Understanding Quadratic Functions Using Real World Problems and IT

Nakhshin A. Karim, Bsc, Msc
Department of Mathematics and Statistics
Zayed University
Abu Dhabi, United Arab Emirates
Nakhshin_karim@zu.ac.ae

Abstract

The concept of function is crucial to a great extent in modern mathematics and is considered a major barrier to many mathematics students. Students have difficulty interpreting information related to functions in general, and quadratic functions in particular. Quadratic Function is one of the topics which are covered in a course which is compulsory for a large number of students in the General Education Program of Zayed University. This program leads to different majors, including Mathematics Education, Business, Information Technology, and other majors.

The challenge in teaching Quadratic Function in a course like this is mostly based on the fact that many students think that Quadratic Function is a difficult topic to understand and learn, and some teachers would agree with them that it is difficult to teach.

In this paper, I demonstrate real world problems aimed to improve the students understanding of Quadratic Functions; life problems on this topic support developing student's knowledge, critical thinking, quantitative reasoning, and analytical skills. This paper also includes examples of the techniques used with graphing of quadratic function, the algebra, and inverses of the same function.

International move to improve mathematics curriculum have supported new goals for student's learning which highlights problem solving skills, reasoning, ability to work in groups and individually, and use of technology. Knowing that information technology plays considerable role in achieving the above goals, teaching students the concept of Quadratic Functions can be smoothly achieved by using Information Technology in solving real world problems.

Introduction

In this paper I will explore, present and discuss the syllabus and pedagogy of COL 111, Mathematical Modeling with Functions course, which is a course in the Colloquy for Integrated Learning at Zayed University. The emphasis in this course is on applications of Quantitative Reasoning in the context of real-world problems; the main objectives of the course are to provide students with material appreciate the role and importance of quantitative reasoning plays in the world; also to provide students with an appreciation of the nature and value of mathematics and to enhance critical thinking skills.

To achieve above aims and one of the objectives of this course is to introduce students to the concept of mathematical functions using real-world problems and there mathematical models.

This delivers students with real representation of the application of mathematical function to inspire and prompt their understanding and learning.

The syllabus includes Linear Functions, Exponential Functions, and Quadratic Functions.

I will take Quadratic Functions to show different methods of presentation and solution of functions. One way is by providing the students with real-world applications and from there move to theory, solutions, and interpretation of solution, teach students algebraic manipulation like factoring, changing from a general form to the vertex form of quadratic functions and vice-versa. The other way is by introducing the real-world problem, and use software to graph the function, from the graph explain and answer questions related to the problem, also use the same software to find solutions of the function.

COL 111 (Mathematical Modeling with Functions)

This course is designed to provide students with a broad general education in quantitative reasoning and critical thinking. It is also provide a foundation for the development of their ability to function competently and confidently in majors' programs. The focus of the course is on analytical reasoning and thinking to solve real world problems in business, finance, economics, computer science, education and the natural sciences.

The content of the course is delivered through classroom activities to introduce the students to the various topics. For some topics or case studies, data can be obtained from primary sources connected with other courses, such as Environmental Science, and Health Science. In each area, knowledge, analytical skills, critical thinking and understanding is developed using relevant examples for discussion, analysis and interpretation in class with follow up exercises or assignments of a similar nature to be done individually or in groups outside the classroom.

The content of the course is summarized in 5 Units:

Unit 1: In this unit we introduce mathematical modeling, we allow the students to think mathematically, and use technology to show some elementary mathematical modeling.
Unit 2: This unit is on Connecting data, graphs and functions, look for data relationships, using data to check rates of change, to look for trends, increasing, decreasing, both or neither, look for a patterns and relationships.
Unit 3:Linear Functions, Rate of Change of a Function, Function Notation for the Rate of Change, A General Formula for the Family of Linear Functions, Formulas for Linear Functions, Alternative Form for the equation of a Straight Line.
Unit 4: Exponential Functions, more details of Growth Factors and percentage Growth Rates, General Formula for the Family of Exponential Functions, Relation to Compound Interest function, Comparing Linear and Exponential Functions.
Unit 5: Quadratic Functions, more on Explorations in Quadratic Functions, The Equation of Quadratic Function in General Form, The Equation of Quadratic Function in Standard Form, Changing from Standard Form to General Form, Changing from General Form to Standard Form, and Zeros and y-intercept of Quadratic Functions.

