Disrupting linear models of mathematics teaching|learning

Barbara Graves, Associate Professor
Christine Suurtamm, Associate Professor
Faculty of Education, University of Ottawa, Canada
bgraves@uottawa.ca suurtamm@uottawa.ca

Abstract

In this workshop we present an innovative teaching, learning and research setting that engages beginning teachers in mathematical inquiry as they investigate, represent and connect mathematical ideas through mathematical conversation, reasoning and argument. This workshop connects to the themes of teacher preparation and teaching through problem solving. Drawing on new paradigms to think about teaching and learning, we orient our work within complexity theory (Davis & Sumara, 2006; Holland, 1998; Johnson, 2001; Maturana & Varela, 1987; Varela, Thompson & Rosch, 1991) to understand teaching|learning as a complex iterative process through which opportunities for learning arise out of dynamic interactions. Varela, Thompson and Rosch, (1991) use the term co-emergence to understand how the individual and the environment inform each other and are "bound together in reciprocal specification and selection" (p.174). In particular we are interested in the conditions that enable the co-emergence of teaching|learning collectives that support the generation of new mathematical and pedagogical ideas and understandings.

The setting is a one-week summer math program designed for prospective elementary teachers to deepen particular mathematical concepts taught in elementary school. The program is facilitated by recently graduated secondary mathematics teachers to provide them an opportunity to experience mathematics teaching|learning through rich problems. The data collected include questionnaires, interviews, and video recordings. Our analyses show that many a-ha moments of mathematical and pedagogical insight are experienced by both groups as they work together throughout the week.

In this workshop we will actively engage the audience in an exploration of the mathematics problems that we pose in this unique teaching|learning environment. We will present our data on the participants' mathematical and pedagogical responses and open a discussion of the implications of our work.

Introduction

A recent body of research on the teacher's role in the development of learners' mathematical understanding (Ball, 2003; Ball & Bass, 2002; Ball & Even, 2004; Boaler, 2002) has revealed that teaching through inquiry poses substantial challenges for teachers. For elementary teachers, the research suggests that they lack sufficient knowledge of mathematics to effectively implement inquiry-oriented mathematics programs (Ball, 1988, 1990; Ball, Lubienski & Mewborn, 2001). For secondary teachers, while they are more likely to have a background in mathematics, concerns exist about the procedural nature of their mathematics knowledge, and their inexperience in inquiry settings (Stigler & Hiebert, 1999). To complicate matters, few curricula are organized around meaningful problems. Among the research literature, some studies suggest that once teachers have experienced learning new mathematics through inquiry-oriented pedagogies, they start to incorporate the use of investigation and open-ended problem solving in their own teaching (Makar & Confrey, 2004; Shifter, 1998). Other researchers report that it is possible for teachers to deepen their understanding of mathematics and mathematics teaching through the activity of teaching itself (Hill & Ball, 2004; Ma, 1999; Segall, 2001). It has also been reported that when teachers engage in meaningful collaboration with other teaching colleagues, they demonstrate increased mathematical knowledge and more effective mathematics teaching (Crespo, 2006; Lachance & Confrey, 2003; Stein, Silver, & Smith, 1998; Wilcox, Schram, Lappan, & Lanier, 1991).

There are, however, a number of research studies which suggest that even when teachers have had some inquiry-oriented learning experiences and acknowledge this approach as supporting their mathematics learning, there remain visible tensions between inquiry-oriented and more

traditional approaches to teaching mathematics (Graves, Suurtamm, & Benton, 2005; Hart, 2004; Jacobs, Hiebert, Givvin, Hollingsworth, Garnier, & Wearne, 2006). This is not surprising given that the changes that are being asked of teachers are neither easy nor straightforward. The movement from established practices of teaching which traditionally include a great deal of direct instruction to an inquiry approach that requires the interactions between teachers and learners be more dialogic, challenges teachers' beliefs and attitudes about learning, and what constitutes mathematics (Stein, Silver, & Smith, 1998; Suurtamm & Graves, 2007). With respect to teaching, these challenges are revealed when teachers struggle to decide when to explain, when to listen, and how much time to leave for ideas to emerge. When they worry about what question to ask next, we hear in their discourse familiar metaphors of linearity that underlie many established educational perspectives.

