

The use of notebooks in mathematics instruction. What is manageable? What should be avoided? A field report after 10 years of CAS-application.

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Abstract

Computer Algebra Systems (CAS) have been changing the mathematics instruction requirements for many years. Since the tendency of using CAS in mathematics instruction has been rising for decades and reports have often been positive, the implementation of notebook classes seems to be the consequent next step of mathematics instruction supported by computers. Experiences that have been made with the use of CAS in PC-rooms can be transformed directly into the classroom. Hence the use of CAS is no longer limited to certain rooms. The permanent availability of the notebook with installed CAS offers the chance to realize these concepts that have already been approved with the use of CAS so far. The following speech shall show what these concepts could look like and that the use of notebooks is not only the further development of teaching in PC-classes. Examples from personal experience in teaching will especially show meanders and thought-provoking impulses in order to support teachers finding their way into teaching mathematics instruction in notebook classes successfully. Please allow me to point out two things in the beginning: (1) Yes, I am a vehement supporter of the use of notebooks (and the use of CAS in particular) in mathematics instruction. (2) No, I do not believe that teachers who have chosen another path (or at least partly) are teaching badly.

0. Explanation

This English text is a short summary of a German paper that was published at the 100th MNU-Congress in Regensburg in April 2009. Main parts concerning the practical experience had to be abbreviated. The whole English version is available at <http://www.acdca.ac.at> (Website of the Austrian Center for Didactics of Computer Algebra)

1. Basic conditions

At my school, a Bundeshandelsakademie (BHAK in short, a commercial highschool) CAS have been inserted into mathematics instruction for more than two decades. In the beginning there were only a few classes held with the support of CAS due to the permanent overcrowding of PC-rooms.

After an adaption of the school now there are four PC-rooms available for approximately 350 students. Each of the rooms offer about 25 PCs and in addition to that there are also one media-and-network-room as well as one economical training enterprise where the real activity of a company is being re-enacted. Furthermore the school is wired with almost 1000 network connections which means that access to the school's intranet as well to the internet is possible from more or less every room. In particular every class is equipped with a teacher's PCc, a beamer and a printer. The decision to introduce notebook classes at my school has been made about ten years ago. Today every student of the BHAK possesses their own notebook that represents an indispensable part of the tuition. The notebooks are used in nearly all the subjects with different focuses, of course.

In this context it is important for me to emphasize that the notebooks are not the centre of tuition but only an additional device. Notebook-arrangements regulate the students' behaviour within the school's intranet (as well as the access to the internet). These arrangements are visible at <http://www.hakhorn.ac.at> within the part of "Verhaltensvereinbarungen".

2. Our expectations concerning the use of notebooks

The positive experiences we (that is the teachers of mathematics instruction at the BHAK Horn) have made with the use of Derive in mathematics instruction so far, gave reason to the hope that we would now get the opportunity to pursue the tuition that have been done in the PC-rooms continuously with computers and CAS. Up to that point we found the fact that tuition should be done without computers except most of the times for one or two hours per week very limiting. During the computer lessons we could observe a higher level of autonomy, willingness to do teamwork as well as more opportunities for discovering studying (not least via the internet) and finally a higher awareness of problems.

3. The first trials

A clear detachment between manual skills and the use of CAS had been approved in tuition so far. Especially the formulation of the basics of mathematics was done without the use of computers. Computer support was only accepted in succession when it came to application-oriented tasks. We wanted to stick to that procedure.

In tuition we set value on a good deal of application tasks that were answered mainly by using CAS.

4. The result – we got what we had deserved

Whereas the lead time for teachers has been rising considerably compared to the time before the notebook classes had been introduced (and there was no tendency of change recognizable), the expected changes within the tuition itself could not be determined.

Instead of a more open and student-oriented tuition the forms of tuition have almost not been changed at all.

Due to the additional time for the acquisition of the CAS handling we had to reduce time for the training of manual skills.

A partly distinctive decrease in the students' performance together with a rising discontent about the tuition was finally leading to a reorientation of mathematics instruction at our school.

In the following I want to present the theses on which our ideas were based as well as the deviated results that have been defining mathematics instruction at our school for many years (as we think: successfully).

5. What we changed – Theses for successful mathematics instruction with notebooks

Do you take all demands for mathematics instruction seriously likewise, the excessive demand of both students and teachers seems to be preprogrammed.

- What should the practical component aim at? (in a commercial school probably most likely at business and economic topics).
- What should the ability of studying aim at? (economic studies makes different demands on mathematics instruction than studies of mathematics or engineering).
- How far should application tasks for already acquired mathematical methods be searched for?
- How strongly should traditional topics of general education (catchword: definition – sentence – proof) shape the subject “applied mathematics”?

Thesis 1: You can not have everything!

One of the first questions you should ask in time before the actual beginning of tuition planning is surely the question for the direction of mathematics instruction. It seems indispensable to carry out a clear assessment of the above mentioned aspects. The arguments for resp. against particular focuses in tuition are manifold, therefore it is not possible to carry out a general directive.