Quadratic Functions

I would start this important topic of the course, likewise any other topic by introducing a real world problem which can showcase the Quadratic Functions. Through the problem I can explain, demonstrate and explore the basic properties of this function, also sketch the graph of the function which is great help for students to understand all aspects of Quadratic Functions and its properties.
A quadratic Function is a polynomial of degree two ($f(x) = ax^2+bx+c$) where a, b, and c are real numbers with $a \neq 0$; its graph in " U " shape curve that is called a parabola.
All parabolas have an axis of symmetry called axis and the point where the axis intersect with the parabola is called the vertex of the parabola or the function.
The Quadratic Function $f(x) = ax^2+bx+c$, has an extreme point (-b/2a, f(-b/2a)), when a > 0, the extreme point is a minimum point and the parabola opens upward. If a < 0, the extreme point is maximum and parabola open downward.
Also, an important point to be raised is the domain and rang of the function, the domain of the polynomial functions and quadratic function is one of them is R, while the range can be calculated as follows:

$Y = f(\frac{-b}{2a})$ which is either the highest point(Maximum point) or the lowest point (Minimum point)

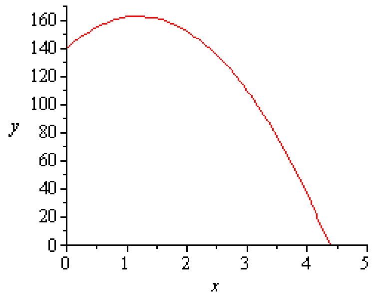
If a > 0 the range is [y min , infinity)
If a < 0 the range is (- infinity , y max).
Side by side with introducing the concept via world problem, using a soft ware to graph and solve the quadratic function is very important and make the understanding of this topic much easier. In this course I am using MAPLE as a tool.

<u>Example Of Application on Quadratic Functions</u>

A boy stand at a top of a 135 Foot high building, and throw a stone upword with an initial velocity of 38 feet per second , the stone travel up ward for a while then eventually be pulled by gravity down to the ground. The height of the stone above the ground is given by the function,

$F(t) = -16t^2 + 38t + 140$

Although the path of the stone is up and down, the graph of its height as a function of time is a concave down parabola, as per the graph below:



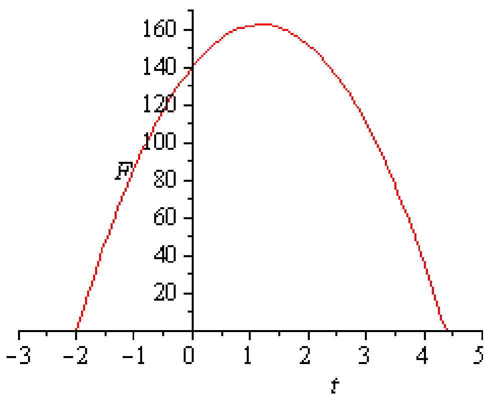
The leading coefficient **a** in the quadratic function indicates the orientation of the parabola. When **a** is negative, to be concave down, while when **a** is positive the parabola concave up.

