Proportional Reasoning Models in Developing Mathematics Education Curricula for Prospective Elementary School Teachers

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Abstract

A study of pre-service primary school teachers in Singapore and the United States revealed superior performance by the Singaporeans on proportional reasoning problems. Analysis of solutions showed the Singapore future teachers were more likely to use unitary and benchmark approaches than were their American counterparts. Conclusions include suggestions for programs intended to improve the performance of prospective elementary school teachers on proportional reasoning problems.

Introduction

Proportional reasoning is a benchmark in students' mathematical development (De Bock, Van Dooren, Janssens, & Verschaffel, 2002) and classroom data continue to demonstrate that students often perform less well on proportional reasoning problems than on other performance measures (Kaput & West, 1994, Van Dooren, De Bock, Hessels, Janssens, Verschaffel, 2005). Since proportional reasoning is a focus of the school mathematics curriculum in the elementary school grades, the capabilities of prospective elementary school teachers in solving proportional reasoning problems are critical for improvement efforts.

The importance of proportional reasoning in the mathematics curricula for prospective elementary school teachers is accentuated in national standards documents (National Council of Teachers of Mathematics, 2000; Ministry of Education, 2000). In the case of Singapore, one of the highest scoring countries on international mathematical comparisons, the teaching of proportional reasoning at the primary level was previously considered an integral part of 'Ratio and Proportion' in national standards documents. However, the latest national standards document, which reflects the revised 2007 mathematics syllabus, emphasizes the instructional importance of proportional reasoning by stipulating that it is now a fundamental aspect of the study of numbers, including whole numbers, fractions and decimals (Ministry of Education, October 2005, 2006d).

Despite many years of national attention in standards documents and other curriculum policy references, the performance of prospective elementary school teachers on proportional reasoning items remains problematic (Stacey, 1989; Swafford & Langrall, 2000). The current study compares the performance of prospective elementary school teachers on proportional reasoning problems in Singapore and in the United States. Since Singapore students have achieved an international reputation for high mathematics achievement and US students typically score less well on international comparisons, it was hypothesized that prospective elementary school teachers' performances and approaches on proportional reasoning problems would correspond to the differences reflected in international student performances. Such correspondences could then provide insights about, and models for, enhancing the performance future primary school teachers on proportional reasoning problems.

The present study investigated how prospective primary school teachers in both countries solved word problems which could be solved by using a proportion. These types of word problems are common in proportional reasoning or ratio and proportion sections of mathematics textbooks for future primary school teachers (Chan, 2007; Billstein, Libeskind, and Lott, 2007). Typically in these textbooks, a section on proportional reasoning is included as an application of students' work with whole numbers or with rational numbers.

Since whole and rational numbers, including mixed numbers, are major components in mathematics courses for prospective elementary school teachers, these two types of numerical representations were selected for inclusion in the proportional reasoning problems for this study. The Book Pages (BP) problem was one of the two problems investigated in this study. The problem statement for the BP problem contained only whole numbers and solutions to this problem could be carried out using only whole number operations.

The Tofu problem was the second problem investigated in this study. The problem statement for this problem contained only rational numbers and solutions to this problem could be carried out using only arithmetic operations with rational numbers.

Participants in this study were all enrolled in mathematics content or mathematics methods courses for prospective elementary school teachers. In the case of the Singapore participants, 24 were in their second year and 40 were in their third year of study in a four-year degree with certification program. Since the 24 Singapore second-year students had already completed a unit on teaching Ratio and Direct Proportion as part of their required mathematics methods course, all 64 Singapore participating prospective elementary school teachers had studied the concept of proportionate reasoning as well as methodology for teaching this form of reasoning at the primary levels in Singapore mathematics classrooms.

All participants worked on the problems for this study as an in-class assignment as part of their Singapore or US mathematics content / methods course. There was no time limit for students to complete their work on the problems and each student worked independently on the problems. In the instructions for completing the problems students were advised to show their work as they would to a class of elementary school students and to clearly indicate the solutions.

Results

For the BP problem, 64 Singapore and 167 US prospective teachers submitted solutions. An analysis of these solutions showed that 92% of the Singapore and 72% of the US future teachers submitted correct answers. The most commonly submitted approach by Singapore and US participants was to determine the amount read in one minute and to multiply the amount by 80 to get the amount read in 80 minutes. Notably, this approach, often called the unitary method, is a common approach taught to Singapore primary school pupils to handle problems involving proportional reasoning (Collars, Koay, Lee, Ong, & Tan, 2006, p.31, Method 2). Fifty-two percent of the Singapore participants used this approach and 97% of these solutions were correct, while 24% of US participants used this approach and 95% of these US participants solved the problem correctly with the unitary method. For Singapore participants the next most frequently used approach for solving the BP problem was to determine the amount read in ten minutes and multiply by 8 to get the amount read in 80 minutes. Twenty-seven percent of the Singapore participants used this approach and all solved the problem correctly. Again, this approach, frequently known as the benchmark method, is another method that is commonly taught in Singapore primary mathematics classrooms to deal with problems on proportional reasoning (Collars, Koay, Lee, Ong, & Tan, 2006, p.31, Method 1). Only one percent of US solutions used the benchmark method and, like the Singaporeans, all these solutions were correct. Koay and Lee (2006, p.38) provided insight into connections between the benchmark and unitary methods for solving proportions when they noted that when pupils have a "tendency [to commit] additive errors [in applying the benchmark method, teachers may want to] encourage them to use Method 2 [unitary method]".