Thesis 2: stay simple!

An intensive occupation with the question of which manual skills and abilities graduates have to improve in any case is indispensable before the use of notebooks.

The notebook changes the contents of mathematics instruction for sure, as well as it might have been the case with the introduction of the pocket calculator, the GTR (graphic pocket calculator) or the CAS. I can remember with amazement that when I was a student manual calculating of radicals was taught despite pocket calculators. The necessity of such contents can be discussed.

In this context I want to refer to a citation by Helmut Heugl, a pioneer in computer-assisted mathematics instruction, which is relevant not only for the common use of CAS (no matter if at the PC or with a handheld) but gains much more importance when thinking about notebook classes:

If It is not necessary to use computers, it is necessary not to use computers.

In order to realise this request especially when using the notebooks permanently one should add the following:

.... as far as one knows what they want.

The direction of mathematics instruction has a big influence of this point of using computers, since the matters of tuition defines when the computer should not be “necessary”.

Even if the mathematical matter shows no need for the use of computers it can nevertheless be meaningful from a didactic point of view. The pro and contra of the computer-assistance should be considered with responsibility, whenever in doubt it should be tried if one could gain additional value with the computer (you can always go back).

Thesis 3: The fact that graduates have less manual skills due to the use of computers is irrelevant for their ability of studying.

One of the loudest arguments against the use of notebooks in mathematics instruction (resp. against the CAS-application in general) concerns the fact that due to the rising implementation of technology arithmetic skills are pushed back. This argument is not new, but it does not become more correct only by repeating it again and again. It is rather true that the moving of teaching matters away from the mere calculating is leading to the fact that certain manual skills are not being trained and hence are not being mastered in the usual way. Students from computer-assisted mathematics classes could make up for manual skills in a short time (as far as this is even necessary) they seem to have advantages in other areas (model-construction, interpretation of results,...)

Thesis 4: The subject mathematics must sell better!

The meaning of natural sciences and mathematics has been pointed out in regular periods nevertheless mathematics importance in (the Austrian) public but also among the non-natural science subjects seems rather little.

“I was always pretty bad at maths!” is still considered as socially accepted statement, lacking knowledge about Goethe, Schiller or the ignorance of the updated orthography are admitted by contrast rather seldom.

The importance of mathematics for other subjects (apart from physics and chemistry) is also challenged at this point.

The right way – and here the cooperation with the colleagues teaching economic subjects is necessary – would have to be the treatment of tasks based (!) on economic consideration. Not the other way round! This would need the understanding of colleagues from other subjects that mathematics plays a certain part in their own subjects. To gain this insight can among other things be an aim of mathematics instruction with practical relevance.

In this context one should naturally not forget another argument against the practical relevance of mathematics instruction:

“Mathematics is more than just application!”

From the point of view of a vocational school the argument can be seen rather coolly since there are only 10 lessons a week (divided into four years of teaching) and focal points have to be set. Apart from this main mathematical concepts can be trained when dealing with the basics for these tasks anyway.

Thesis 5: Mathematics instruction currently still strongly follows the mathematic matters of the curriculum rather than the tasks of education and teaching!

(The general and didactic tasks of education as well as the tasks of education and teaching for the subject “mathematics instruction and applied mathematics” can be seen in the document “Handelsakademie (Anlage 1) / Lehrplan 2004” (S. 3ff, 30ff) in the category Lehrpläne at <http://www.hac.cc>)

This statement applies with regards to content perfectly to

Thesis 6: The use of notebooks can support the movement to the direction of general (didactic) educational goals!

We have already mentioned some possible reasons for the orientation at the matters of the curriculum. They may be based in traditional views of mathematics instruction (catchword: “Acquisition of the craft!”)

If you are of the opinion that the matters of mathematics instruction should move into the direction of general educational goals, as they can be found in the curriculum (modelling, arguing, interpretation, critical challenge,...) you will surely find the use of notebooks an important tool. Once the transition has been made, mathematics instruction has clearly gained surplus as we think.

Thesis 7: Even if carried out skilfully the use of notebooks does not lead to additional knowledge but to a different kind of knowledge!

As pointed out above the notebook can play a major role in moving the matters of mathematics instruction. It has to be mentioned that this movement can in big parts also be reached without the use of notebooks.

We want to warn you at this point from the notion that the introduction of notebook classes would automatically lead to an advancement of marks in the subject mathematics instruction. This hope has not come true – at least in our case. It is true that our students’ approach to mathematics seems now more open and fearless and also other desirable characteristics have improved.

As comfort the fact remains that marks have not become worse either. Mathematics instruction has just become “different”- maybe not better but as we think in no case worse either!

6. If we could start anew – our advice for new comers resp. teachers switching

Contemplated retrospectively our ideas of mathematics instruction with notebooks were in the beginning misleadingly driven from the notion to have quasi the all-in-one device suitable for every purpose in front of us now. The thought of being able to make everything better that was so far and additionally integrate new ideas into mathematics instruction necessarily had to result in the above mentioned failure. This is a basic realisation from our beginnings:

Failure has to be allowed!