With this problem and its graph, properties of quadratic function which was mentioned earlier can be explained, for that I use Maple to illustrate important points on the graph:

$$F := t \to -16t^2 + 38t + 140$$

$$t \to -16\ t^2 + 38t + 140$$

plot(F(t),t = -3..5, F = 0..170)



Looking at the graph would gives us a chance to explore the idea of y-intercept, x-intercept (zeros), axis of symmetry, and max or min approximately. Finding exact values can be done via answering few questions related to the problem.
How high is the boy from the ground when he throw the stone from the top of the building. Here, I explore the idea of independent and dependent variables, the time it takes the stone to reach the ground is independent, and the height of the stone dependent, as it depends on the time.
The height of the boy from the ground is as the height of the stone when it was in his hand before he throws it away, or at the time of zero second. In quadratic function its the Y-Intercept, on the graph it is the point where the curve cuts the y-axis. To find y-Interceot in Maple, use f(0):
F(0) 140
The height of the stone from the ground is 140 feet. To find how tall is the boy?
The boy is 140 – 135 = 5 feet tall.
Another question could be asked, when the stone hits the ground?
When the stone hits the ground, means the high of the stone is zero, or value of F(t) is 0, what will be the value of t which gives y zero?
X-Intercept or zeros of the same function in the problem is representing the point at which the stone hits the ground. On the graph zeros are points where the curve cuts the x-axis. `
Solve the equation for the value of t:
fsolve (F(t),{t})

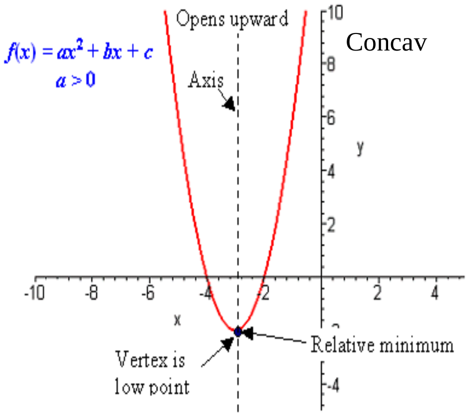
{t = -2.0000000000} , {t = 4.375000000}

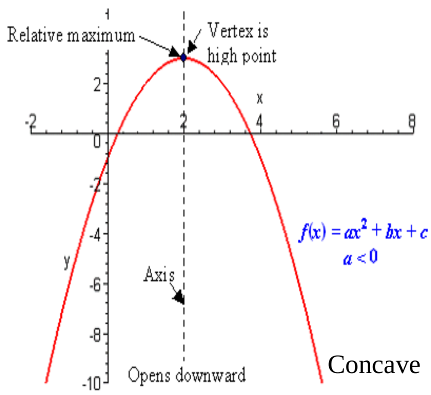
The two values are the roots of the quadratic function, in this problem we consider the positive value and ignore the negative value. So after 4.4 seconds the stone hit the ground.
Using the same command to find after how many seconds does the stone reach different heights. Example, when the stone will be at the height of 145 feet?

fsolve (F(t)= 145, {t})

{t = 0.1398090866}, {t= 2.235190913}

After 0.14 second on the way up, then on the way down at 2.23 seconds the stone will be at the height of 145 feet.
One of the important points on the graph of parabola is the extreme points(Maximum or Minimum). When the stone will be at the highest point and how high?
This gives a good opportunity to explain different forms of quadratic function, in this problem the function is given in general form $f(x) = ax^2+bx+c$, change to standard form (vertex form) to find the max or min point, also a chance to illustrate how do we know if the graph of the function has a maximum point or a minimum point; again the value of **a** indicate (when **a** +ve) the parabola has a minimum point and (when the value of **a** -ve) the parabola has a maximum point.





Changing from general form to standard form, can be done algebraically, in this course I use Maple, as my aim is to have better chance for interpretation of those important points and their relation to our life.
with(student):
completesquare(F(t))

$$-16(t-\frac{19}{16})^2+\frac{2601}{16}$$

Compare between the approximate coordinate of the maximum point on the graph with the exact coordinate which you find from the standard form $(\frac{19}{16},\frac{2601}{16})$. Also what does the values in the order pair of maximum point means in respect to the problem we have, or in any other quadratic function.
When the quadratic function is expressed in the form y = f(x – h) + k we see that the graph is in the standard form ; has been shifted from the origin with the coordinates of the vertex being (0, 0) to a position with the vertex at (h, k).
In this form we can state the vertex immediately; it has coordinates (h, k).
The standard function is shifted h to the right and k upwards.

