CAS and calculation competence of students

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Abstract

The use of new tools for mathematics at school wins increasingly Importance. It follows from this that they are consquences as well as on aims and contents of mathematics at school us like on methods in the lessons.

It is not unusual, that students and parents and also university professors are to be feared, that the calculation competence is decreasing with the use of CAS. In the lesson should be showed a possible way to developing such competences in the beginning phase of the learning process in Algebra. The examples refer to a level of school mathematics for students in the middle school age. The methods tell apart phases with and without CAS and shows a didactic principles of mathematics lessons in the case of use of CAS handheld technology.

Workshop-summery

The question of the use of technology isn't, as the history points, a question the "whether" but a question the "as". In the history this always called scepticism of some teachers and mathematicians.

In Saxony, one state of the Federal Republic of Germany, the use of graphic calculators is obligatory from the 8^{th} form on in the gymnasium (high school). It is also necessary in the central school leaving examinations.

Introduce technology in math education has different methodical reasons.

- explorative learning – experimentation
- visualisation
- motivation
- calculator help
- change of assignment culture
- cross-curricular teaching and learning.

In the lesson I will look at the role of technology in the context of development of elementary calculation rules exclusively. I would like to ignore all the other aspects of use of technology.

Technology support the reform of teaching of mathematics.

But a lot of teachers and also some parents have doubts about the meaning of new technology in math education.

"Why do we need technology?" "Shall this what?" "The students forget the mental arithmetic. It is a big danger."

This isn't always simple.

Even if one has not aversion against the technology use in the mathematics lesson, the appreciation of sensible use of the technology is difficult from time to time as the following example shows.

The teacher provided the task:

The sum of the squares of three natural numbers, succeeding one another, is 590. Find the three numbers.

She expected the following solution way:

$$n^2 + (n+1)^2 + (n+2)^2 = 590$$

$$n^2 + n^2 + 2n + 1 + n^2 + 4n + 4 = 590$$

$$3n^2 + 6n + 5 = 590 \quad \text{I} - 590$$

$$3n^2 + 6n - 585 = 0 \quad \text{I} \div 3$$

$$n^2 + 2n - 195 = 0$$

$$x_{1;2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$$

$$= -1 \pm \sqrt{1 - (-195)}$$

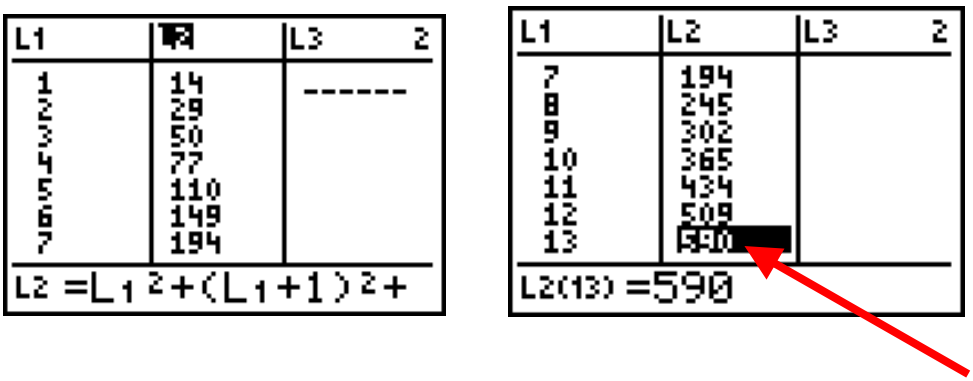
$$= -1 \pm \sqrt{196}$$

$$x_1 = 13 \,(\text{trifft zu})$$

$$x_2 = -15 \,(\text{entfällt})$$

Result: 13, 14 and 15. are the three numbers

A student worked on the task with the list menu of his graphic calculator and got the result very fast:



The reaction of the teacher was interesting now: **"Yes, this is correct. But now You hat to do this correctly again!"**

A basic concept of the mathematicians was also in the history, mathematics exempt of annoying algorithmic calculation.

Now it is a problem that every teacher, every school book author, every school administrator and also parents up to the acceptance must run through a cognition way. These phases of the cognition way can approximately be described as follows.

- developing fundamental interest
- discovering CAS as a hand tool for oneself
- use as a demonstration equipment in the hand of the teacher
- use as a calculation aid (most at well known problems)
- use as experimenting tool

→ at recognizing each the necessity of change of the mathematics lesson

In the school praxis often we can observe the classical way to introduce a new mathematical method.
In the first phase the teacher explain the new method with one or more examples.
In next step is the students practise it without technology on many examples.
If the students have solve enough examples, the teacher demonstrate the way with technology.
The problem is that students don´t experience the technology as a instrument in the realization process. They only know the power of technology as a calculator machine.
But in our point of view it is more effective to use the technology in the first phase of the realization process. In the next phase students should practise the mathematical methods without technology till a reasonable size. In the 3rd phase the mathematical methods should be applicable to practical problems or an other context. Technology is a useful tool in this phase.

