Project work *Is the Legacy of Ancient Greece and Rome really the Cradle of European Civilization?*

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Abstract
In this paper the project for 15-year-old students with the title *Ancient Greece and Rome* and the sub-title *Is the Legacy of Ancient Greece and Rome really the Cradle of European Civilization?* is introduced. It shows how to connect mathematics with art, history, physics, geography and philosophy by studying ancient Greek scientists and their achievements. Collaborative teaching is introduced. The major aim of the project was to show mathematics as a part of human civilization and to follow its development through history. Some topics from theory of numbers and geometry were studied. One part of the project was also a theatre performance, which should make the students aware of the difficulties of many dedicated mathematicians to find the answers to some problems from the ancient times.

Introduction

In this presentation we will describe a cross-curriculum project work, which was done with the firs-year students from the secondary school called "gimnazija". In Slovenia the secondary school lasts for four years and ends with an external examination. Afterwards most students continue their education at university. According to the syllabus students have four lessons of mathematics per week for four years. Last year we started with a new syllabus, which includes project work and team teaching as a new teaching method. For this project the topic was chosen which was interesting for different subjects, with mathematics as the central subject. The title was *Ancient Greece and Rome* and the sub-title *Is the Legacy of Ancient Greece and Rome really the Cradle of European Civilization?*

The project lasted for three days. During the first day the students were introduced into the science, especially mathematics, of the ancient times. The mathematics teacher taught in a team together with teachers of art, history, physics, geography and philosophy. During the second day students worked in different workshops. In one group they learned about the cuisine in the Roman time and they made "Roman" bread. Some groups studied and represented sport disciplines in the Olympic games from the ancient time and from the modern world. One group visited Greek and Roman remains in our town and the other compared the speech of Pericles with the speeches of different modern politicians. We offered three different mathematical workshops, but the most attractive one was a theatre performance which made the students aware of the difficulties of many dedicated mathematicians to find the answers to some problems from ancient times. On the last day the students introduced the project to their parents.

Mathematics as the central subject

"Arhimedes will be remembered when Aishilus is forgotten, because languages die and mathematical ideas do not. "Immortality" may be a silly word, but probably a mathematician has the best chance of whatever it may mean." (G. H. Hardy, A mathematician Apology)

Maybe that's why we chose mathematics as the central subject. Naturally, all the teachers of mathematics liked it, because we do not have enough time for teaching history of mathematics during our regular lessons. The major aim of the project was to show mathematics as a part of human civilization and to follow its development through the history. 15-year-old students have enough knowledge to understand the development of theory of numbers and geometry, at the appropriate level, of course. But in the project we also found time for additional topics, especially in the theory of numbers.

The project gave us the opportunity for collaborative teaching. This was very useful when learning about ancient scientists, because they were not only mathematicians but also philosophers, astronomers, etc. In the previous years our students heard about Pythagoras and his theorem, about Thales, when they discussed similarity and about Euclid, when Euclidean algorithm was treated. But nothing more. Consequently, one of our aims was also to present some other mathematicians, which the students had to arrange in the chronological order.

Mathematics and art

The art teacher opened our session. She compared the ancient painting and architecture with the modern art. The connection between mathematics and art was the golden ratio. The first clear definition of it was given 300 B.C. by Euclid, but it must have been known at least 200 years before in the Pythagorean time. The golden ratio is the best answer to our heading question *Is the Legacy of Ancient Greece and Rome really the Cradle of European Civilization.* The golden ratio presents

aesthetically pleasing qualities in art from the ancient to the modern times. The mathematics teacher then explained the golden ratio and the art teacher gave some examples like Parthenon (432 B.C.), of course, and the painting Modulator (1948) by Le Corbusier.
The art teacher finished the lesson with a very famous painting The school of Athens (1511) by the renaissance painter Raphael. This painting and the figures in it was a starting-point for our further work.

Mathematics and history

The history teacher was indispensable in studying history of mathematics. She described the government of Polycrates, which was so tyrannical that Pythagoras escaped from Samos. She presented the position of woman in the ancient time. By knowing their role in society we can better understand the importance of Pythagoreans, who allowed women to function on equal terms in their organization.
The history teacher reminded our students of the Punic war, because it was fatal for Archimedes, whose last words were "Do not disturb my circles" before he was killed by Romans. There was another murder of a mathematician some centuries later, which was described by the history teacher. Hypathia, the first notable woman in mathematics, was killed by Christian fanatics.
In the painting The school of Athens, we can also find Alexander the Great. He was known to the students from history lessons, but they did not know that Aristotle was his teacher. This is the example of team teaching. Teachers, who are together in the classroom, can supplement each other.

Mathematics and philosophy

Mathematics was closely connected with philosophy in the ancient time, so the philosophy teacher was very helpful in our project. However, profound philosophical questions can be very difficult for 15-year-old students. In this case the teacher had to substitute the theory with the anecdotes and other stories. But the teachers warned the students that these stories are based on poorly documented historical records. The mathematics and philosophy teachers explained together Plato's philosophy and Platonic solids, because both topics are connected. It is the same with Zeno of Elea and his paradox "Achilles and the tortoise". We can connect this paradox with modern mathematics because of the infinite process. In the 17th century it was shown that the infinite convergent series can have finite limit, which is the sum of the series.
Platonic solids also had influence on later mathematicians and physicists. In the 16th century Kepler used them in his solid model of the solar system. In the 18th century Leonhard Euler proved that the numbers of vertices minus numbers of edges plus number of faces of polyhedra (Platonic solids are included) is 2. In the 19th century the symmetry groups of Platonic solids were studied. The statement from the 20th century, that minerals and viruses have the shape of regular polygons, helps scientists to study their nature.

