Modelling tasks for learning, teaching, testing and researching

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Abstract

The article deals with a special kind of modelling tass. These problems are used for learning and researching as well. So the results of an empirical study on mathematical modelling of pupils in secondary schools are presented. Pupils of forms 8-10 were observed working on open, realistic problems. These observations were recorded and evaluated. The goal of the presented part of the study is a detailed look at the control processes of modelling problems. In this context changes between real life control and mathematical control during the control phases are studied and evaluated. We describe in detail the sub phases of controlling and explain their connection with modelling process. The problems used in this project can also be used in math lessons, so this kind of research can put teachers and researchers together. These tasks are suitable to support ongoing in-service development and teacher education.

Problems

Open reality-related problems can be used to analyse model-building and problem-solving processes. The following problems are examples of open and fuzzy problems with reality references used for teaching and researching.


How much will it cost to plaster this house?

The house-plastering problem

In order to characterize the house-plastering problem, I use the description of a problem as initial state, target state and transformation, borrowed from the psychology of problem solving (Bruder 2000, p. 70). The problem's initial state is unclear because the relevant information is missing. Also unclear is the transformation from initial state to target state, which students can employ. However, the final state is clearly defined, for instance, by asking for a price.

How many people are caught up in a traffic jam 180 km long?

The traffic jam problem

Learning and Research

Two pupils at a time were monitored while they worked on their problems (e.g. the house plastering problem). The students were asked to undertake the task in pairs – without any further help. The students' work was recorded using a video camera.

For evaluation, the entire video data were transcribed. Within the framework of open coding with three raters, the individual expressions of the pupils were allocated conceptual terms, which were discussed and modified during several runs through the data. These terms for individual text passages were then assigned to the following categories: planning, data capture, data processing and checking.

The process category 'planning' describes text passages in which the pupils discuss the path to complete the task or which – in the broadest sense – relate to the path of completing the task. The process category 'data acquisition' describes text passages in which pupils procure data for their further work on the problem. This can involve guessing, counting, estimating, measuring or recalling intermediate results that had been achieved earlier. The process category 'data processing' describes the calculation with concrete values. This can be done either with or without a calculator. For all problems the pupils were provided with a (conventional scientific) calculator. The process category 'checking' includes text passages in which the data processing, data acquisition or planning is questioned or controlled.

The choice of categories was done in such a way that the categories could be allocated in a consistent manner, independently from the problem. During this phase the preliminary categories were therefore combined and modified. Following this allocation, the entire transcripts were coded using the now finalized categories. Later we found that one rater was able to code all categories with adequate confidence on his/her own. The degree of agreement was checked by performing a sample correlation analysis (see Bortz et al. 1990, p. 460f), which showed statistically significant concordance at the 0.05 level. As a result, I then coded all of the remaining transcripts on the basis of the developed categories.

In the following, we present three observations, which were examined by analysing the central components of control processes.

Observation I: Global control

First two comprehensive school pupils of grade 8 are observed. They concerned themselves approx. 22 minutes with the task and obtained from a mathematical point of view a very exact result. The control procedures of the pupils contained the solution plan of the problem. Thereby also, larger sections of planning are taken into the view. The pupils did not control single computations and data capture. This was however in most cases not necessary.

Observation II: local control

Second two secondary modern school pupils of grade 8 are observed. They concerned themselves approx. 15 minutes with the task and obtained from a mathematical point of view an inexactly result. The pupils controlled their work only locally. For instance, they controlled only the last calculation. There are phases with data capture control, but they do not look at

the final result. They controlled the plausibility only in cases of partial results.
Observation III: multiple controls
Third two secondary modern school pupils of grade 9 are observed. They concerned themselves approx. 9 minutes with the task and obtained from a mathematical point of view an inexactly result. During the work with the problems, many control procedures take place. Many different kinds of controls are observed, e.g. controls of the data processing, data capture or planning. The control processes have local and global aspects.
To summarize the control behaviour of the pupils is extremely different. The idealized modelling cycle and the problem solving process are only partial suited to describe and differentiate between these control processes adequately.

Teaching and testing

Pupils can encounter difficulties while working on modelling problems in many places. Therefore, the modelling skills of pupils should be diagnosed exactly before a lecture series on modelling problems. By diagnosis, we understand systematic capture of individual conceptions, abilities, knowledge and learning ways. (s. Abel et al. 2006, p. 10, Büchter/Leuders 2005, p. 167). For this purpose, modelling tasks can be reduced to subtasks, which focus on special steps of the modelling cycle. By the following specifications, it can be determined whether pupils acquired these subsidiary skills.

Subsidiary skills	**specification**
simplify	Pupils separate important and unimportant information of a real situation.
mathematise	Pupils translate real situations into mathematical models (e.g. Term, equation, graph, diagram, function)
calculate	Pupils work with the mathematical model.
interpret	The pupils transfer the information gathered in the model to the real situation.
validate	The pupils verify the information gathered in the model at the real situation. They compare and evaluate different mathematical models for a real situation.
Evaluate	The pupils judge critically the used mathematical model.
Apply	The pupils assign a suitable real situation to a mathematical model and/or find to a mathematical model a suitable real situation.

Table: subsidiary skills and specification (s. Ministerium für Schule 2004, p. 28)

The specifications in the table have a double function. On the one hand, they allow the specification of the subsidiary skills of modelling and on the other hand, they are used to diagnose these subsidiary skills.

Development of diagnostic tasks

The production of tasks fitting to single subsidiary skills is not easy - in particular for subsidiary skills of modelling. By reducing of modelling tasks on subsidiary skills, the authenticity of the task can be lost. However, the authenticity is for modelling activities an indispensable condition.
By using diagnostic tasks, it is important to learn much about the ways of thinking of the pupils. Then we can learn something about the strengths and the difficulties of the pupils. The tasks should be open and the pupils should generate self-productions to prevent standard-answers. Another important aspect is the validity of the task relating to the relevant subsidiary skills (see Abel et al. 2006, P. 12, Büchter/Leuders 2005, P. 173).

A possibility for the development of diagnostic tasks to subsidiary skills of modelling is the limiting of existing modelling tasks by giving of further information. In the following, some examples of tasks are presented.

Example 1 (validate)

Katja and Toni want to compute, how many people are caught up in a traffic jam 180 km long. They assume 10 m space on the road for a vehicle and considered the following calculations

$3 \cdot 18000 \cdot 4 =$
$3 \cdot 18000 \cdot 2 =$

Compare the two calculations and evaluate them.

Example 2 (simplify)

Katja and Toni want to compute, how many people are caught up in a traffic jam 180 km long. The pupils were held to reflect for themselves, and consequently prepare a list of necessary facts. For which of these information's would you decide? Why?

- Vehicle length
- weather
- type of vehicle
- petrol consumption
- state
- distance to the next vehicle
- number of lanes
- day of week
- season
- age of driver
- number of passengers
- time of day
- road works
- holiday time

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