Nineteen percent of the Singaporean solutions to the BP problem used a single ratio or a variation of one ratio in the solution and 67% of these solutions were correct. In contrast, none of the US solutions used a single ratio or a variation of one. Twenty-two percent of US participants determined the time needed to read one page and then multiplied by 80. This approach was successful 51% of the time. Three percent of Singapore participants used this approach and 100% of these approaches led to the correct solution. Another twenty-two percent of the US solution used a table, list, or chart, and 56% of these solutions were correct. However, none of the Singapore participants used a table, list, or chart. Twenty percent of the US solutions set up a proportion and 97% of these solutions were correct, while none of the Singapore participants set up a proportion to solve the BP problem.

A possible explanation for the absence of proportions in the solutions to the BP problem by Singapore pre-service teachers may be the perception that setting up a proportion is an algebraic solution. In the Singapore context, such an approach is not encouraged in teaching primary mathematics, as algebraic solutions of proportional reasoning problems are only introduced at the secondary levels (Ministry of Education, 2006a, 2006b, 2006c).

Among the US solutions to the BP problem there were 6% in which subtractive reasoning was used to obtain an incorrect solution, while no Singapore solutions used subtractive reasoning.

A final 6% of the US solutions contained four other approaches and each of these alternatives resulted in a correct solution.

For the Tofu problem, 63 Singapore and 25 US prospective teachers submitted solutions. An analysis of the solutions showed that 84% of the Singapore and 56% of the US future teachers submitted correct answers. The most commonly submitted approach by Singapore participants was the unitary method, i.e. to determine the amount needed for 1 kg and then multiply by the mixed number $4\frac{2}{3}$. In all 54% of the Singapore solutions used this approach and 91% of these solutions were correct. None of the US participants used this approach. However, 100% of the US solutions set up proportion with unknown component and solved for the unknown and 56% of these solutions were correct. In contrast, 2% of the Singaporeans used this approach correctly, a result possibly attributable to the perception held by many Singapore future teachers that this approach is algebraic and therefore not appropriate for primary school mathematics.

Seventy-eight percent of the Singapore solutions calculated $\frac{4\frac{2}{3}}{5\frac{1}{3}} \times 3\frac{1}{2}$ or variations of this calculation, and 87% of these solutions were correct. Another 13% of the Singapore solutions determined the amount needed for $\frac{1}{3}$ kg and then multiplied by 14 or variations, i.e., approaches that are basically variations of the benchmarking method. All these solutions were correct. The remaining 9% of the Singapore solutions included setting up a proportion with an unknown component, using $\frac{5\frac{1}{3}}{3\frac{1}{2}} \times 4\frac{2}{3}$ or submitting no work, a partial solution, or no solution. Of these solutions only the one that set up a proportion was correct.

Conclusions

The differences in the percent of correct solutions to the two proportional reasoning problems (92% vs. 72% and 84% vs. 56%) provides evidence that Singapore pre-service primary school teachers outperform their US counterparts on these type of proportional reasoning problems. These performance differences also provide evidence that proportional reasoning problems containing rational numbers may be more difficult than those containing whole numbers.

One of the more notable findings of this study concerned the greater use of the unitary and benchmark approaches by Singapore participants and the great success enjoyed by participants who used these methods. In the case of the BP problem, more than twice as many Singapore participants (52%) used the unitary approach as compared to 24% of the US participants. Yet, the percentage of correct solutions for those using this approach (97% for the Singaporeans and 95% for the Americans) was quite similarly very high. For those participants using the benchmark approach for the BP problem, the results were even more striking: 27% of Singapore future teachers used this approach as did only 1% of their US counterparts and 100% of those using this approach achieved correct solutions.

On the Tofu problem, 54% of the Singapore solutions used the unitary approach and 91% of these solutions were correct, while none of the US participants used this approach. Another 27% of Singaporeans used the benchmark approach or a variant of this approach and 100% of these solutions were correct, while once again, none of the US participants used either this approach or a variation on the Tofu problem.

Given the 20% and 28% differences in the percent of correct solutions by these prospective primary school teachers with Singapore future teachers consistently outperforming their US counterparts, there are grounds for incorporating the successful approaches used by the Singapore participants into mathematical content studied by US pre-service elementary

school teachers. Particularly, the unitary and the benchmark approaches, commonly used successful approaches in the solutions by the Singapore prospective primary school teachers, were less frequently used by the participating US pre-service teachers. Consequently, efforts to improve the performance of prospective primary school teachers on proportional reasoning problems are apt to benefit from instructional programs that include the unitary and benchmark approaches to solving these problems.

The general decline in the percent of correct solutions by both countries prospective teachers on proportional reasoning problems that involved rational numbers as opposed to whole numbers is another notable finding from this study. This finding reinforces the results by Lim-Teo, Chua, Cheang, and Teo (2007) that even when future primary school teachers are seemingly well-versed in the unitary approach to solving proportional reasoning problems, they are often unable to come to terms with changes in problem difficulty as a result of rational numbers replacing whole numbers in solutions.

Strengthening a pre-service teacher's understanding of proportional reasoning problems is a more complex endeavor than merely ensuring that the unitary and benchmark approaches and rational number operations are emphasized in their programs of study. Nevertheless, these are apt to be key components of improvement efforts to ensure that teachers are prepared to teach proportional reasoning at the elementary level.

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