Theoretical framework

We orient our work within complexity theory (Davis & Simmt, 2003, 2006; Davis & Sumara, 2006; Holland, 1998; Johnson, 2001; Maturana & Varela, 1987; Varela, Thompson & Rosch, 1991) with respect to how we conceptualize and enable teaching|learning situations. Educational research on knowledge and learning that is oriented by complexity science focuses on the dynamic interactions emerging from diversity and variation, and suggests the value of viewing the interactions of teachers and learners as adaptive and co-emergent learning systems. From this perspective we resist viewing teaching and learning as separate entities and prefer to see teaching|learning as a mutually constitutive dynamic process in which the teacher is also the learner, and the learner is also the teacher. Varela, Thompson and Rosch, (1991) use the term co-emergence to understand how the individual and the environment inform each other and are "bound together in reciprocal specification and selection" (p.174). In particular we are interested in the conditions that enable the co-emergence of learning collectives that support the generation of new mathematical and pedagogical ideas and understandings. In addition, complexity theory provides models and metaphors of non-linearity that help us argue against the linear sequence of prescribing the next step which is so deeply entrenched in educational practice. In non-linear terms next steps are always contingent and co-emergent. This is the case not only in how we understand the teaching|learning dynamic but reflects our own research process. While we begin with specific research questions and a research design, we do not ignore emergent themes, participants or issues, but rather incorporate these into our study as they evolve with the research context.

The study

This research study began in August 2004 and is now in its sixth year. Each year of the study begins with a one-week summer mathematics program for prospective elementary teachers to deepen particular mathematical concepts taught in elementary school. The facilitators in this one-week program are recently graduated secondary mathematics teachers. Our objectives are twofold: to enable prospective elementary teachers to enhance their view of mathematics before they begin their teacher education program; and to provide beginning secondary math teachers an opportunity to teach mathematics through rich problems and inquiry. During five days, the prospective teachers have many opportunities to immerse themselves in mathematical discussion and argumentation as they work on interesting problems, and share solutions, strategies, and representations. They engage with a variety of materials including manipulatives and technological tools in order to explore mathematical ideas in numerous contexts, both within and outside the classroom. During the five days the facilitators actively participate in the inquiry process as they pose problems, provide encouragement, listen attentively and respond to mathematical ideas. In this way, the summer math program provides a teaching|learning experience in which prospective elementary teachers can expand their mathematical understanding and beginning secondary math teachers have the opportunity to experience engaging learners in mathematical inquiry.

Who are the prospective elementary teachers? In our recruitment invitation we state that "Many prospective elementary teachers confess to anxiety about doing and teaching math" and we invite them to join us in "a series of meaningful math activities in a convivial and supportive learning environment, [to] develop a better understanding of the mathematics that [they] will be teaching." Each year we accommodate 75 participants, the majority of whom either express that they are anxious about math or that they have forgotten, or never understood their previous school mathematics. There are also a small number who accept the invitation because they enjoy doing mathematics and welcome the opportunity to do more.

We collect data from the prospective teachers during the one-week program and throughout their teacher education program by means of questionnaires, and focus group interviews. Our focus is on how their mathematical understandings as well as their attitudes and beliefs about mathematics and mathematics learning become transformed in different settings.

Who are the beginning secondary math teachers? In the spring prior to the summer math program, we invite teacher candidates currently in secondary math to apply for the position of facilitator for the summer program. We receive approximately 20 – 25 applications each year, and we personally interview all applicants. We choose facilitators based on their expertise in mathematics, their openness and enthusiasm to think about mathematics and mathematics teaching and learning in different ways, and their conviction that mathematics is something that everyone can learn.

We begin our work with the facilitators during the week before the summer math program to collaboratively explore the mathematics in a number of rich problems and to prepare for the upcoming week. Facilitators come to the preparation week with a variety of expectation about what this teaching experience will be like. An important goal during this week is to begin a discussion and exploration of what it means to facilitate mathematics learning in a meaningful way and to think about posing problems, and listening to solutions rather than telling and explaining. The data collected during the preparation week include daily journals, and video-recordings of their work together. During the week of the summer program we meet with the facilitators for debriefing sessions at the end of each day. This provides all of us the opportunity to listen to everyone's experiences, engage further in mathematical investigations, and to understand how their views of mathematics and mathematics teaching|learning are evolving. The data collected during this week consist of daily journals and video-recordings of the de-briefing sessions.

Workshop format

In this workshop we will describe the summer math program and our research framework and will actively engage the audience in an exploration of the mathematics problems that we pose in this unique teaching|learning environment. In this context we hope to develop a substantial discussion around the nature of rich problems as well as the nature of problem posing. We will then share our data on the participants' mathematical and pedagogical responses to these problems and open a discussion of the implications of our work.