Mathematics and geography

During the project the geography teacher helped students to mark the birth places and other important places of the ancient scientists on the map. Then she gave a short illustration of ancient geography with stress on Ptolemy. Ptolemy for geographers is like Euclid for mathematicians. Together with the physics teacher they explained Eratosthenes' measurement of the Earth's circumference.

Mathematics and physics

The physics teacher added explanations of physics achievements. When talking about Archimedes, for example, we mentioned his excellent approximations for π and $\sqrt{3}$, his proof that the volume and surface area of the sphere is two thirds of the cylinder of the same height and diameter. The physics teacher added the information on many Archimedes' inventions. We explained Heron's Formula for the area of a triangle and the physics teacher explained his inventions and achievements.

Students' activities

We have mentioned some students' activities already. They formed the timeline and marked the important places on the map. They got handouts with the presentation and with the tasks that they did on their own:

- Archimedes: To estimate his approximations for π and $\sqrt{3}$
- Diophantus: To find one integer solution of the equation $12x + 42y = 6$
- Eratosthenes: To write the Sieve of Eratosthenes for numbers up to hundred
- Euclid: To find the first seven numbers of the form $N_n = 2^{n-1} \cdot \left(2^n - 1\right)$. Which of them are perfect numbers (the number which is equal to the sum of its proper divisors)?

- Hero of Alexandria: Water flows to the reservoir from four different pipes. The first pipe should fill up the reservoir in one day, the second in two days, the third in three and the fourth in four days. How long would it take to fill up the reservoir if all pipes are switched on?
- Plato: To check the Euler's formula on the example of Platonic solids
- Pythagoras: To prove, that 1184 and 1210 are amicable numbers (the sum of the proper divisors of one number is equal to the other)
- Thales: To prove the Thales' theorem (the diameter of a circle always subtends a right angle to any point on the circle).
- To arrange all mathematicians and other mentioned scientists in to the chronological order

More difficult tasks, like different proofs of Pythagoras' theorem, were solved with the help of maths teachers.

Influence of ancient Greek mathematics on modern mathematics

Not only Greek letters, in modern mathematics we also use many Greek words, like hypotenuse, ellipse, hyperbola and parabola. We know that Pythagoras found the name for the longest side of a right triangle. The conics got the name by Apollonius of Perga. We can find many mathematical terms, which have their origin in Greek language: axiom, arithmetic, asymptote, diameter, dodecahedron, graph, orthogonality, polynomial, sphere, trapezium, etc.

For every topic discovered in time of ancient mathematics we can find its impact in modern mathematics. For example in theory of numbers. It seems that perfect numbers and amicable numbers are unuseful, but for Pythagoras they were very important. In the 20^{th} century it was discovered that these numbers are important for the assessment of capacity of computers. An other example can be found in geometry. Euclid gave five postulates. Mathematicians until 19^{th} century tried to modify the fifth one, because it seemed less obvious than the others. So the Non-Euclidian geometries were discovered. The influence of ancient mathematics can also be found in music. In ancient Greek times it was recognized that consonant musical sounds relate to simple number ratios and the oldest system of the scale construction is the Pythagorean scale. It is a base for the equally-tempered scale we use in European music today.

"There is no doubt that anybody who grew up in a western or mideastern civilization is a pupil of the ancient Greeks, when it comes to mathematics, science, philosophy, art and literature. The phrase of the German poet Goethe-"of all peoples the Greeks have dreamt the dream the best"-is only a small tribute to the pioneering efforts of the Greeks in branches of knowledge that they invented and denominated."[1]

Workshop

One of the workshops was maths theater. Students performed a play with the title THE LAST FERMAT THEOREM. Of course, we did not prove it, but only showed its history and some other problems from Theory of numbers that interested the ancient Greeks and are still open now. Three groups of students were simultaneously on the stage discussing mathematics. The students from the first group presented Pythagoras and other mathematicians from 300 B.C. (dressed in white sheets), in the second group there were Fermat, Descartes and Pascal with white collars and wigs and in the third group there were mathematicians from Princeton University at the end of 20^{th} century, admiring Andrew Wiles. The ancient mathematicians introduced some problems like Pythagorean triples, perfect and amicable numbers, twin primes. The "mathematicians" in other groups added the knowledge achieved during their time, Fermat was looking for the solutions of the equation $x^n + y^n = z^n$, Wiles was proud of his proof, the others complained that certain problems have not been solved yet. The students wrote the scenario by themselves, they also included some non-mathematical content. The aim of this performance was to develop the awareness of mathematics as a science, which is constantly developing, however some things that were once proved, will always be true. We think that some other topics like geometry, calculating the values of π or trigonometric functions, could also be presented in such a way.

Students did not learn a lot of mathematics but the aim was certainly achieved.

REFERENCES

[1] Livio, M., 2002, *The Golden ratio,* London: Review.
[2] *Greek mathematics.* (Online, Januar 2009). http://en.wikipedia.org/wiki/Greek_mathematics, (Cited: 12 April 2009)
[3] Berlinghoff. W. P., 2008, *Matematika skozi stoletja.* Ljubljana: Modrijan.
[4] Wollenberg, S., 2006, *Music and mathematics: an overview.* From the book *Music and Mathematics. From Pythagoras to Fractals.* Oxford: Oxford University Press.