Can Early Algebra lead non-proficient students to a better arithmetical understanding?

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Abstract

In mathematics curricula teachers often find the more or less implicit request to link the taught subjects to the previous knowledge of the students, for example using word problems from everyday life. But in today's multicultural and multisocial society teachers can no longer assume that the children they teach have a more or less equal background and thus everyday live can have a very different meaning for different children. Furthermore there is evidence that good previous knowledge in arithmetic can hinder the approach to other mathematical subjects, like algebra. In this paper I want to provide a brief overview on how previous knowledge in arithmetic can affect student's access to algebra and therefore present an early algebra teaching project which introduces elementary school children to algebraic notation by measurement in an action-oriented way. Thereby the chosen approach to algebra explicitly does not come back to the student's previous arithmetical knowledge but additionally may support non-proficient students in obtaining more insight in the structure of calculations and hence may help them to have more success in solving calculations and word problems.

Introduction

In the German national curricular standards ("Bildungsstandards"), the guideline for the curricula of the German federal states you can read the following:

"Der Mathematikunterricht der Grundschule greift die frühen mathematischen Alltagserfahrungen der Kinder auf, vertieft und erweitert sie und entwickelt aus ihnen grundlegende mathematische Kompetenzen. Auf diese Weise wird die Grundlage für das Mathematiklernen in den weiterführenden Schulen und für die lebenslange Auseinandersetzung mit mathematischen Anforderungen des täglichen Lebens geschaffen."(KMK, p. 6)

"The mathematical education in primary school takes up, deepens and extends early mathematical everyday life experiences and develops basic mathematical competencies from those experiences. Thus the foundation is laid for learning mathematics in higher classes and for lifelong examination of the mathematical requirements of everyday life."(translation by the author)

Everyday life in mathematical education

There are two contrasting ways to combine everyday life with mathematics: Looking at everyday life and trying to find mathematical content or learning mathematical concepts and applying those to ones everyday life.

The former, which seem to be more in line with the quotation above, you can easily find in primary school textbooks. The German textbook "*Das Zahlenbuch*" for 4th graders for example shows a map of Germany to motivate distances (p. 10), a handicraftsman to motivate calculating with money (p. 22) and a recreational lake to motivate calculating with decimal numbers (p. 71). There is also a double page about Christmas (pp. 122/123) and Easter (pp. 124/125) and a page about the benefits of mathematics (p.126) showing among others a doctor, a retiree and a consumer advisor, all talking about why they need mathematics. In the textbook you also can find a lot of word problems which are linked to the alleged everyday life of children, like car inspections (p.66) or buying lentils (p.73).

Looking at the textbook brings up some questions: Is this everyday life of all children in our multisocial and multicultural society? Can you really find everyday life that all children have in common? Is it necessary to base mathematical education upon everyday life at all?

There is no doubt that the mathematical background of children, the similarities and differences which arise by reason of children growing up in different quarters of a town to the point of totally different cultural backgrounds should definitely be part of mathematical education. But if you look at the background of children in a today classroom, one can easily see that there are a lot of differences and that it is hard to find a similarity for all of the children. The one everyday life which fits for all children in classroom does exist.

Instead there is to find a way to look at the everyday life of every child in the classroom. A way of doing this can be the latter mentioned above, teaching mathematical concepts and letting the child apply those to its everyday life. But the question "How can you use this in your everyday life?" is hardly to find in textbooks and classrooms.

Some reasons for putting everyday life on hold

Teaching mathematical concepts without coming back to the student's previous knowledge of everyday mathematics and applying those concepts to everyday life later can be a way to cope with the different social and cultural backgrounds of primary school children. This is all the more important because, at least in Germany, children with migrational background are disadvantaged in the educational system (see Auernheimer 2003).

But there are some other reasons, why it can be a good way not to build on children's previous knowledge while teaching mathematical concepts. McNeil (2004) observed, that "the activation of existing knowledge can interfere with the acquisition of new information". She explicitly refers to pre-school knowledge as well. El'konin (1975) differentiates between theoretical scientific and empirical knowledge. Empirical knowledge designates knowledge children extract from their everyday experiences, while theoretical, scientific knowledge is knowledge on a higher level.

"The adult – the teacher – is the key figure and helps the child to develop ways of operating with objects through which he can discover their essential properties – those which constitute genuine concepts." (El'konin 1975, p. 48)

Hasemann & Stern (2002) addressed their research to the question how to foster mathematical understanding of lower achievers. They worked with 2nd graders on different programmes and drew following conclusion:

"Die Auswertung der Tests ergab, dass bei schwächeren Kindern das alltagsnahe Programm eindeutig am wenigsten bewirkte, während bei den Kindern, die das abstrakte Programm durchlaufen haben, der größte Leistungszuwachs zu beobachten war." (Hasemann & Stern 2002, p. 222)

„The evaluation of the tests shows, that the program close to everyday life definitely had the lowest effect on low-achievers while children who worked on the abstract program showed the biggest learning progress."(translation by the author)

Thus we are looking for an abstract teaching program that gives children tools that can aid them with solving mathematical problems of everyday life and also with solving the word problems in their text books. Thereby it is important that abstract does not mean doing it without concrete materials. If young children shall cope with abstract knowledge this knowledge has to be taught in an action-oriented way.

