properties of special figures can be determined, patterns can be discovered and the different structural elements of the pinboard can be assembled.

The Material

The pinboards are made of birchwood.

A pinboard contains

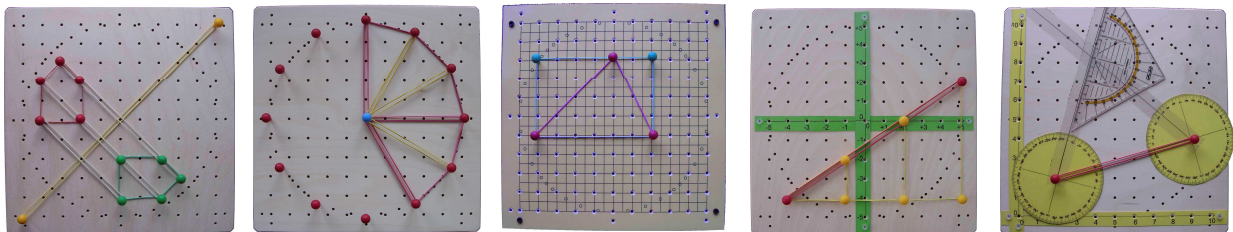
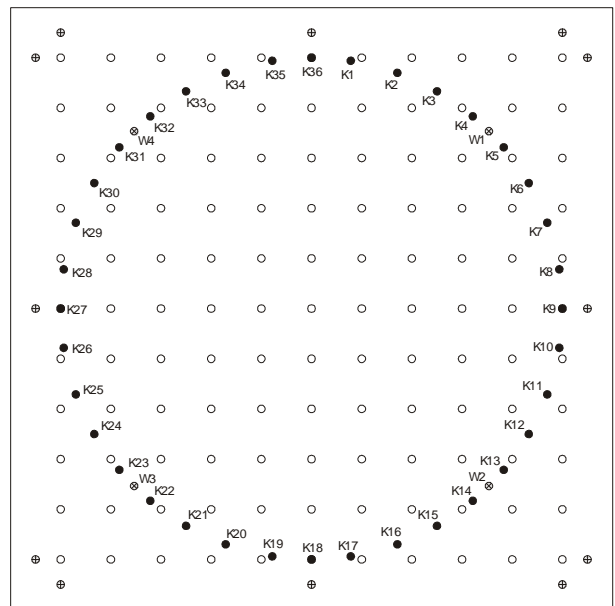
- 11 x 11 gridpoints (o) in the vertical and horizontal distance of 2 cm,
- 36 six points (●, K1 to K 36) on a circle line with the diameter 10 cm,
- 4 points (W1 to W4) on the circle line and on the bisecting line,
- 12 points outside the grid, which can be used to fix coordinate axes and other additional material.

A Class set of pinboards (typically one board for two students), pins in five colors and rubberbands are stored in a wooden box, which contains the same pattern of holes on each side.

Additives like coordinate axes, lines, discs to

measure angles can be produced by the students. The necessity to work accurately is self-evident.

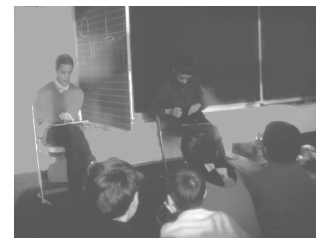
Thus the material can be used for application areas in geometry, functions, fractions and others.



Example 1:Material based introduction of a new concept

Obtaining an agreement to a coordinate system

One wing of a blackboard is moved forward to make sure the two students on either side cannot see the pinboard of the other one. The class is watching. The initial question is:



Imagine you have found a figure on the pinboard you really like. Your friend in another city has the same kind of pinboard. You want to describe the figure for him by phone, so that he also can build the figure. Let us test how it could work with a figure of about 6 corners.

While several students are trying to communicate the position of the pins, the class is realizing successful and less successful strategies. The student, for example, who has one pin in the middle of the first row of the pinboard, gives the order “Put the pin in the middle of the board.”, which makes the other student put it in the intersection of the diagonals. The audience realizes, how wrong positions are perpetuated, as a student continues to describe the position of one pin related to the position of the previous pin. Reflecting on the observations, the students describe reasons for problems in communications and criteria for successful strategies. Many students use the grid structure, for example “two steps to the left, three steps above”. It becomes evident, that a point of reference has to be agreed upon. So it is decided that the left grip point of the lowest row is to become the point of reference. Now it makes sense to introduce the notation for coordinates.as the number of steps, which had to be moved from this point to the right and above. Paper stripes are fixed on the pinboard and labelled to visualize the new concept. Hanni wrote a text showing that

she did not only experience the situation on the cognitive level:

Pinboards

Next time we are playing this game we got to do it directly like Rouven and Pascal. Before you choose a point, at which you have to go back again and again. Because, if you always move from one point to another, you lose quickly and easily. I really like these new stripes for the pinboards. You can always say $(4/3)$ or $(9/10)$, this is easier. I enjoy the fun game, for instance if something different happens as it should.

After the agreement on the coordinate system, the students draw the axes on paper stripes, label them and hammer holes in the stripes. Very often the first attempt does not bring the necessary result. The stripes cannot not be fixed, because the holes in the stripes are in a wrong position.. Thus the students have an evident self-control. The necessity to measure and draw precise is here part of material based work.