References

Ball, D. L. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics (8)*1. 40 – 48.

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal 90*(4), 449-466.

Ball, D. L. (Ed.). (2003). *Mathematical proficiency for all students: Toward a strategic research and development program in mathematics education*. Santa Monica, CA: RAND Institute.

Ball, D. L. & Bass, H. (2002). Bridging practices: Intertwining content and pedagogy in teaching and learning to teach. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 82-104). Westport, CT: Ablex Publishing.

Ball, D. L., & Even, R. (2004). The International Commission on Mathematical Instruction (ICMI) – The fifteenth ICMI study: The professional education and development of teachers of mathematics. *Journal of Mathematics Teacher Education, 7*(3), 279-293.

Ball, D. L,, Lubienski, S., & Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of Research on Teaching* (4th ed.). New York: Macmillan.
Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education, 33*(4), 239-258.
Crespo, S. (2006). Elementary teacher talk in mathematics study groups. *Educational Studies in Mathematics 63*(1), 29-56.
Davis, B., & Simmt, E. (2003). Understanding learning systems: Mathematics education and complexity science. *Journal for Research in Mathematics Education, 34*(2), 137-167.
Davis, B., & Simmt, E. (2006). Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. *Educational Studies in Mathematics 61*(3), 293-319.
Davis, B. & Sumara, D. (2006). *Complexity and education: Inquiries into learning, teaching, and research*. Mahwah, NJ: Lawrence Erlbaum Associates.
Graves, B., Suurtamm, C., & Benton, N. (2005). Learning and teaching mathematics in communities of inquiry: Is it enough to transform practice? *International Commission on Mathematical Instruction – Study 15*. May 2005. Brazil.
Hart, L. C. (2004). Beliefs and perspectives of first-year, alternative preparation, elementary teachers in urban classrooms. *School Science and Mathematics, 104*(2), 79-88.
Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education, 35*(5), 330-351.
Holland, J. H. (1998). *Emergence: From chaos to order*. Reading, MA: Helix Books.
Jacobs, J. K., Hiebert, J., Givvin, K. B., Hollingsworth, H., Garnier, H., & Wearne, D. (2006). Does eighth grade mathematics teaching in the United States align with the NCTM *Standards*? Results from the TIMSS 1995 and 1999 video studies. *Journal for Research in Mathematics Education, 37*(1), 5-32.
Johnson, S. (2001). *Emergence: The connected lives of ants, brains, cities, and software*. New York: Scribner.
Lachance, A., & Confrey, J. (2003). Interconnecting content and community: A qualitative study of secondary mathematics teachers. *Journal of Mathematics Teacher Education, 6*(2), 107-137.
Ma, L. (1999) *Knowing and teaching elementary mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates.
Makar, K., & Confrey, J. (2004, February). *Modeling fairness of student achievement in mathematics using statistical software by preservice secondary teachers.* Paper presented at the Fourteenth Study of the International Commission on Mathematics Instruction (ICMI-14), Dortmund, Germany.
Maturana, H. R., & Varela, F. J. (1987). *The tree of knowledge: The biological roots of human understanding.* Boston: Shambhala.
National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.
Segall, A. (2001). Re-thinking theory and practice in the preservice teacher education classroom: Teaching to learn from learning to teach. *Teaching Education, 12*(2), 225-241.
Schifter, D. (1998). Learning mathematics for teaching: From a teachers' seminar to the classroom. *Journal of Mathematics Teacher Education, 1*(1), 55-87.
Stein, M. K., Silver, E. A., & Smith, M. S. (1998). Mathematics reform and teacher development: A community of practice perspective. In J. G. Greeno and S. V. Goldman (Eds.), *Thinking practices in mathematics and science learning* (pp. 17-52). Hillsdale, NJ: Lawrence Erlbaum Associates.
Stigler, J. & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: Free Press.
Suurtamm, C., & Graves, B. (2007), *Curriculum Implementation in Intermediate Mathematics (CIIM): Research Report.* Toronto, ON : Queen's Printer.
Varela, F. J., Thompson, E., & Rosch, E. (1991). *The embodied mind: Cognitive science and human experience*. Cambridge, MA: The MIT Press.
Wilcox, S., Schram, P., Lappan, G., & Lanier, P. (1991). The role of a learning community in changing preservice teachers' knowledge and beliefs about mathematics education. *For the Learning of Mathematics, 11*(3), 31-39.