An unconventional way of teaching Early Algebra

In the first years of school children usually spend a lot of time with calculating with natural numbers. They also adopt strategies that cannot be transferred to calculations with decimal numbers or fractions. With a teaching experiment in the 60s Davydov (1975) chose a different approach to mathematical education. His idea was teaching the properties of numbers while already using the common algebraic symbolizations and before introducing numbers at all. Therefore he chose an action-oriented way by using direct comparison of magnitudes like length, area, volume, mass, time and so on. The children used concrete material like water containers for comparing volume and balance scales for comparing mass and learned to write down their findings with inequations. The question, how big the difference between the compared magnitudes was, lead to equations. Aided by the concrete material, the children learned to manipulate and to interpret different linear equations. After the children have learned dealing with the equations properly numbers are implemented by introducing a unit. This way of implementing numbers not only works for natural numbers but for the whole real numbers.

Davydov's idea was taken on by the MeasureUp-Program (see Dougherty & Venenciano 2007), which showed that children can successful deal with abstract equations, achieve a deep understanding of properties of numbers and use them effectually for solving word problems.

Early Algebra as a guideline for word problems

Certainly starting mathematical education without using numbers would be a big change for the German school system and would hardly become accepted by teachers and parents. But the idea of teaching the abstract properties of numbers by the aid of concrete comparison of magnitudes while firstly excluding numbers deserves a closer look in terms of its usefulness for helping children deal with word problems and mathematical problems of everyday life. The main questions are: Will the MeasureUp-Program work for school children of different grades although the already have been introduced to numbers and arithmetical operations? Can they transfer the knowledge about abstract equations to mathematical problems of everyday life? And can this program lead non-proficient students to a better arithmetical understanding?

In a first project we modified the MeasureUp-Program for the use in a few weeks lasting teaching-experiment in grade three and fife. After the children have been introduced to the comparison of length, area and volume and the use of letters they learned how to set up and manipulate equations. To connect the abstract equations without numbers with word problems we gave the children word problems that contained letters instead of numbers. We the asked the children to make up word problems that are appropriate to given letter equations. Therewith we keep up our intention to firstly teach mathematical concepts and applying those to everyday life not till the children can handle the concept properly.

First results

Children of a 5th grade were given the equations L - R = U and N + M = B – J and asked to invent fitting word problems. Below we want to give some examples. The equation L – R = U resulted in the following word problems:

Lena geht in den Laden und will 10 Buntstifte von Pelikan kaufen. 10 Stifte = R. Doch es gibt noch so viele schöne andere, dass Lena noch mehr kauft. Sie kauft 23. Was für ein Wert hat U? Wie viele Stifte kauft Lena mehr? (Angelina)

Lena walks into a shop and wants to buy ten coloured crayons. 10 crayons = R. But there are so much other pretty ones, that is why Lena buys some more. She buys 23. What is the value of U? How much crayons more did Lena buy?(translation by the author)

Although the children learned to use letter equations only in the context of geometric magnitudes like length, area and volume Angelina chose the context of money for their word problems. We assume they chose money because it plays a major role in their everyday life and therewith a much bigger role than geometric magnitudes. The details on the brand of the crayons and the reason why she bought more are evidence that here we see an episode that really has happened or could happen in her life. The word problem fits to the equation which is revealed by Angelina as she is relating some of the letters to the values. Other children only used numbers or only used letters:

Kim hat 20 Blumen, sie verliert 5. Wie viele hat sie noch? (Axel)

Kim has 20 flowers. She loses 5. How many are left over? (translation by the author)

Horst hat L Boote geschnitzt. Ihm fallen R Boote ins Wasser. Wie viele hat er noch übrig?

Horst carved L boats. R boats are falling into the water. How many are left over? (translation by the author)

The above examples show that the children not only use the letters for magnitudes but also for numbers of objects.

The equation N + M = B – J resulted in the following word problem:

Lara geht zu Faberkastell und will einen Radiergummi von 2,00 € kaufen und einen Bleistift von 3,00 €. Sie hat aber nur 5,50 € mit. Reicht das Geld und wenn ja, wie viel bekommt sie zurück? (Lana)

Lara walks to Faber-Castell and wants to buy an eraser of 2.00 € and a pencil of 3.00 €. She only has 5.50 € with her. Is this enough money and if yes, how many money will she get back? (translation by the author)

The word problem fits to the equation. Lana invents values for N and M (2.00 € and 3.00 €) and B (5.50 €) and wants to know how big J is. She as well implicitly writes down, why it is important for her to know how big J is: she wants to know, if she has enough money for her buying.

Perspective

The next step is to explore if children will and can use their knowledge about abstract symbolic equations for solving word problems only containing numbers and no letters. First observations showed that low-achieving children who have not been able to solve a word problem directly came back to abstract symbolic equations. For example a low-achieving 3rd grader's first reaction after reading the word problem "A street has length 845 m. Hans has already walked 220m. How far does he still have to go?" was "I want to do that with letters."

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