Example (8th form, age of 14)
Aim: Set and resolve of brackets
Discovery phase with technology:
There is given the following number puzzle with the term: $(a \cdot 2 + 5) \cdot 50 + m - 365$
The variable a shall stand for your age, the variable m for your weight.
Multiple a by 2 and add up 5. Multiple the whole by 50 and adds up m. Subtract 365.
If the students do so and call the result, the teacher is able to calculate at once the age and the weight of the students. How is it possible?
If You give the term into a CAS, the result is 100a+m-115. If You add up now 115, you find the age in the first both numbers and the weight in the last two numbers of the result.
Such number puzzles are for students more motivating as calculating terms stubbornly. It is the Aim for the students to develop even such puzzles. But the students can not resolve any brackets at the beginning of the 8th class.
At availability of CAS the following task is provided:

Before a bracket can stand a plus, a minus or a multiply dot. You should find rules to resolve brackets by experimenting with the CAS orders "expand" or "factor". After the working phase in grozps You should give a presentation for the class.
An introduction to the CAS orders usually isn't necessary; Teenagers disclose themselves also the operation of a mobile telephone in shortest time.
The results which discover the students themselves finally have to be remembered more easily than predefined knowledge (activity level of the individual in the process of appropriation).
Depending on the situation the students will develop the need for grounds or confirmations of her assumptions. The teacher decides how far such grounds, plausibility considerations or proofs in the situation are necessary. Of course a a proof would always be necessary in subject science. In the mathematics lesson from time to time one can leave it with the confirmation of the teacher, too.
Exercise phase without technology:
Tasks are now practiced to obtaining the standard demanded in the teaching curriculum or of the teacher. Use of technology is not convenient in this phase of work.
It is possible to use CAS for result control, though. E.g. escapes the usual "announcements" of the homework results with that. The teacher only says: "Please ask your questions". The students could already check with the help of the CAS whether the calculations are right.
Application phase with technology:
It goes therefore now to set and resolve brackets in other mathematical and factural connections.
The students develop corresponding terms and work on themselves. The students are appointing the technology as a calculation tool (Black box).
It is undisputed that classic calculation competence loses her place value here.
On the other side there are increasing "new" competences which are possible only in the CAS age.
These are such competences

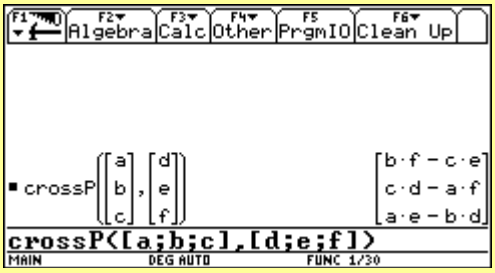
- Competence to find terms,
- Structure reconnaissance competence,
- Test competence,
- Visualization competence,
- Competence to use technology suitably,
- Competence to use computer work suitably for the formulation.

(Heugl, Herget, Kutzler, Lehmann)
The way of the change of phases with and without technology is used e.g. in the following part from a german school book at the introduction of the vector product (only the discovery phase with technology is shown here):

The multiplication of two elements of a number range is one element of this number range too in all number ranges known to us. If one multiplies two natural numbers, the product is a natural number. If one multiplies two real numbers, the product is a real number too. But the dot product of two vectors is a real number and not, like perhaps expect, a vector.

There is:
Sum: Vector +Vector = vector
Difference: Vector - Vector = vector
Multiplication by a real number: a * vector = vector
Dot product: Vector * vector = real number.

For mathematics it was necessary also to define a product which one a multiplication "vector*vector = vector " describes. You find it with the CAS order "crossP".



Working tasks:
1. Find a possible rule for a multiplication of vectors so that is: vector * vector = vector.

Examine your multiplication for qualities like commutativity and associativity. Interpret your result geometrically.
2. Find one vector which vertically stands on a given vector.
3. Find one vector which vertically stands on the two given vectors.
4. The picture in the introduction example shows the application of the order "crossP" on two vectors. The result is a vector again.
Examine what result the order "crossP" at application to two vectors would produce.
Check your assumption with the help of technology. Try to find a rule according to which one can calculate the coordinates of the product vector.
A negative example is the following table of contents contain in an other textbook.

1. Repeat
2. Limes of a function
3. Steadiness of a function
4. Difference quotient
5. Derivative of a function
6. Derivative of the power function
7. Derivative of a logarithm and exponential function
8. Derivative with CAS

Only the last passage contains parts with technology.

Summary:
The use of technology in math education don´t avert developing of calculation competences and literacy. On the contrary the use of technology is a basis for profounder comprehension of the mathematical methods so as calculation algorithm. In Saxony and Thuringia occurs in the last years surveys to compare calculation competences of students learning with or without technology. The results of the population learning with the use of technology were provable better.
But the kind of use had to follow an pedagogical concept. This kind of use can´t be a chance one. Technology should be integrated in the discovering phase of the learning process of calculation algorithm and should not be only a addition on the end of the learning process.

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