Example 2: Intelligent Exercise

From a square to kites

If you move one corner of the maximum square on the pinboard towards the opposite corner along the diagonal, symmetric quadrangles are generated. So a structured series of tasks evolves.

Which percentage of the area of the square has an emerging quadrangle?

Beginning with the first quadrangle, students find different approaches:

- They use the pinboard and a sheet with grid lines (100 small squares) and count the small squares.
- They dissect the square, determine the area of segments and subtract these from the area of the square
- They calculate areas of triangles and quadrangles by formulas.

At least after determining the first two results 90 % and 80 % students presume that by moving the corner one more step the area decreases by 10 % of the area of the initial square. Now the question is, if this presumption is plausible or can be confirmed for other quadrangles in the series. If a result does not fit to the series of other results, a check of the procedure by the students makes sense.

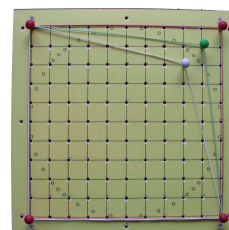
Further problems help to leave the material base:

Imagine that in the middle between two holes in the diagonal is another hole. What does the series of results look like?

Imagine the pinboard would be continued at the top and at the left. Determine the percentage of the area of the quadrangle emerging, when you move the upper right corner one step on the diagonal to the right. Solving this question by an appropriate dissection of drawing is a challenge. Regarding the area of the quadrangles as a function of the length of the diagonal helps to get a general solution.

Characteristics of this learning environment are:

- A low barrier at the beginning facilitates an access to the problem for every student.
- Different approaches, especially hands on activities for the students. Students individually decide if they use the material.
- By using the sheet with a hundred small squares, basis concept of percentages are enhanced.
- Self-control is possible because the results belong to a structured series.
- The limitation of the material and appropriate problems facilitate a transfer from the material based work to mental work.
- The structured series of problems contributes to building a concept of functions.

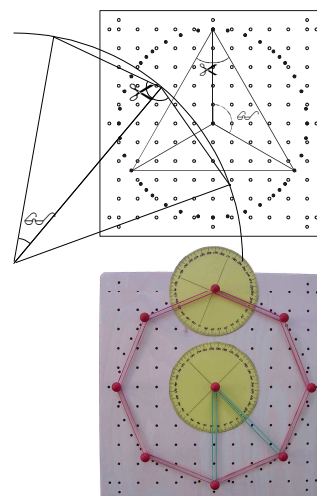


Example 3: Experimental approach – vertical integration:

The relation between the interior angles and the central angles of regular polygons

“Investigate the relation between the interior angle α of a regular polygon and the central angle β of a partial triangle.”

The pinboards allow one to measure the angles in the following regular polygons: triangle, square, hexagon, octagon, nonagon and dodecagon. Here is a typical result of the investigation:



Number of Corners	3	4	5	6	7	8	9	10	11	12
Interior angle α	61	90		119		133	142			150
Central angle β	121	90		60		44	41			31

After the data collection from all student groups first presumptions emerge:

- 1) The larger the number of corners, the larger the interior angle.
- 2) If the number of corners doubles, the interior angle also doubles, example $3 \rightarrow 6, 61 \rightarrow 119$
- 3) The larger the number of corners, the smaller the central angle.
- 4) The sum of the interior angle and the central angle seems to have almost the same value for different regular polygons.

A Graph is a next approach to the subject showing for example that presumption 2) is false.

When angles are measured in this certain context, a diagram of asymptotic curves occurs. There has to be a maximum of an interior angle and a minimum of the central angle because of the geometric properties. There are further presumptions concerning the missing values. The symmetry of the values can be discovered.

The determination of the missing data by the students drawing the polygons on paper is suggested, which brings the next challenge: How can one draw a rectangular pentagon? If students don't find an approach through calculating the interior angle, matches or sketches help to develop an imagination of the properties of a rectangular pentagon. By calculating the central angle it is possible to draw the missing regular polygons.

Yet the measurement is still imprecise. How is it possible to prove presumption 4) and determine the exact value? The pinboard may help to develop a figure for the proof.

As in the learning environment in example 2 all the students find an approach by hands-on activities and measuring angles. After some time some students will still measure angles whereas others will have proceeded to further tasks depending on the individual progress or on the progress of the work group.

In contrary to dynamic geometry programs the imprecise measurement and the limitation given by the material is a chance to understand why a general argumentation is needed to prove a general proposition.

The Workshop

After a short introduction to present impressions of classroom situations, the workshop participants will have the chance to experience the material and the learning tasks working on 10 stations like the examples two or three in this article. An exchange of the experiences will prepare a general discussion on topics like the following:

- Does the learning situation provide an access to the subject for every student in the learning group?
- Can the student work on the learning task on different levels?
- Is it possible, to discover mathematical structures or patterns?
- How can students be supported in the transfer from material based learning to mental work?
- How can students document working- and learning processes?
- Which chances are given for individual products of students?
- How can material based learning activities be combined with the aquirement of general mathematical competencies?