In the same way as students must be given the right to take wrong paths on their search for solutions and then change their minds teachers must be given the opportunity to learn from wrong concepts and subsequently make it better. Of course this needs a lot of understanding on the part of the school supervisory board, especially headmasters are in demand here when wrong concepts are to be accepted without detrimental consequences for teachers and tuition and if necessary stand behind the idea that new studying environments need new ways of studying and sustainable concepts have to be worked out. In some cases this is unfortunately only possible via trial and error.

The expenditure of time for preparation and revision in the introduction of notebooks should in no way be underestimated. It was true for the mere CAS-application that common forms and concepts of tuition could often not be kept the way they were, but this is true on a higher level for the tuition using notebooks. Thus the advice must be (not only for the beginning but in general):

Don’t rock the boat!

Trying to adjust the whole mathematics instruction to the new tuition environment does not seem meaningful. This trial quickly leads to a massive congestion of the teachers as well as the students and does not lead to any positive effect.

It seems rather advantageous to slowly change from traditional tuition to the desired aims. In any case is it sufficient to take just a few new tasks and it is not necessary at all to create new tasks which are compatible to the new demands by oneself permanently. Mainly in the beginning it is more than enough to modify well known tasks in a way that they fit into the new concept. Students will very well recognise the changes but the tasks will somehow be familiar to them which is a quite comforting situation. A quick transformation swamps teachers as well as students and is only good for creating negative motivation from the very beginning.

For later on it is also a good advice to look for compatible tasks first before creating tasks by oneself. Of course it is satisfying and enjoyable for a teacher when a new task seems successful and makes its way also in tuition but one invests a lot of time into such work. Selective research offers enough material compatible with mathematics instruction due to the presented ideas, apart from the internet there are also certain (school)books that concern themselves with this form of tuition in particular (see bibliography).

Discard something old for every new matter

The use of notebooks does not increase the amount of time available for mathematics instruction. The decision for new focal points (e.g. arguing, modelling, ...) is hence indispensably connected with the decision against other matters that have been taught so far. Considering which matters should be shortened or even discarded in order to give new ideas sufficient (temporal) room is probably one of the most important points for teachers preparing for notebook tuition. If this reorganisation fails you are on the best way to frustrating and discouraging tuition which may very well lead quite quickly to re-establishing well-tried traditional concepts and renounce the use of notebooks.

Concerning the organisation of your tuition it seems advisable to focus on some indications again and again:

No experiments in tuition if you are insecure (there are enough unexpected things happening anyway)

Be sure of the contents you want to convey and make sure to be familiar with the technology you use if you try out something new. Whenever teachers are insecure they are engaged with themselves and the introduction of their media so much that they are unable to commit themselves to the students and support them.

Never allow the notebook (or the CAS) to become the centre of tuition

The notebook itself will not enhance or revolutionise mathematics instruction neither has the pocket calculator. It just offers new opportunities that one should use extensively. Nevertheless the notebook is just an additional tool in order to present circumstances in a better way of to shorten steps in the calculation. In no case mathematics instruction is allowed to become a user training course for particular software products. It has been approved to use software just from the very beginning next to common tuition. Students learn how to handle it quite quickly in general. Most of the times they acquire the necessary skills themselves. The regular tuition should only pay attention to the handling problems if a great number of students is concerned. (note: this is not true for exams; here everything should be done to reduce handling problems – see the article “Tasks for tests and A-levels using CAS” by Heidi Metzger-Schuhäker in this conference transcript.)

Notebook excuses are forbidden!

There will always be students who have no working notebook at hand. Experience has shown us that you should not consider this circumstance, in the worst case students without notebooks should work together with other students for the time of the mending (or maybe there are arrangements with the dealers who offer replacements for that period of time resp. the school owns some replacements). Homework can be done on other computers during that time it is rarely the case that no computer is available at all for students whose notebook has to be mended. Tuition should not pay too much attention to such situations.

Finally one major point for the introduction of notebook classes should be mentioned which may be decisive for the success of the tuition:

Above all, try to prevent frustration concerning marks in the first grades!

Certainly the information contained in this speech does not represent the silver bullet to the use of notebooks in mathematics instruction. But maybe one or the other thought-provoking impulse or one or the other information can be helpful for teachers to support their own way planning their mathematics instruction tuition in classes with notebooks or at least avoid some difficulties.

Altogether I want to point out in the end that we at the BHAK Horn meanwhile see ourselves encouraged to adapt and transform our tuition in the above mentioned form after difficulties in the beginning. (Even though I have to admit that the rapid technical development presents new changes for tuition again and again).

We are with Apostolos Doxiadis (Greek mathematician and author) of the opinion to have shown our students that actual mathematics is infinitely more interesting than solving quadratic equations or calculating the volume of bodies.

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