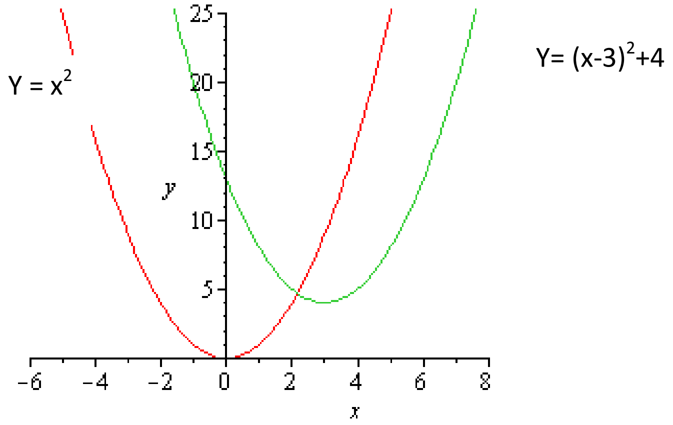
f:= $x \to x^2$

$$\boldsymbol{x \to x^2}$$

f1:= $x \to (x-3)^2 + 4$

$$\boldsymbol{x \to (x-3)^2 + 4}$$

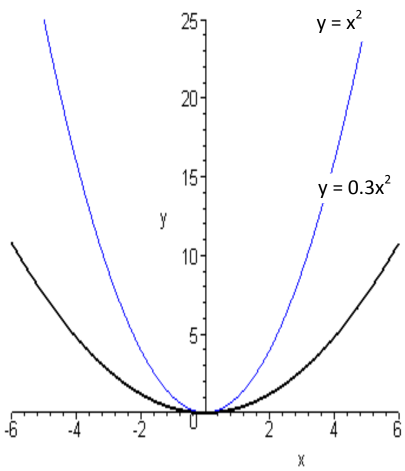
plot ([f(x), f1(x)] , x = - 6..8, y = 0..25)

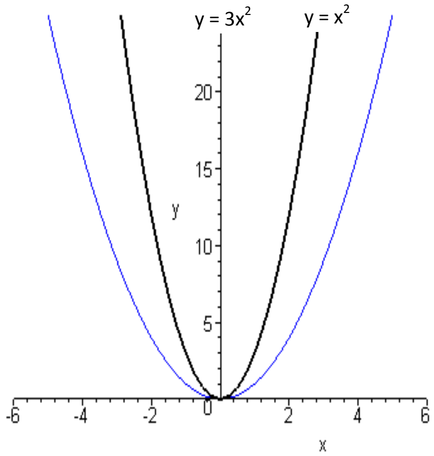


The stretch factor, *a*, affects the flatness of the graph of a function.

For example:

If $0 \leq a \leq 1$, the graph of a function is flattened
If $a \geq 1$ the graph of a function is squeezed upwards.





Effects/Conclusion

No doubt real life problem solving helps in making mathematical concepts more understandable; in order to successfully achieve this, through the above simple applications I was trying to show students how the quadratic functions can be used to find answers to questions that usually come across their minds and form barrier to their understanding, give them answers that are tangible, can be felt, and sense the real benefit of using such mathematical tool.

Thus, with the above example, along with the use of Maple as a tool to answer many questions in the application and others related to inverse of the same function , the approach to develop students critical thinking, quantitative reasoning, and knowledge has effect and cause the objectives to be achieved.

References

1. Maple 12, Learning Guide.

2. Course notes, Department of Mathematics and Statistics/Zayed university/UAE